A Library to Support the Development of Applications that Process Huge Matrices in External Memory

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Abstract: This paper presents a new library, named *TiledMatrix*, to support the development of applications that process large matrices stored in external memory. The library is based on some strategies similar to cache memory management and its basic purpose is to allow that an application, originally designed for internal memory processing, can be easily adapted for external memory. It provides an interface for external memory access that is similar to the traditional method to access a matrix. The *TiledMatrix* was implemented and tested in some applications that require intensive matrix processing such as: computing the transposed matrix and the computation of viewshed and flow accumulation on terrains represented by elevation matrix. These applications were implemented in two versions: one using *TiledMatrix* and another one using the *Segment* library that is included in GRASS, an open source GIS. They were executed on many datasets with different sizes and, according the tests, all applications ran faster using *TiledMatrix* than *Segment*. In average, they were 7 times faster with *TiledMatrix* and, in some cases, more than 18 times faster. Notice that processing large matrices (in external memory) can take hours and, thus, this improvement is very significant.

1 INTRODUCTION

Matrix processing is a central requirement for many applications as image analysis, computer graphics applications, terrain modelling, etc. In many cases, the matrix is huge and it can not be stored/processed in internal memory requiring external processing. For example, in terrain modeling, the recent technological advances in data collection such as *Light Detection and Ranging* (LIDAR) and *Interferometric Synthetic Aperture Radar* (IFSAR) have produced a huge volume of data and most computers cannot store or process this huge volume of data internally. Since the time required to access and transfer data to and from external memory is generally much larger than internal processing time (Arge et al., 2001), external memory algorithms must try to minimize the number of I/O operations. That is, these algorithms should be designed based on a computational model where the cost is the number of data transfer operations instead of CPU time. One of these models was proposed by Aggarwal and Vitter (Aggarwal and Vitter, 1988).

In this context, some external memory libraries such as *LEDA-SM* (Crauser and Mehlhorn, 1999), *Segment* (GRASS, 2011) and *STXXL* (Dementiev et al., 2005) have been developed to reduce and optimize the access operations in external memory. The *STXXL* and *LEDA-SM* libraries provide external memory data structures such as vectors, queues, trees, and basic funtions for searching and sorting. The *Segment* library, on the other hand, provides a data structure to store large matrices in external memory.

This work presents a new efficient library, named *TiledMatrix*, to support the development of applications that process large matrices stored in external memory. This library is based on the use of some special data structures to manage hard disk accesses and, thus, to reduce the number of I/O operations. The basic idea is to subdivide the matrix into blocks whose size allows that some blocks can be stored in internal memory and these blocks are managed as a cache memory. The proposed library was implemented and tested in some applications as computing the matrix transposition and the *viewshed* and flow accumulation on huge terrains represented by digital elevation matrix. As the tests showed, the applications can run much faster using *TiledMatrix* than using the *Segment* library. In average, the performance was 7 times better and, in some cases, more than 18 times faster. It is worth to notice that, processing large matrices (in external memory) can take hours and, thus, this improvement is very significant.

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2 RELATED WORK

In situations where an application needs to process a huge volume of data that does not fit in internal memory, the data transference between the internal and external memories often becomes a bottleneck. Thus, the algorithms for external memory should be designed and analyzed considering a computational model where the algorithm complexity is evaluated based on I/O operations. A model commonly used was proposed by Aggarwal and Vitter (Aggarwal and Vitter, 1988) and it defines an I/O as the transference of one disk block of size B between the external and internal memories. The performance is evaluated considering the number of such I/Os and the algorithm complexity is related to the number of I/Os performed by fundamental operations such as scanning or sorting N contiguous elements stored in external memory where $scan\ (N) = \theta\ (N/B)$ and $sort\ (N) = \theta\ (\frac{N}{B} \log_{M/B} \frac{N}{B})$ and M is the internal memory size.

Some libraries such as *LEDA-SM* (Crauser and Mehlhorn, 1999), *STXXL* (Dementiev et al., 2005) and *Segment* (GRASS, 2011) have been created to support the implementation of external memory algorithms.

LEDA-SM library was designed as an extension of *LEDA* (Mehlhorn and Näher, 1995) library for handling huge data sets. It provides implementations of some I/O efficient algorithms as sorting and graph algorithms and data structures such as external memory stack, queue, heap and B^+-tree.

