Isotropy Analysis of Optical Mouse Array for Mobile Robot Velocity Estimation

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Abstract: This paper presents the isotropic analysis of an optical mouse array for the velocity estimation of a mobile robot. It is assumed that there can be positional restriction on the installation of optical mice at the bottom of a mobile robot. First, the velocity kinematics of a mobile robot with an array of optical mice is obtained, and the resulting Jacobian matrix is analyzed symbolically. Second, the isotropic, anisotropic, and singular optical mouse placements are identified, along with the corresponding characteristic lengths. Third, the least squares mobile robot velocity estimation from the noisy optical mouse velocity measurements is discussed. Finally, simulation results for the isotropic placement of three optical mice are given.

1 INTRODUCTION

For the velocity estimation of a mobile robot, several attempts have been made to use optical mice that were originally invented as an advanced computer pointing device. In fact, an optical mouse is an inexpensive but high performance motion detection sensor with a sophisticated image processing engine inside. Optical mice installed at the bottom of a mobile robot, as shown in Fig. 1, can detect the motions of a mobile robot traveling over a plane surface. The mobile robot velocity estimation using optical mice is free from the problems of typical sensors: wheel slip in encoders, the line of sight in ultrasonic sensors, and heavy computation in cameras.

A pair of optical mice was proposed as a simple but viable means for the mobile robot velocity estimation in the presence of wheel slip (Lee and Song, 2004; Bonarini et al., 2004). Using redundant velocity measurements of two optical mice, a simple procedure for error detection and reduction in the mobile robot velocity estimation was developed (Bonarini et al., 2005). The redundant number of optical mice was proposed to reduce the effect of the noisy velocity measurements of optical mice and to handle their partial malfunction (Kim and Lee, 2008). Using the geometrical relationship among optical mice, the calibration for systematic errors and the selection of reliable velocity measurements were presented (Hu et al., 2009).

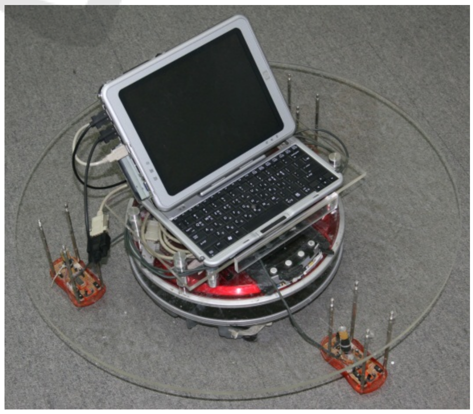

Figure 1: A prototype of three optical mouse array for the mobile robot velocity estimation (Kim and Lee, 2008).

For a mobile robot with a circular base, a regular polygonal array of optical mice can be a natural and desirable choice of the optical mouse placement. For instance, a pair of optical mice are placed to be symmetric about the center of a mobile robot. And, $N(\geq 3)$ optical mice are placed in a regular N-gonal array with its geometrical center coincident with the center of a mobile robot (Kim and Lee, 2008). However, there can be some restriction on the installation of optical mice, owing to a non-circular

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base of a mobile robot or other structures pre-installed on its base. With positional restriction on installation, a non-regular polygonal array of optical mice can be a better choice, compared with its regular polygonal counterpart (Cimino and Pagilla, 2011).

The performance of an optical mouse array for the mobile robot velocity estimation can be evaluated based on its Jacobian matrix. The Jacobian matrix maps the velocity of a mobile robot to the velocities of optical mice, which is a function of the installation positions of optical mice. Through the Jacobian matrix, the unit sphere in the optical mouse velocity space can be mapped into an ellipsoid in the mobile robot velocity space. For the optimal placement of optical mice, the volume of the ellipsoid can be one measure, and also its closeness to a sphere, so-called the isotropy, can be another measure. The concept of isotropy has been adopted for the optimal design of serial and parallel manipulators (Ranjbaran et al., 1995; Angeles, 1997; Chablat and Angeles, 2002; Zanganeh and Angeles, 1997; Fattah and Ghasemi, 2002), as well as, omnidirectional mobile robots (Saha *et al*., 1995; Kim and Moon, 2006).

This paper presents the isotropy analysis of an optical mouse array for the mobile robot velocity estimation. It is assumed that there can be positional restriction on the installation of optical mice at the bottom of a mobile robot. This paper is organized as follows. Section 2 obtains the velocity kinematics of a mobile robot equipped with optical mice, and Section 3 analyzes the resulting Jacobian matrix symbolically. Sections 4, 5, and 6 identify the isotropic, anisotropic, and singular optical mouse placements, along with the corresponding characteristic lengths. Section 7 discusses the least squares mobile robot velocity estimation from the noisy optical mouse velocity measurements. Section 8 gives simulation results for the isotropic placement of three optical mice. Finally, the conclusion is made in Section 9.

