Static Output-feedback Control with Selective Pole Constraints
Application to Control of Flexible Aircrafts

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Abstract: A non-smooth optimization approach is considered for designing constant output-feedback controllers for linear time-invariant systems with lightly damped poles. The design requirements combine H_∞ performance requirements with regional pole constraints excluding high frequency lightly damped poles. In contrast to the usual (full) pole-placement (FPP) problem, the problem dealt here is one of Selective Pole Placement (SPP). The latter design problem is frequently encountered in the control of aircraft with non-negligible aeroelastic modes which are too fast to be handled by the control surface actuators. As in the FPP case, the pole constraints are embedded in the design criterion using a transformation on the system model which modifies the H_∞-norm of the closed-loop system via a barrier function that is related to the closed-loop poles damping. Dissimilar to the closed-loop solution that is designed for the FPP, in the SPP case, numerical calculations of the gradient of the cost function is needed. The proposed method is applied to a flight control example of a flexible aircraft.

1 INTRODUCTION

The static output-feedback control problem has attracted the attention of many in the past (Bernstein et al., 1989)-(Yaesh and Shaked, 1997). The main advantage of static output-feedback is the simplicity of its implementation and the ability it provides for designing controllers of prescribed structure such as PI and PID. As in other control related fields, PI and PID controllers are widely applied in the aerospace industry. When aircraft or missiles possess flexible modes which are within or close to the desired useful system bandwidth, one may either try to damp the dynamic modes or just try to provide the control system means to avoid excitation of these modes. The latter is the common case, and it is widely encountered in practice due to bandwidth and slew rate limitations of the control surface actuators (e.g. electrical or hydraulic servo systems) and due to possibly large uncertainties in the parameters (i.e. natural frequency and damping) which characterize the flexible modes. The uncertainty in the natural frequency is the result of modelling errors, which in turn are caused by data inaccuracy of the mass distribution model. Since the damping may possess a nonlinear behavior (e.g. large damping for large input amplitudes and small one for small amplitudes), one has to take into account the whole range of damping coefficients. One may conclude in such cases that the controller should minimize a performance criterion subject to pole-placement constraints (e.g. damping coefficient) of the rigid modes poles. The rigid pole modes for which pole-placement requirements are applied can be differed from the flexible modes poles, by their natural frequency. Namely, poles which possess a natural frequency above some pre specified bound are classified as belonging to flexible modes and are not to be re-placed. Noting that performance (e.g. bandwidth) requirements as well as robust stability requirements (e.g. gain and phase margin) can be achieved using H_∞-norm minimization, one may apply one of the available tools that enable the design of static output-feedback controllers ((Burke et al., 2006),(Apkarian and Noll, 2006)). In such designs, the closed-loop damping of the dominant poles can not be guaranteed and, therefore, one may end up with an under damped closed-loop.

In (Yaesh and Shaked, 2012) the H_∞-optimization problem with pole-placement constraints has been solved by adopting the non-smooth optimization approach of (Burke et al., 2006) to deal with pole constraints. There, the pole-placement requirement has been used to modify the H_∞-norm cost function using a barrier function (i.e. large penalty when constraints

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are violated) and the gradient of this barrier function has been evaluated in closed-form, allowing efficient use of (Burke et al., 2006). In the present paper, the application of the pole-placement requirements is restricted only to the poles which are classified as the rigid modes of the plant whereas the flexible modes remain untouched as much as possible. The design for this selective requirement on the poles is the subject of the present paper.

Since the chosen H_∞-optimization method for static output-feedback design is the one of (Burke et al., 2006) some short (and not complete) survey of other methods may be in place. In this context, one should mention that the static output-feedback synthesis problem is known to be non-convex, and that many algorithms have been presented that combine convex methods with iterative solutions. One can mention, at this context, the algorithm in (Iwasaki, 1999) which, under some assumptions, is found to converge in stationary infinite horizon examples without uncertainty. The static output-feedback synthesis problem is characterized in (Iwasaki, 1999) by inequalities which are bilinear in the variable matrices. Therefore, standard convex programming procedures could not be used in the past to solve the problem, even in the case where the system parameters were all known, and various methods were proposed to deal with this difficulty (see e.g. (Peres et al., 1999) and (Leibfritz, 2001)). Another approach is the one of direct search which has been explored in (Simon, 2011) and (Esquivel et al., 2011). Recently, a non-smooth optimization approach to H_∞-optimization has been suggested in (Apkarian and Noll, 2006) and (Burke et al., 2006). These methods utilize recently developed quadratic programming techniques for non-smooth functions.