STXXL is an open source library for handling large data sets and it provides algorithms for data scanning, sorting, merging, etc and several external memory containers.

Segment is a library included in *GRASS* (an open source GIS) (GRASS, 2011) which was developed for handling large matrices in external memory. Basically, it subdivides the original matrix into small submatrices (called *segments*) which are stored in external memory and some of these segments are kept in internal memory and managed as a *cache* memory.

It is important to notice that all libraries listed above, except *Segment*, do not allow operations with matrices in external memory. And, as described below, although *TiledMatrix* has a similar purpose as *Segment*, the former includes some additional features that makes it more efficient than the latter (see experimental results in section 4).

3 *TiledMatrix* LIBRARY

TiledMatrix was designed to support the development of applications that require the manipulation of huge matrices in external memory. And, as is known, any application requiring external memory processing should be designed focusing the I/O operations reduction and, in general, this reduction is achieved either by reordering the data considering the access sequence (Arge et al., 2001; Fishman et al., 2009; Haverkort et al., 2007) or by adapting the algorithm to use a more efficient access pattern (Haverkort and Janssen, 2012; Meyer and Zeh, 2012). In this context, the *TiledMatrix* library was developed to allow easy adaptation of algorithms originally designed for internal memory to process huge matrices stored in external memory. The idea is to make the matrix accesses transparent to the application, that is, matrix acesses will be managed by *TiledMatrix*.

More specifically, a huge matrix is subdivided in small rectangular blocks (*tiles*) whose size allows that some blocks can be stored in internal memory in a data structure named *MemBlocks*. Thus, when a given matrix cell needs to be accessed, the block containing that cell is loaded in *MemBlocks* and then, any further access to cells in that block can be done efficiently. Since it is not possible to store all blocks in internal memory, the *MemBlocks* structure is managed as a cache memory adopting a replacement policy. The library provides the following policies: LRU - *Least Recently Used* (Guo and Solihin, 2006), FIFO - *first in first out* (Grund and Reineke, 2009) and random selection (Chandra et al., 2005).

The library was developed as general as possible and it gives many options to the application developer. It is possible to define the block size (that is, the number of rows and columns), the *MemBlocks* size and also, it is possible to select the replacement policy. The LRU policy was implemented using a *timestamp* assigned to each block in memory. Thus, when a cell in a block B is accessed, the B *timestamp* is updated to be greater than the others and, therefore, when a block needs to be evicted, the block with smallest *timestamp* is chosen. In the FIFO policy it was used an array to store the order in which the blocks were loaded. Also, to reduce the writing operations, for each block in memory is used a *dirty* flag to indicate if the block needs to be written back to the disk when it is evicted.

Additionally, in order to reduce the number of I/O operations during the block transference to/from the disk, the library uses the LZ4 algorithm (Lz4, 2012) for fast data compression. The idea is to compress the block before writing it in the disk and decompress when it is loaded into the memory. As the tests showed, this strategy makes the library more efficient and, in many cases, the applications run two times faster when compression is used. Figure 1 presents

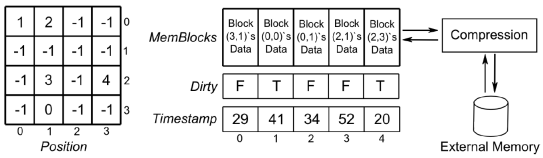


Figure 1: Example of data structures used in *TiledMatrix*.

the data structures used by *TiledMatrix* library: in this example, the external memory matrix was divided in 16 square blocks and the *MemBlocks* array is able to store 5 blocks; the replacement policy used was the LRU. The matrix *Position* stores the position where the corresponding matrix block is stored in the *MemBlocks* array (the value -1 indicates that the corresponding block is not loaded into the memory). Then, if a cell in block (0,1) is accessed, the *Position* matrix indicates the block is stored in the slot 2 of *MemBlocks* and thus, that block does not need to be (re)loaded into memory. On the other hand, supposing the *MemBlocks* is already full, if a cell in the block (1,2) is accessed, that block needs to be loaded and, in this case, a block stored in *MemBlocks* is evicted to open room for the "new" block. According the *Timestamp* array, the block (2,3) has the smallest timestamp. Thus, it will be evicted; but, since its *dirty flag* is set *true*, it is compressed, written to disk and the corresponding cell in *Position* matrix is set to -1. Next, the block (1,2) is loaded, decompressed, stored in the *slot* 3 of *MemBlocks* and the *Position* matrix is updated to indicate the block position in *MemBlocks*.