2 VELOCITY KINEMATICS

The velocity of a mobile robot traveling on a plane can be estimated using the velocity measurements of $N(\geq 2)$ optical mice installed at the bottom of a mobile robot. Fig. 2 shows three coordinate frames that are used for the description of a mobile robot and the i^{th} optical mouse. Let O_W, X_W, and Y_W denote the origin, the x and y axes of the world coordinate frame, respectively; let O_R, X_R, and Y_R denote the origin, the x and y axes of the mobile robot coordinate frame, respectively; and, let O_i, X_i, and Y_i, $i = 1, \cdots, N$, denote the origin, the x and y axes of the i^{th} optical mouse coordinate frame, respectively. For simple description, the following assumptions are made. 1) Two origins, O_W and O_R, are coincident with the center, denoted by O, of a mobile robot. 2) The origin, O_i, $i = 1, \cdots, N$ is coincident with the installation position, P_i, of the i^{th} optical mouse. 3) The world coordinate frame is aligned with the mobile robot coordinate frame, with which the i^{th} optical mouse coordinate frame is also aligned. The position vector, $\mathbf{p}_i = [x_i\ y_i]^t$, $i = 1, \cdots, N$, of the i^{th} optical mouse can be expressed by

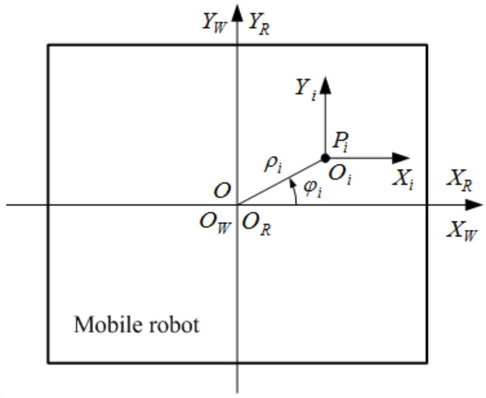

Figure 2: Three coordinate frames for a mobile robot and the i^{th} optical mouse.

$$\mathbf{p}_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix} = \begin{bmatrix} \rho_i \times \cos\varphi_i \\ \rho_i \times \sin\varphi_i \end{bmatrix},\ i = 1, \cdots, N \qquad (1)$$

where ρ_i and φ_i are the polar coordinates of the installation position P_i of the i^{th} optical mouse.

Let v_{rx} and v_{ry} be two linear velocity components of a mobile robot long the x axis and the y axis, respectively, and ω_r be its angular velocity component about the center O of a mobile robot. And, let that v_{ix} and v_{iy}, $i = 1, \cdots, N$, be the lateral and longitudinal velocity measurements of the i^{th} optical mouse. The velocity relationship between a mobile robot and the i^{th} optical mouse can be presented by

$$v_{rx} - \omega_r \times y_i = v_{ix},\ i = 1, \cdots, N \qquad (2)$$

$$v_{ry} + \omega_r \times x_i = v_{iy},\ i = 1, \cdots, N \qquad (3)$$

From (2) and (3), the velocity kinematics of a mobile robot with an array of N optical mice can be obtained by

$$\mathbf{A}\mathbf{v}_r = \mathbf{v}_s \qquad (4)$$

In the above, $\mathbf{v}_r = \left[v_{rx}\ v_{ry}\ \omega_r\right]^t \in \mathbf{R}^{3\times1}$ represents the velocity vector of a mobile robot, $\mathbf{v}_s =$

$[\mathbf{v}_1^t\ \mathbf{v}_2^t\ \cdots\ \mathbf{v}_N^t]^t \in \mathbf{R}^{2N\times 1}$ represents the velocity vector of N optical mice, with $\mathbf{v}_i = [v_{ix}\ v_{iy}]^t$, $i = 1,\cdots,N$ being the velocity measurement of the i^{th} optical mouse; and, $\mathbf{A}$ represents the Jacobian matrix mapping $\mathbf{v}_r$ to $\mathbf{v}_S$, given by

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & -y_1 \\ 0 & 1 & x_1 \\ 1 & 0 & -y_2 \\ 0 & 1 & x_2 \\ \vdots & \vdots & \vdots \\ 1 & 0 & -y_N \\ 0 & 1 & x_N \end{bmatrix} \in \mathbf{R}^{2N\times 3} \tag{5}$$

Note that the expression of $\mathbf{A}$ is very simple as a function of the positions of N optical mice, $\mathbf{p}_i = [x_i\ y_i]^t$, $i = 1,\cdots,N$.