The method of the present paper is successfully applied to the control problem of a flexible aircraft where the rigid mode control is designed to avoid both modification and excitation of the flexible modes.

Notation: Throughout the paper the superscript 'T' stands for matrix transposition, $\mathcal{R}^n$ denotes the n dimensional Euclidean space and $\mathcal{R}^{n\times m}$ is the set of all $n\times m$ real matrices. For a symmetric $P\in\mathcal{R}^{n\times n}$, $P>0$ means that it is positive definite. The notation $col\{a,\ b\}$ for vectors a and b represents the augmented vector $[a^T\ b^T]^T$. For square $A\in\mathcal{R}^{n\times n}$, $\lambda(A)$ denotes its eigenvalues whereas $\alpha(A)$ denotes its spectral abscissa(Apkarian and Noll, 2006). For two matrices A and B of the appropriate dimensions we denote the matrix product in the usual sense by AB and their Kronecker product by $A\bigotimes B$. We also denote by $\mathrm{Tr}\{A\}$ the trace of a matrix A. For a complex scalar $z=x+iy$ where $i^2=-1$. Also note that the integer $i=1,2,...$ is also used as index. Distinguishing between these two uses will be by the context. We also denote $\bar{z}=x-iy$ which is not to be confused by e.g. $\bar{A}$ which is just a notation of a real-valued matrix. In this paper we provide all spaces $\mathcal{R}^k$, $k\geq 1$ with the usual inner product $<\cdot,\cdot>$ and with the standard Euclidean norm $||\cdot||$. We denote by $\mathcal{L}_2$ the space of square-integrable functions. For a transfer-function matrix $G(s)=\bar{C}(sI-\bar{A})^{-1}\bar{B}+\bar{D}$ where $\bar{A}$ is Hurwitz, we denote by $||G||_\infty$ its H_∞ -norm. Note that $||G||_\infty<\gamma$ with w and z being, respectively, the input and output signals of G, corresponds both to $||z||_2^2-\gamma^2||w||_2^2<0$ for all $w\in\mathcal{L}_2$ and $sup_{\Omega\in\mathcal{R}}\bar{\sigma}[G(i\Omega])$ where $\bar{\sigma}$ denotes the maximum singular value. The gradient with respect to a matrix $X\in\mathcal{R}^{n\times m}$ of a scalar function $f(x)$ is defined by $\frac{\partial f(X)}{\partial X}:=\{G_{ij}\}$ where $G_{ij}=lim_{\delta\to 0}\frac{f(X+e_ie_j^T\delta)-f(X)}{\delta}$ where e_i is the i'th unit column vector. The directional derivative of $f(X)$ along the direction $Y\in\mathcal{R}^{n\times m}$ which is defined by $lim_{\delta\to 0}\frac{f(X+Y\delta)-f(X)}{\delta}=Tr\{Y^T\frac{\partial f(X)}{\partial X}\}$. Also note that in case of $f(X)-f(X+\Delta)=Tr\{S\Delta\}+o(\Delta)$, it is readily obtained that $[\frac{\partial f(X)}{\partial X}]_{ij}=lim_{\delta\to 0}\frac{Tr\{Se_ie_j^T\delta\}+o(\delta)}{\delta}=S_{ji}$. Namely, $\frac{\partial f(X)}{\partial X}=S^T$. We finally note that in order to avoid confusion, the output vector of the linear system is denoted by the boldface $\mathbf{z}$ while a complex scalar is denoted just by z.