3.1 *TiledMatrix* versus *Segment*

It is important to mention that, although the *Segment* library, included in GRASS (GRASS, 2011), is based on similar concepts as *TiledMatrix*, in the implementation of *TiledMatrix* it was used some different strategies that makes it more efficient than *Segment*.

Firstly, *Segment* does not use data compression to reduce disk I/O. Also, *Segment* allows only the use of LRU replacement policy but, in the two libraries, this policy was implemented based on different methods for blocks marking. In *Segment*, the blocks are marked with a *timestamp* which is updated as follows: when a new block is accessed, its timestamp is set to zero and all other blocks timestamp are incremented by one; thus, if a block needs to be evicted, the block with higher timestamp is selected. As one can see, if there are n blocks in the memory, each step of the block marking takes $O(n)$ time. On the other hand, in *TiledMatrix*, when a block is accessed, only that block timestamp is increased; thus, the block having the smallest timestamp is evicted. In this case, the block marking requires a constant time.

Another important difference between *TiledMatrix* and *Segment* is the process to determine if a block is already loaded in internal memory and, if yes, where it is stored. In *TiledMatrix*, when a cell in a block needs to be accessed, it is fast to determine if the block is loaded in internal memory (and where it is stored) since it just needs to access the corresponding block position in the *Position* matrix. Thus, this operation can be executed in constant time. But, in *Segment*, it is necessary to sweep the array that stores the blocks in internal memory to check (and access) the block; To try to reduce this overhead, the library keeps the position of the last block loaded. Again, in the worst case, this operation requires linear time.

Therefore, in *TiledMatrix*, an access to a block loaded in memory, in all cases, takes $O(1)$ CPU time while in *Segment* it can take $O(n)$ CPU time. Notice that this performance improvement in the block access operation makes the *TiledMatrix* more efficient than the *Segment* because the block access is a basic operation which is executed many times.

4 EXPERIMENTAL TESTS

In order to evaluate the *TiledMatrix* performance, some experimental tests were carried out to compare *TiledMatrix* against the *Segment* library. The tests were based on three applications that require intensive matrix processing: computing the transposed matrix and the computation of viewshed and flow accumulation on terrains represented by elevation matrix. For each application it was selected one algorithm that was adapted for external memory processing, that is, to process the matrix stored in disks. Thus, for each algorithm was generated two versions: one using *TiledMatrix* to manage the access to matrix stored in external memory and another one using *Segment*.

The algorithms used in the tests are described below. They were implemented in C++, compiled using gcc 4.5.2 and the tests were executed in a *Core i5* computer, 4GB of *RAM* memory, 1TB *Sata* Hard Disk running *Ubuntu Linux* 12.10 64bits operating system. In all applications the external memory libraries were configured to use, at most, 3GB of RAM. The results of these tests are presented in section 4.4.

4.1 Matrix Transposition

The algorithm for matrix transposition is based on the trivial method where the input matrix is read sequentially (row by row) from the disk and each cell *(i,j)* is stored in the position *(j,i)* in a temporary matrix *M*

handled by the library *TiledMatrix* or *Segment*. Then, the transposed matrix M is written to the disk.

4.2 Viewshed

Given a terrain T represented by an elevation matrix, let O be a point in the space (the *observer*) from where other terrain points (the *targets*) will be visualized. Both observer and target can be at a given height above the terrain. Usually, it is assumed that the observer has a range of vision ρ, the *radius of interest*, which means that the observer can see points at a given distance ρ. Thus, a target T is visible from O if and only if the distance of T from O is, at most, ρ and the straight line from O to T is always strictly above the terrain, that is, there is no point in this line whose height is smaller or equal than the elevation of the corresponding terrain point. See Figure 2.

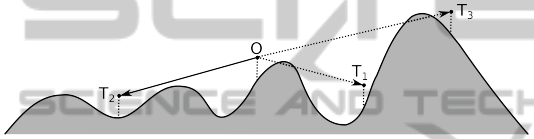


Figure 2: Target's visibility: T_2 is visible, but T_1 and T_3 are not .

The *viewshed* of O corresponds to all terrain points that can be seen by O; formally,

$$viewshed(O) = \{p \in T \mid \text{a target on } p \text{ is visible from } O\}$$

where the radius of interest ρ is left implicit. Since the terrain is represented by raster DEMs, the viewshed can be represented by a square $(2\rho+1) \times (2\rho+1)$ matrix of bits where 1 indicates that the corresponding point is visible and 0 is not. By definition, the observer is in the center of this matrix.