3 SYMBOLIC ANALYSIS

In the velocity kinematics of (4), the velocity vector of a mobile robot, $\mathbf{v}_r$ is composed of two linear and one angular components, while the velocity vector of N optical mice, $\mathbf{v}_s$, is composed of a total of $2N$ linear components. To eliminate the physical inconsistency among velocity components, the characteristic length, denoted by L, can be introduced (Angeles, 1997):

$$\hat{\mathbf{A}}\hat{\mathbf{v}}_r = \mathbf{v}_s \tag{6}$$

where $\hat{\mathbf{v}}_r = [v_{rx}\ v_{ry}\ (L \times \omega_r)]^t \in \mathbf{R}^{3\times 1}$ and

$$\hat{\mathbf{A}} = \begin{bmatrix} 1 & 0 & -\frac{1}{L}\times y_1 \\ 0 & 1 & \frac{1}{L}\times x_1 \\ 1 & 0 & -\frac{1}{L}\times y_2 \\ 0 & 1 & \frac{1}{L}\times x_2 \\ \vdots & \vdots & \vdots \\ 1 & 0 & -\frac{1}{L}\times y_N \\ 0 & 1 & \frac{1}{L}\times x_N \end{bmatrix} \in \mathbf{R}^{2N\times 3} \tag{7}$$

Note that all elements of $\hat{\mathbf{A}}$ are physically dimensionless.

From (7), $\hat{\mathbf{A}}^t\hat{\mathbf{A}}$ can be written as

$$\hat{\mathbf{A}}^t\hat{\mathbf{A}} = N \times \begin{bmatrix} 1 & 0 & -\frac{C_y}{L} \\ 0 & 1 & \frac{C_x}{L} \\ -\frac{C_y}{L} & \frac{C_x}{L} & \frac{R^2}{L^2} \end{bmatrix} \in \mathbf{R}^{3\times 3} \tag{8}$$

where

$$C_x = \frac{1}{N}\sum_{i=1}^{N} x_i \tag{9}$$

$$C_y = \frac{1}{N}\sum_{i=1}^{N} y_i \tag{10}$$

$$R = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i^2 + y_i^2)} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\rho_i^2} \tag{11}$$

In the above, C_x and C_y, respectively, represent the averages of the x and y coordinates of the position vectors, $\mathbf{p}_i = [x_i\ y_i]^t$, $i = 1,\cdots,N$, of N optical mice, and R represents the root mean square of the distances of N optical mice from the center O of a mobile robot.

Using (8), the characteristic polynomial of $\hat{\mathbf{A}}^t\hat{\mathbf{A}}$ is given by

$$(\lambda - N)\left\{\lambda^2 - N\left(1 + \frac{R^2}{L^2}\right)\lambda + N^2\left(\frac{R^2}{L^2} - \frac{C_x^2 + C_y^2}{L^2}\right)\right\} = 0 \tag{12}$$

From (12), three eigenvalues of $\hat{\mathbf{A}}^t\hat{\mathbf{A}}$, denoted by λ_1, λ_2, and λ_3, are obtained by

$$\lambda_1 = \frac{N}{2}\left\{\left(1 + \frac{R^2}{L^2}\right) + \sqrt{\left(1 - \frac{R^2}{L^2}\right)^2 + \frac{4(C_x^2 + C_y^2)}{L^2}}\right\} \tag{13}$$

$$\lambda_2 = N \tag{14}$$

$$\lambda_3 = \frac{N}{2}\left\{\left(1 + \frac{R^2}{L^2}\right) - \sqrt{\left(1 - \frac{R^2}{L^2}\right)^2 + \frac{4(C_x^2 + C_y^2)}{L^2}}\right\} \tag{15}$$

It can be shown that there hold the following inequality relationships among three eigenvalues, λ_1, λ_2, and λ_3:

$$\lambda_3 \le \lambda_2 (= N) \le \lambda_1 \tag{16}$$

regardless of the values of C_x, C_y, and R^2, as well as L. Note that λ_1 and λ_3 are the largest and smallest eigenvalues of $\hat{\mathbf{A}}^t\hat{\mathbf{A}}$, while λ_2 is its middle eigenvalue which remains constant as N.

4 ISOTROPIC PLACEMENT

For the $2N \times 3$ Jacobian matrix $\hat{\mathbf{A}}$ with $N \geq 2$, the condition number can be defined by

$$\kappa = \frac{\sigma_1}{\sigma_3} = \sqrt{\frac{\lambda_1}{\lambda_3}} \tag{17}$$

where σ_1 and σ_3 represent, respectively, the largest and smallest singular values of $\hat{\mathbf{A}}$, and λ_1 and λ_3 represent, respectively, the largest and smallest eigenvalues of $\hat{\mathbf{A}}^t\hat{\mathbf{A}}$. Note that the condition number κ can have values from unity to infinity. The Jacobian matrix $\hat{\mathbf{A}}$ is isotropic when $\kappa = 1$, and $\hat{\mathbf{A}}$ is singular when $\kappa = \infty$.