2 PROBLEM FORMULATION

We consider the following linear system

$$\begin{array}{l}\dot{x}=Ax(t)+B_1w(t)+B_2u(t),\ x(0)=x_0\\ y=C_2x(t)\end{array}\qquad(1)$$

with

$$\mathbf{z}(t)=C_1x(t)+D_{12}u(t)\qquad(2)$$

where $x\in\mathcal{R}^n$ is the system state vector, $w\in\mathcal{R}^q$ is the exogenous disturbance signal, $u\in\mathcal{R}^\ell$ is the control input, $y\in\mathcal{R}^m$ is the measured output and $\mathbf{z}\in\mathcal{R}^r\subset\mathcal{R}^n$ is the state combination (objective function signal) to be regulated. The matrices A,B_1,C_1,C_2 and D_{12} are constant matrices of appropriate dimensions.

We seek a controller

$$u=Ky,\qquad(3)$$

where K is a constant gain matrix, that achieves a certain performance requirement. The design of K should comply with the following requirements:

- **The H_∞-performance requirement:** Assuming that the exogenous disturbance signal is energy bounded (i.e. $w\in\mathcal{L}_2$), a prescribed disturbance attenuation factor γ should be guaranteed, so that $J_\infty < 0$ where

$$J_\infty := ||\mathbf{z}(t)||_2^2 - \gamma^2 ||w(t)||_2^2 \tag{4}$$

- **The closed-loop poles damping ratio constraint:** To define this design constraint one first defines the maximum natural frequency ω_R (the subscript R stands for "rigid") so that when a pole $s \in \mathcal{C}$ satisfies $|s| < \omega_R$, it is classified to be a mode of the rigid body dynamics. Then pole-placement requirement is defined to ensure that the eigenvalues $\lambda_j = r_j e^{i\theta_j}, j = 1,2,..n$ with natural frequency smaller than ω_R of $A + B_2KC_2$ should possess large enough damping ratios:

$$min_{r_j \le \omega_R, j=1,2,...,n}\{\zeta_j\} \ge \zeta_{min} \tag{5}$$

where $\zeta_j := cos(\theta_j)$.

3 PROBLEM SOLUTION

The requirements of the previous section are on the closed-loop system which is obtained by substituting (3) into (1). The closed-loop system is

$$\begin{aligned} \dot{x} &= (A + B_2KC_2)x(t) + B_1w(t) := \bar{A}x + \bar{B}w,\ x(0) = x_0 \\ \mathbf{z} &= (C_1 + D_{12}KC_2)x(t) := \bar{C}x \end{aligned} \tag{6}$$

To solve the above problem of combined H_∞-performance and closed-loop damping requirement we should first put the latter in a tractable form. Denoting, to this end, $cos(\theta) = \zeta_{min}$, the latter pole-placement requirement is equivalent ((Chilali and Gahinet, 1996)) to $f_D(z) < 0$ where

$$f_D(z) = \begin{bmatrix} \sin(\theta)(z+\bar{z}) & -\cos(\theta)(z-\bar{z}) \\ \cos(\theta)(z-\bar{z}) & \sin(\theta)(z+\bar{z}) \end{bmatrix} = W^T z + W\bar{z} \tag{7}$$

and where

$$W = \begin{bmatrix} \sin(\theta) & \cos(\theta) \\ -\cos(\theta) & \sin(\theta) \end{bmatrix} \tag{8}$$

We next invoke the result of (Arzelier et al., 1993) which states, for our case, that (7) is satisfied if and only if there exists $P > 0$ so that

$$(W \bigotimes \bar{A})P + P(W \bigotimes \bar{A})^T < 0 \tag{9}$$

Moreover, it was shown in (Chilali and Gahinet, 1996) that the structure of P is block diagonal with equal blocks, namely that the existence of $P > 0$ which satisfies (9) is equivalent to the existence of $X > 0$ so that

$$(W \bigotimes \bar{A}) \begin{bmatrix} X & 0 \\ 0 & X \end{bmatrix} + \begin{bmatrix} X & 0 \\ 0 & X \end{bmatrix} (W \bigotimes \bar{A})^T < 0 \tag{10}$$

where

$$\bar{A}_W = W \bigotimes \bar{A} = \begin{bmatrix} W_{11}\bar{A} & W_{12}\bar{A} \\ W_{21}\bar{A} & W_{22}\bar{A} \end{bmatrix} = \tag{11}$$

$$\begin{bmatrix} \sin(\theta)\bar{A} & \cos(\theta)\bar{A} \\ -\cos(\theta)\bar{A} & \sin(\theta)\bar{A} \end{bmatrix}$$