Then, the visibility of a target on a cell c_t can be determined by checking the slope of the line connecting the observer and the target and the cells' elevation on the rasterized segment. More precisely, suppose the segment is composed by the cells $c_0, c_1, \cdots, c_t$ where c_0 and c_t are respectively the cells corresponding to the observer and target projections. Let α_i be the slope of the line connecting the observer to the cell c_i, that is, $\alpha_i = \frac{\zeta(c_i)-(\zeta(c_0)+h_o)}{dist(c_0,c_i)}$ where $\zeta(c_0)$ and $\zeta(c_i)$ are, respectively, the elevation of the cells c_0 and c_i and $dist(c_0,c_i)$ is the "distance" (the number of cells) between these two cells. Thus, the target on c_t is visible if and only if the slope $\frac{\zeta(c_t)+h_t-(\zeta(c_0)+h_o)}{dist(c_0,c_p)}$ is greater than α_i for all $0 < i < t$. If yes, the corresponding cell in the viewshed matrix is set to 1; otherwise, to 0.

Notice that the viewshed computation demands an intensive matrix processing and, for terrains whose matrix does not fit in internal memory, it is necessary a very large number of external memory accesses since the matrix is accessed non-sequentially.

There are many methods for viewshed computation (Franklin and Ray, 1994; Haverkort et al., 2007) and, in particular, (Franklin and Ray, 1994) proposed a linear time method where the visibility is computed along a ray (segment) connecting the observer O to the center of a cell in the boundary of a square of side $2R+1$ centered at O. Thus, this ray is counterclockwise rotated around the observer following the cells in that boundary (see Figure 3).

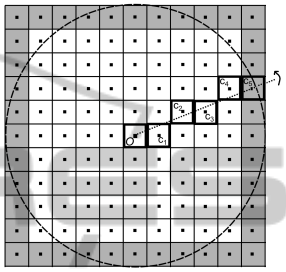


Figure 3: Sweeping the terrain.

This algorithm can be easily adapted for external memory processing using either the library *TiledMatrix* or *Segment*. The basic idea is, when a matrix cell needs to be accessed, this access is managed by one of these libraries as previously described.

4.3 Flow Accumulation

Another important group of applications requiring intensive matrix processing is the drainage network computation on terrains. Informally, the drainage network of a terrain delineates the path that water flows through the terrain (the flow direction) and the amount of water that flows into each terrain cell (the flow accumulation). In other words, the flow direction consists in assigning directions to terrain cells such that these directions predict the path taken by an overland flow running in the terrain (Tarboton, 1997). The flow direction in a cell c can be defined as the direction of the lowest neighbor cell whose elevation is lower than the elevation of c. Additionally, the flow accumulation of a cell c is the number of other cells whose flow achieves c (informally, the flow accumulation means the amount of flow running through each cell) supposing that each cell receives a rain drop. Figure 4 shows an example of a flow direction matrix and the flow accumulation matrix computed from it.

It is important to notice that the flow direction computation needs to include some steps to treat depressions and flat areas on the terrains and, in general,

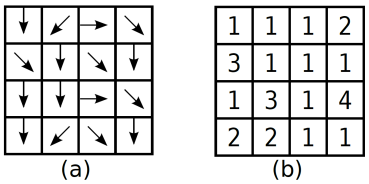


Figure 4: Drainage network computation: (a) flow direction; (b) flow accumulation.

these steps are not very simple and use some complex data structures. Thus, the adaptation of flow direction algorithms for external processing is not very easy since it is necessary to make some modifications in the algorithm implementation. Therefore, in the tests, we decided to use only a flow accumulation algorithm because it could be easily adapted for extenal processing using *TiledMatrix* or *Segment*.

In this case, given the flow direction matrix computed by using the algorithm RWFlood (Magalhães et al., 2012), which is based on graph topological sorting where the idea is to process the flow direction as a graph where each matrix cell is a vertex and there is a directed edge connecting a cell c_1 to a cell c_2 if and only if c_1 flows to c_2. Initially, all vertices in the graph have 1 unit of flow. Then, in each step, a cell c with in-degree 0 is set as visited and its flow is added to $next(c)$ flow where $next(c)$ is the cell following c in the graph. After processing c, the edge connecting c to $next(c)$ is removed (i.e., $next(c)$'s in-degree is decremented) and if the in-degree of $next(c)$ becomes 0, $next(c)$ is also similarly processed. More precisely, for the tests, we adapted the algorithm presented by Haverkort and Janssen (Haverkort and Janssen, 2012) to use *TiledMatrix* and *Segment*.