The placement of N optical mice is said to be isotropic, if the isotropy of the $2N \times 3$ Jacobian matrix $\hat{\mathbf{A}}$ can be achieved:

$$\hat{\mathbf{A}}^t\hat{\mathbf{A}} = N \times \mathbf{I}_3 \tag{18}$$

where $\mathbf{I}_3$ represents that 3×3 identity matrix. Note that $\hat{\mathbf{A}}^t\hat{\mathbf{A}}$ has three identical eigenvalues of magnitude N, that is, $\lambda_1 = \lambda_2 = \lambda_3 = N$, so that the condition number becomes unity, $\kappa = 1$. From (8) and (18), the isotropy conditions for $\hat{\mathbf{A}}$ are given by

$$C_x = \frac{1}{N}\sum_{i=1}^{N} x_i = 0 \tag{19}$$

$$C_y = \frac{1}{N}\sum_{i=1}^{N} y_i = 0 \tag{20}$$

$$\frac{R^2}{L^2} = \frac{1}{L^2} \times \frac{1}{N}\sum_{i=1}^{N}(x_i^2 + y_i^2) = 1 \tag{21}$$

In the above, (19) and (20) indicate that the geometrical center of N optical mice coincides with the center O of a mobile robot. And, (21) indicates that the squared value of the characteristic length should be equal to the average of the squared distances of N optical mice from the center O.

Using (1), (19), and (20) can be written as:

$$\sum_{i=1}^{N} \mathbf{p}_i = 0 \tag{22}$$

Let S_N be the isotropic set of the position vectors of N optical mice, satisfying (22):

$$S_N = \left\{ \mathbf{p}_i, i = 1, \cdots, N \;\middle|\; \sum_{i=1}^{N} \mathbf{p}_i = 0 \right\} \tag{23}$$

For given $N(\geq 2)$ optical mice, Fig. 3 shows the isotropic sets of N position vectors, $\mathbf{p}_i, i = 1, \cdots, N$. In the case of $N = 2$ shown in Fig. 3(a), the isotropic set S_2 can be parameterized by two variables, (y_1, θ); in the case of $N = 3$ shown in Fig. 3(b), the isotropic set S_3 can be parameterized by four variables, (y_1, x_2, y_2, θ); and, in the case of $N(\geq 4)$ shown in Figs. 3(c) and 3(d), the isotropic set S_N can be parameterized by $2(N-1)$ variables, $(y_1, x_2, y_2, \cdots, x_{N-1}, y_{N-1}, \theta)$. Note that the rotation of the isotropic set of N position vectors by the angle θ with respect to the center O of a mobile robot are also isotropic:

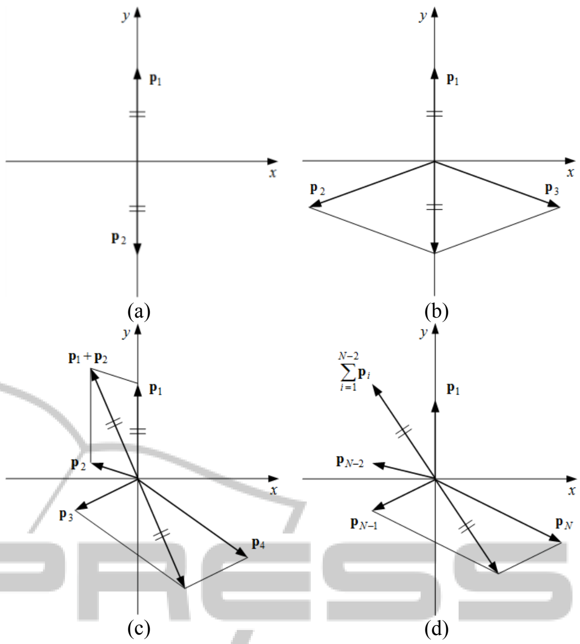

Figure 3: The isotropic set of the position vectors of N optical mice: (a) $N = 2$, (b) $N = 3$, (c) $N = 4$, and (d) $N \geq 4$.

Since the union of two isotropic sets is also isotropic, new isotropic sets for $N(\geq 4)$ position vectors can be obtained from existing isotropic sets known already:

$$S_N = S_{N_1} \cup S_{N_2} \tag{24}$$

where $N = N_1 + N_2$, $2 \leq N_1, N_2 \leq N - 2$. For $N = 5$ optical mice, Fig. 4(a) shows the isotropic set S_5, which is obtained as the union of S_2 and S_3. However, note that (24) cannot produce all possible isotropic sets of N position vectors, since $\sum_{j=1}^{N_1} \mathbf{p}_j = \sum_{j=N_1+1}^{N} \mathbf{p}_j = 0$ is sufficient but not necessary for $\sum_{i=1}^{N} \mathbf{p}_i = 0$. It should be mentioned that the simplest isotropic set of N position vectors is a regular polygon, for which

$$\rho_1 = \rho_2 = \cdots = \rho_N \tag{25}$$

$$\varphi_2 - \varphi_1 = \varphi_3 - \varphi_2 = \cdots = \varphi_1 - \varphi_N = \frac{360°}{N} \tag{26}$$

Fig. 4(b) shows the isotropic placement of N=5 optical mice, which from a regular pentagon.

Once the isotropic placement of N optical mice, denoted by $\mathbf{p}_i^* = [x_i^* \; y_i^*]^t$, $i = 1, \cdots, N$, is determined from (19) and (20), the value of the characteristic length L, required for the isotropy of the Jacobian matrix $\widehat{\mathbf{A}}$, can be determined, from (21):

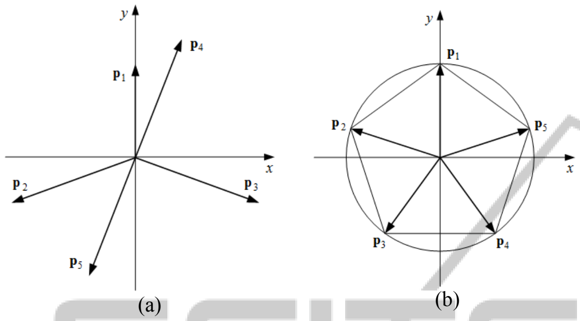

Figure 4: Two isotropic sets of $N = 5$ position vectors: (a) $S_2 \cup S_3$, and (b) a regular pentagon.

$$L^* = R^* = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i^{*2} + y_i^{*2})} \tag{27}$$

which is called as the optimal characteristic length. Note that the optimal characteristic length L^* is the root mean square of the distances of N position vectors, $\mathbf{p}_i^*, i = 1, \cdots, N$, from the center O of a mobile robot.

5 ANISOTROPIC PLACEMENT

For a given placement of N optical mice, it may be impossible to achieve the isotropy of the $2N \times 3$ Jacobian matrix $\widehat{\mathbf{A}}$. Seen from (15), the smallest eigenvalue of $\mathbf{A}^t\mathbf{A}$, λ_3, can be zero, and thus we consider the condition index, defined by

$$\gamma = \frac{\lambda_3}{\lambda_1} \tag{28}$$

which is the inverse of the condition number κ of $\mathbf{A}^t\mathbf{A}$, given by (17). Note that the condition index γ can have values between zero and unity, where $\widehat{\mathbf{A}}^t\widehat{\mathbf{A}}$ is isotropic when $\gamma = 1$, and $\widehat{\mathbf{A}}^t\widehat{\mathbf{A}}$ is singular when $\gamma = 0$. For the placement of N optical mice, it is desirable to make the value of γ as large as possible.

Using (13) and (15), (28) can be written as

$$\gamma = \frac{A - \sqrt{B}}{A + \sqrt{B}} \tag{29}$$

with

$$A = 1 + \frac{R^2}{L^2} \tag{30}$$

$$B = \left(1 - \frac{R^2}{L^2}\right)^2 + \frac{4\left(C_x^2 + C_y^2\right)}{L^2} \tag{31}$$

Setting $\frac{\partial \gamma}{\partial L}$ equal to zero, we have

$$2\frac{\partial A}{\partial L} \times B = A \times \frac{\partial B}{\partial L} \tag{32}$$

with

$$\frac{\partial A}{\partial L} = 2L \tag{33}$$

$$\frac{\partial B}{\partial L} = 4(L^2 - R^2) \times L + 8\left(C_x^2 + C_y^2\right) \times L \tag{34}$$

Plugging (30), (31), (33), and (34) into (32), it follows that

$$(L^2 - R^2) \times \left\{R^2 - \left(C_x^2 + C_y^2\right)\right\} = 0 \tag{35}$$

As will be shown later,

$$R^2 \neq C_x^2 + C_y^2 \tag{36}$$

unless $\widehat{\mathbf{A}}^t\widehat{\mathbf{A}}$ is singular. From (35) and (36), using (11), the condition for maximizing the value of γ is obtained by

$$L^2 - R^2 = 0 \tag{37}$$

which results in

$$L^\# = R^\# = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i^2 + y_i^2)} \tag{38}$$

which is called as the suboptimal characteristic length. It should be noted that the expression of the suboptimal characteristic length $L^\#$, given by (38), is the same as that of the optimal characteristic length L^*, given by (27).