4.4 Performance Evaluation

As stated before, the *TiledMatrix* was evaluated comparing the execution time of the three applications using two libraries: *TiledMatrix* and *Segment*. Also, to evaluate the impact of block compression for the *TiledMatrix* performance, we used two versions of this library: one including compression and another one does not. In all cases, the libraries *TiledMatrix* and *Segment* were configured to use LRU as the replacement policy and the block size was set as following: for matrix transposition and *viewshed*, it was used square blocks with 1000 x 1000 cells and in flow accumulation, blocks 250 x 250 cells.

The matrices used in the tests were obtained from SRTM website and they represent 30 meters resolution terrains corresponding to regions from the United States of America. For the matrix transposition, we used different matrix sizes from a same region and, since the processing time for computing the *viewshed* and the flow accumulation depends on terrain topography, in these two cases, we used different terrain sizes from two different regions: one corresponding to a very hilly terrain and the other, to a smoother one. More precisely, in the flow accumulation, the input are the flow direction matrices computed by using the algorithm RWFlood (Magalhães et al., 2012) using these terrain datasets. And, in the *viewshed* computation it was used a radius of interest covering the whole terrain and the observer was positioned 50 meters above the terrain.

Notice that, although an input matrix can fit in internal memory, maybe the processing algorithm can not be executed internally because it needs to use some additional data structures. Thus, considering that the terrain matrices are represented using 2 bytes per cell, a terrain dataset with N cells requires $2N$ bytes. In the tests, the matrix transposition algorithm uses only one matrix (with $2N$ bytes) to store the input data. On the other hand, the viewshed algorithm uses two matrices: the (input) terrain matrix with $2N$ bytes and the (output) viewshed matrix with N bytes (one byte per cell), thus requiring $3N$ bytes in total. And, finally, the flow accumulation algorithm uses 3 matrices: the input flow direction matrix (N bytes), the indegree matrix (N bytes) and the accumulated flow matrix ($4N$ bytes) requiring $6N$ bytes in total.

The tests results are presented in the Tables 1 and 2 and also, in the graphs in the Figures 6 and 5. According the results, all applications ran faster using *TiledMatrix* than *Segment* and, in general, they were 7 times faster with *TiledMatrix*. But, in some cases, the *TiledMatrix* can make the application more than 18 times faster (for example, flow accumulation on 3.4 GB terrains). This performance increase can be explained by two reasons: the data transference is reduced because of block compression and the use of more efficient strategies for block management.

Another interesting observation is the speedup achieved when the compression was used in *TiledMatrix*. In all cases, the application performance using *TiledMatrix* was better when compression was enabled than when it was not. In average, the compression usage improved the application performance in 50% and, in some situations, this performance improvement was almost 200%.

4.5 Computing the Block Size

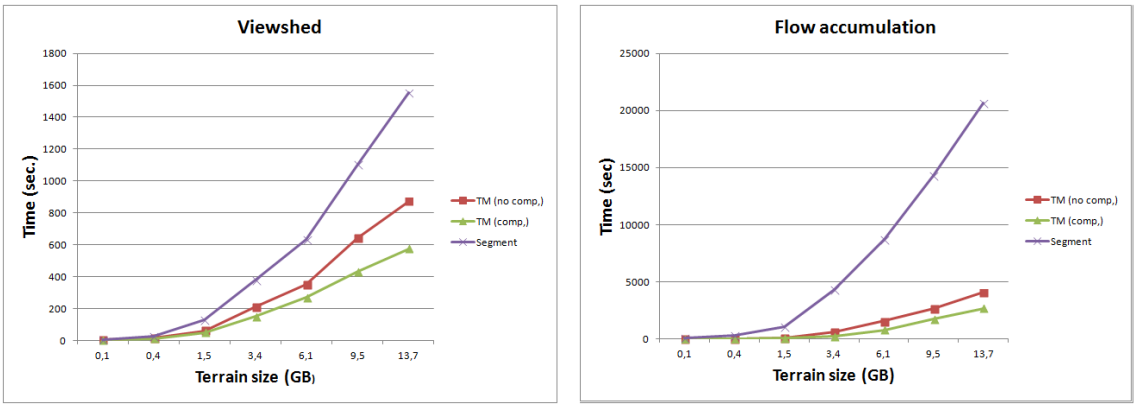
The performance of an external memory algorithm is affected by some elements as: the internal memory size, the block size and shape (number of cells in rows and columns). Ideally, the best case for an exter-

Table 1: Time (in seconds) to transpose a matrix using *TiledMatrix* (with and without compression) and *Segment* libraries.