With the suboptimal characteristic length $L^\#$ known, the maximum value of the condition index γ that can be achieved for a given anisotropic optical mouse placement can be obtained. Plugging (37) and (38) into (30) and (31) and using (29), we obtain

$$\gamma^\# = \frac{L^\# - \sqrt{C_x^2 + C_y^2}}{L^\# + \sqrt{C_x^2 + C_y^2}} \tag{39}$$

which is called as the maximal condition index. Note that the maximal condition index $\gamma^\#$ can have values between zero and unity, where $\gamma^\# = 1$ when $C_x = C_y = 0$, and $\gamma^\# = 0$ when $L^\# = \sqrt{C_x^2 + C_y^2} (= R^\#)$.

6 SINGULAR PLACEMENT

The placement of N optical mice is said to be singular, if $\hat{\mathbf{A}}^t\hat{\mathbf{A}}$ falls into singularity, that is, the smallest eigenvalue of $\hat{\mathbf{A}}^t\hat{\mathbf{A}}$, λ_3, becomes zero:

$$\lambda_3 = 0 \tag{40}$$

for which the condition index becomes zero, $\gamma = 0$. From (15) and (40), we have

$$\left(1+\frac{R^2}{L^2}\right) = \sqrt{\left(1-\frac{R^2}{L^2}\right)^2 + \frac{4\left(C_x^2+C_y^2\right)}{L^2}} \tag{41}$$

which leads to

$$R^2 = C_x^2 + C_y^2 \tag{42}$$

Plugging (9)-(11) into (42) and using (1), we have

$$(N-1)\sum_{i=1}^{N}\rho_i^2 - 2\sum_{j=1}^{N-1}\sum_{k=j+1}^{N}\rho_j \times \rho_k \times \cos\psi_{j,k} = 0 \tag{43}$$

(43) can be rearranged into

$$\sum_{j=1}^{N-1}\sum_{k=j+1}^{N}\left(\rho_j^2+\rho_k^2-2\rho_j\times\rho_k\times\cos\psi_{j,k}\right) = 0 \tag{44}$$

where

$$\psi_{j,k} = \varphi_k - \varphi_j,\ \ j=1,\cdots,N-1,\quad k=(j+1),\cdots,N \tag{45}$$

represents the angle between two position vectors, $\mathbf{p}_j$ and $\mathbf{p}_k$, $j=1,\cdots,N-1$, $k=(i+1),\cdots,N$.

It can be shown that

$$0 \le \sum_{j=1}^{N-1}\sum_{k=j+1}^{N}\left(\rho_j-\rho_k\right)^2 \le \sum_{j=1}^{N-1}\sum_{k=j+1}^{N}\left(\rho_j^2+\rho_k^2-2\rho_j\times\rho_k \times\cos\psi_{j,k}\right) \le \sum_{j=1}^{N-1}\sum_{k=j+1}^{N}\left(\rho_j+\rho_k\right)^2 \tag{46}$$

In the above, the equality holds, when $\cos\psi_{j,k}=1$ or $\cos\psi_{j,k}=-1$, $j=1,\cdots,N-1$, $k=(i+1),\cdots,N$. In the former case, for which

$$\psi_{j,k}=0,\ j=1,\cdots,N-1,\quad k=(i+1),\cdots,N \tag{47}$$

and, from (44), we have

$$\sum_{j=1}^{N-1}\sum_{k=j+1}^{N}\left(\rho_j-\rho_k\right)^2 = 0 \tag{48}$$

which results in

$$\rho_1=\rho_2=\cdots=\rho_N \tag{49}$$

Note that (47) and (49) indicate that N optical mice are placed at the same position on a mobile robot. Next, in the latter case, for which

$$\psi_{j,k}=180^\circ,\ j=1,\cdots,N-1,\quad k=(i+1),\cdots,N \tag{50}$$

and, from (44), we have

$$\sum_{j=1}^{N-1}\sum_{k=j+1}^{N}\left(\rho_j+\rho_k\right)^2 = 0 \tag{51}$$

which results in

$$\rho_1=\rho_2=\cdots=\rho_N=0 \tag{52}$$

(52) indicates that N optical mice are placed at the center O of a mobile robot. At both singular placements of N optical mice, one given by (47) & (49) and the other given by (52), it should be noted that the rank of the Jacobian matrix $\hat{\mathbf{A}}$ drops to two.

7 LEAST SQUARES VELOCITY ESTIMATION

Based the velocity kinematics of (6), the mobile robot velocity can be estimated from the noisy velocity measurements $\mathbf{v}_s$ of N optical mice by

$$\tilde{\hat{\mathbf{v}}}_r = \begin{bmatrix}\tilde{v}_{rx}\\ \tilde{v}_{ry}\\ L\times\tilde{\omega}_r\end{bmatrix} = \hat{\mathbf{A}}^{+}\mathbf{v}_s \tag{53}$$

where

$$\hat{\mathbf{A}}^{+} = \left(\hat{\mathbf{A}}^t\hat{\mathbf{A}}\right)^{-1}\hat{\mathbf{A}}^t \in \mathbf{R}^{3\times 2N} \tag{54}$$

Note that (53) with (54) represents the least squares solution to (6), which minimizes $\left\|\hat{\mathbf{A}}\hat{\mathbf{v}}_r - \mathbf{v}_s\right\|$.

Assume that the placement of N optical mice is isotropic, with the isotropic position vectors $\mathbf{p}_i^* = [x_i^*\ y_i^*]^t$, $i=1,\cdots,N$, and the optimal characteristic length L^*. Plugging (7), (18), and (27) into (53), we have

$$\hat{\mathbf{A}}^{+} = \frac{1}{N}\begin{bmatrix}1 & 0 & 1 & 0 & \cdots & 1 & 0\\ 0 & 1 & 0 & 1 & \cdots & 0 & 1\\ -\frac{y_1}{R^*} & \frac{x_1}{R^*} & -\frac{y_2}{R^*} & \frac{x_2}{R^*} & \cdots & -\frac{y_N}{R^*} & \frac{x_N}{R^*}\end{bmatrix} \tag{55}$$

Using (55), from (53), the estimated velocity of a mobile robot is obtained by

$$\tilde{v}_{rx} = \frac{1}{N}\sum_{i=1}^{N} v_{ix} \quad (56)$$

$$\tilde{v}_{ry} = \frac{1}{N}\sum_{i=1}^{N} v_{iy} \quad (57)$$

$$\tilde{\omega}_{r} = \frac{1}{N}\sum_{i=1}^{N} \omega_{i} \quad (58)$$

where

$$\omega_i = \frac{1}{R^{*2}} \times \left(-y_i^* \times v_{ix} + x_i^* \times v_{iy}\right), \quad i = 1, \cdots, N \quad (59)$$

represents the angular velocity component experienced by the i^{th} optical mouse, which is equivalent to the velocity measurement $\mathbf{v}_i = \left[v_{ix}\ v_{iy}\right]^t, i = 1, \cdots, N$. Seen from (56)-(58), two linear and one angular components of the esimated a mobile robot velocity can obtained by the averages of the corresponding components of N optical mice. Note that such a computational simplicity in the mobile robot velocity estimation is attributed to the isotropic placement of N optical mice.

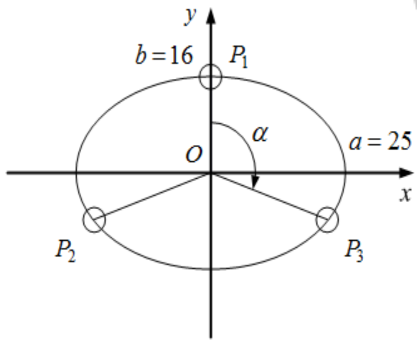

Figure 5: The symmetrical placement of three optical mice along an elliptical path.

Now, let us discuss the role of the characteristic length L in the least squares mobile robot velocity estimation, given by (53) with (54), which involves the inversion of $\widehat{\mathbf{A}}^t\widehat{\mathbf{A}}$. Seen from (8), it is apparent that the selection of L will affect the conditioning of $\widehat{\mathbf{A}}^t\widehat{\mathbf{A}}$, for a given optical mouse placement, $\mathbf{p}_i = [x_i\ \ y_i]^t, i = 1, \cdots, N$. For instance, if L is chosen to be too small, $\widehat{\mathbf{A}}^t\widehat{\mathbf{A}}$ becomes close to singularity. This may lead to numerical instability during the inversion process of $\widehat{\mathbf{A}}^t\widehat{\mathbf{A}}$, so that the accuracy of the estimated mobile robot velocity can be unacceptably poor. On the other hand, the proper selection of L, most preferably $L = L^{\#}$, given by (39), can improve the conditioning of $\widehat{\mathbf{A}}^t\widehat{\mathbf{A}}$, even when a given optical mouse placement is near singular.

8 SIMULATION RESULTS

Suppose that three optical mice($N = 3$) are placed on an elliptical path, given by

$$\Omega = \left\{(x, y) \mid \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1\right\} \quad (60)$$

where the first optical mouse is fixed on the principal axis along the y axis, but the second and third optical mice that are symmetric with respect to the y axis, as shown in Fig. 5 where $a = 25$ cm, $b = 16$ cm, and $0° \leq \alpha \leq 90°$. Using (19) and (20), the isotropic optical mouse placement can be found at $\alpha^* = 120°$, corresponding to $(\varphi_1^*, \varphi_2^*, \varphi_3^*) = (90°, 210°, 330°)$. Note that three optical mice form an equilateral triangle in general, but they will form a regular triangle if the elliptical path Ω becomes circular, that is, $a = b$. And, using (27), the optimal characteristic length is obtained by $L^* = \sqrt{\frac{3}{2N}(a^2 + b^2)} = 20.99$ cm. On the other hand, using (47) and (49), the singular optical mouse placement can be found as $(\varphi_1, \varphi_2, \varphi_3) = (90°, 90°, 90°)$.