Matrix Size		**TiledMatrix**		
row × col	GB	w/o comp.	with comp.	**Segment**
5000^2	0.1	1.34	1.08	2.97
10000^2	0.4	5.47	4.09	11.59
20000^2	1.5	25.49	18.49	62.97
30000^2	3.4	92.18	79.28	251.51
40000^2	6.1	234.32	178.67	486.35
50000^2	9.5	365.38	249.72	800.42
60000^2	13.7	507.63	335.39	1246.18

Table 2: Time (in seconds) to compute *viewshed* a terrain using *TiledMatrix* (with and without compression) and *Segment* libraries.

Terrain			**Viewshed**			**flow accumulation**		
			TiledMatrix			**TiledMatrix**		
Dataset	row × col	GB	w/o comp.	with comp.	**Segment**	w/o comp.	with comp.	**Segment**
1	5000^2	0.1	3.69	3.15	6.33	7.10	3.86	65.25
	10000^2	0.4	14.24	12.35	26.96	26.79	15.29	277.75
	20000^2	1.5	58.50	49.49	115.66	119.7	65.26	1038.15
	30000^2	3.4	182.76	147.16	344.24	612.11	242.78	4336.54
	40000^2	6.1	365.82	278.45	648.47	1685.65	938.57	8542.93
	50000^2	9.5	591.41	426.39	1036.57	2927.60	1927.28	14640.51
	60000^2	13.7	906.19	609.76	1568.51	4736.27	3243.15	22977.13
2	5000^2	0.1	3.90	3.19	7.03	6.82	3.89	65.03
	10000^2	0.4	15.08	12.65	28.76	26.10	15.07	276.51
	20000^2	1.5	61.21	51.46	132.81	113.60	64.98	1032.23
	30000^2	3.4	211.85	151.56	378.65	642.27	237.24	4305.04
	40000^2	6.1	352.41	270.04	631.750	1557.33	798.03	8703.05
	50000^2	9.5	643.91	434.58	1106.22	2624.07	1713.16	14299.41
	60000^2	13.7	872.69	575.72	1552.40	4056.73	2663.41	20625.91



(a) (b)

Figure 5: Running time charts for (a) *viewshed* and (b) flow accumulation computation considering different terrain sizes from Dataset 2.

nal memory algorithm would be to have any block already loaded in internal memory when one of its cells is accessed and a block should be evicted only when it is no longer needed. But, usually, it is not very easy to achieve this case when the algorithm has to process data sets that are too large to fit into internal memory.

Also, when the library needs to store (load) a block to (from) the disk, it is necessary to perform a

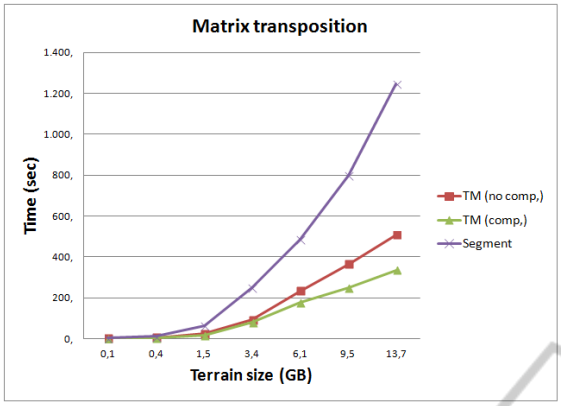


Figure 6: Running time chart for the matrix transposition considering different terrain sizes (in number of cells).

seek operation to reach the block position in the disk and, then, transfer it to (from) the disk. The seek operation cost can be amortized using larger block size, that is, the larger the block size, the smaller the significance of seek operation cost for the data transference. On the other hand, larger blocks size usually increases the cache miss penalty since the time needed to store (load) a large block to (from) the disk is greater.