Next, let us examine the least squares velocity estimation of a mobile robot for a given placement of three optical mice. Assume that a mobile robot is commanded to move forward along the y axis at the velocity of $v_{rx} = 0$ cm/sec, $v_{ry} = 20$ cm/sec, and $\omega_r = 0$ deg/sec. To simulate the noisy velocity measurements of three optical mice, normally distributed random numbers, n_{ix} and n_{iy}, $i = 1,2,3$, with mean 0 and variance 0.2 are added, independently, to the nominal values of the x and y velocity components of each optical mouse. With $\mathbf{v}_r = [0\ 20\ 0]^t$, using (4), the noisy velocity measurements of three optical mice are obtained by

$$\mathbf{v}_s = \mathbf{A}\mathbf{v}_r + \mathbf{n}_s \quad (61)$$

where $\mathbf{n}_s = [\mathbf{n}_1^t\ \mathbf{n}_2^t\ \mathbf{n}_3^t]^t \in \mathbf{R}^{6\times1}$ with $\mathbf{n}_i = \left[n_{ix}\ n_{iy}\right]^t, i = 1,2,3$, represents the random noise vector experienced by three optical mice. For the isotropic optical mouse placement, Fig. 6 shows the velocity measurements of the first optical mouse, and the resulting least squares velocity estimation of a mobile robot, using (53) with (54). Note that a total of 10,000 samples are taken in our simulation, but for better visibility, only 500 samples are plotted in Fig. 6. Overall, it can be observed that the effects of noisy velocity measurements of three optical mice are reduced significantly. The noise levels of two linear components, $\tilde{v}_{rx}$ and $\tilde{v}_{ry}$, of the estimated mobile robot velocity amount to about 58% of those of two linear velocity components of each optimal mouse.

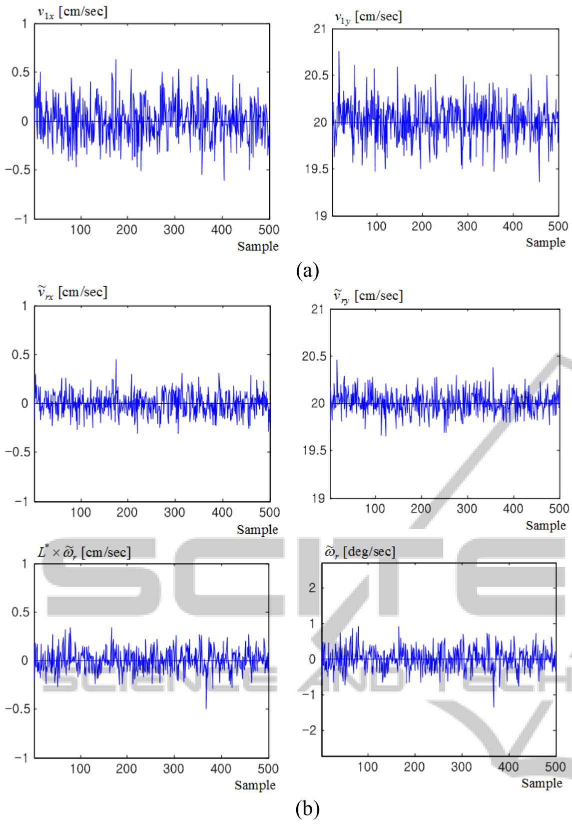

Figure 6: The least squares mobile robot velocity estimation from the noisy optical mouse velocity measurements: (a) the measured velocity components, v_{1x} and v_{1y}, and (b) the estimated velocity components, $\tilde{v}_{rx}$, $\tilde{v}_{ry}$, $L^* \times \tilde{\omega}_r$, and $\tilde{\omega}_r$.

9 CONCLUSIONS

In this paper, we presented the isotropy analysis of an optical mouse array for the mobile robot velocity estimation. Positional restriction on the installation of optical mice at the bottom of a mobile robot is assumed. The main contributions of this paper can be summarized as 1) the symbolic analysis of the Jacobian matrix, mapping the mobile robot velocity to the optical mouse velocities, 2) the identification of the isotropic, anisotropic, and singular optical placements along with their corresponding characteristic lengths, and 3) the application to the least squares mobile robot velocity estimation from the noisy optical mouse velocity measurements. The results of this paper can be helpful especially for the development of personal robot mobile platforms having a non-circular base.

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