Thus, in order to achieve a good efficiency when using *TiledMatrix*, the block size needs to be defined to try to reduce the number of times a same block is evicted and reloaded while achieving a high disk transfer rate. And, it is important to notice that this definition depends on the application because it needs to be done, mainly, based on the access pattern used by the application. To illustrate how the block size can be defined based on the application analysis, let us describe the block size definition for the three application used in the tests.

In the matrix transposition algorithm, the matrix is sequentially accessed row by row, thus the block size should be defined such that it should be possible to store, in internal memory, as many blocks as necessary to cover a whole row. Therefore, since in the tests the memory size available is 3GB, we defined a block with 1000×1000 cells.

In the *viewshed* computation, the block size was defined based on the Franklin and Ray algorithm access pattern. More precisely, as it was showed in (Ferreira et al., 2012), if the internal memory can store $2\left(\frac{\rho}{\omega}+2\right)$ blocks where ρ is the radius of interest and the block size is $\omega \times \omega$ cells then when a block is evicted by *LRU* policy, it will no longer be needed. Thus, as the memory size is 3GB, we defined blocks with 1000×1000 cells to allow that the required number of blocks can be stored in internal memory.

Finally, in the flow accumulation problem, the block size definition is a little more complex since

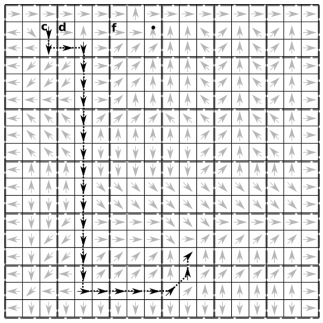


Figure 7: Computing the flow accumulation in a 18×18 matrix (divided in blocks with 3×3 cells). The chain of cells detached with a dotted line is processed when cell c is visited by the row-major access.

there is not a specific access pattern because the topological sorting is based on the flow direction which can be very different for each terrain. Notice that the algorithm used for the topological sorting accesses the matrix in two ways: mainly, the matrix is accessed using a row-major order and, occasionally, the access sequence follows a chain of cells whose input degree is 1. Figure 7 illustrates these accesses: initially, the matrix is processed in row-major order (and all blocks intercepting the first row are loaded in the memory) and when the cell c is processed, since its in-degree is 0, the access sequence needs to follow its flow direction because its neighbor in-degree also becomes 0. This process is repeated for all cells indicated using a dark gray which requires that 8 blocks are accessed (loaded). After processing this chain, the row-major access sequence is restored in cell d.

Notice that when a chain of cells with in-degree 0 is followed, the access sequence could need several blocks loading and this process could force a block eviction that will be accessed again soon. For example, in the Figure 7, the block containing cell d was loaded during the processing of the first row and it will be accessed again after processing the referred chain. Thus, if the processing forces the d's block to be evicted, this block will be reloaded to process d.

Therefore, in the flow accumulation, we used a small block in order to increase the amount of blocks that can be stored internally. According empirical experiments, a good block size for the this problem is 250×250 cells, since bigger sizes led to many block swaps operations and smaller sizes led to very low disk transfer rate.

5 CONCLUSIONS AND FUTURE WORKS

We presented a new library, named *TiledMatrix*, to

support the development of applications that process large matrices stored in external memory. This library uses some special data structures and cache memory management algorithms to reduce the number of I/Os. The basic purpose is to allow that an application originally designed for internal memory processing can be easily adapted for external memory. The library provides an interface for external memory access that is similar to the traditional internal matrices access. An interesting strategy included in this library was the use of data compression to reduce the transference between the internal and external memories.

The *TiledMatrix* was implemented and tested in some applications that require intensive matrix processing such as: computing the transposed matrix and the computation of viewshed and flow accumulation on terrains represented by elevation matrix. These application were implemented in two versions: one using *TiledMatrix* and another one using the *Segment* library. They were executed on many datasets with different sizes and, according these tests, all applications ran faster using *TiledMatrix* than *Segment*. In average, they were 7 times faster with *TiledMatrix* and, in some cases, more than 18 times faster. Also, the tests showed that the compression usage improved the application performance in 50% and, in some situations, this performance improvement was almost 200%.

The *TiledMatrix* source code is available in *www.dpi.ufv.br/˜marcus/TiledMatrix*

As a future work, we intend to evaluate the *TiledMatrix* in some other applications and to develop strategies to define the block size according to the algorithm memory access pattern and the memory size.

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