Writing (10) more explicitly, we obtain

$$\begin{bmatrix} (\bar{A}X + X\bar{A}^T)\sin(\theta) & (\bar{A}X - X\bar{A}^T)\cos(\theta) \\ -(\bar{A}X - X\bar{A}^T)\cos(\theta) & (\bar{A}X + X\bar{A}^T)\sin(\theta) \end{bmatrix} < 0 \tag{12}$$

The latter inequality guarantees the damping requirement. We, therefore, resort to the recently suggested approach of non-smooth optimization (NSO) ((Apkarian and Noll, 2006) and (Burke et al., 2006)) and define, to this end, a cost function which combines the H_∞-performance criterion and the criterion of minimum closed-loop poles damping. The combined cost function is just the H_∞-norm of the closed-loop, whenever the eigenvalues $\lambda_j = r_j e^{i\theta_j}, j = 1,2,..n$ of $\bar{A}$ satisfy either $\zeta_j \ge \zeta_{min}$, or $r_j > \omega_R$. Whenever $\zeta_j < \zeta_{min}$ and $r_j \le \omega_R$ the cost function is increased monotonically with $\zeta_{min} - \zeta_j$, using a barrier function.

We note that one could suggest defining a subset of $\mathcal{C}_R \in \mathcal{C}$ where any $re^{i\theta} \in \mathcal{C}_R$ satisfies either $r > r_R$ or $cos(\theta) > \zeta_{min}$ and then finding a matrix W, replacing the one of (8) by a new W_R, so that (10) will be satisfied by W_R. If such W_R could be found, one could just apply the results of (Yaesh and Shaked, 2012) to design static output-feedback controllers for flexible aircrafts which satisfy the requirements of Section 2 above.

Unfortunately $\mathcal{C}_R$ is not a convex set and, therefore, such W_R does not exist. We, therefore, resort to the analysis of the eigenvalues $\lambda(\bar{A}_W)$ of $\bar{A}_W$. To this end, we invoke the following property of the spectrum of $W \bigotimes \bar{A}$.

Lemma 1. Let $\lambda_j, j = 1,2,...,n$ and μ_1, μ_2 be respectively the eigenvalues of $\bar{A}$ and W. Then, the eigenvalues of $W \bigotimes \bar{A}$ are $\lambda_i \mu_k, j = 1,2,...,n,\ k = 1,2$.

Since, however, W of (8) is an orthogonal matrix, we have $|\mu_1| = |\mu_2| = 1$ leading to

$$|\lambda_j(\bar{A}_W)| = |\lambda_j(\bar{A})|$$

The eigenvalues of $\bar{A}$ which are with natural frequencies smaller than or equal to ω_R are, therefore, mapped to eigenvalues of $\bar{A}_W$ with the same property.

Consider the spectral abscissa (see e.g. (Apkarian and Noll, 2006)) of $\bar{A}$ and define its natural-frequency restricted version, by $\alpha_R(\bar{A}) := \max_{j=1,2,...n} Real(\lambda_j; |r_j| < \omega_R)$ where $\lambda_j = r_j e^{i\theta_j}, j = 1,2,...n$ denote the eigenvalues of $\bar{A}$. The following result is then readily obtained from (9) and Lemma 1:

Lemma 2. *Consider the system $\dot{x} = \bar{A}x$. The inequality (5) is satisfied if and only if $\alpha_R(\bar{A}_W) < 0$.*

We, therefore, consider the following cost function :

$$f(K)=||\bar{C}(K)(sI-\bar{A}(K))^{-1}\bar{B}(K)||_{\infty}+\rho\beta(K)\alpha_R(\bar{A}_W(K)) \tag{13}$$

where $\rho >> 1$ is a scalar, and

$$\beta(K) = \left\{ \begin{array}{l} 0 \text{ if } \alpha_R(\bar{A}_W(K)) < 0 \\ 1 \text{ if } \alpha_R(\bar{A}_W(K)) \geq 0 \end{array} \right\}$$

We note that in the script files that accompany (Burke et al., 2006), $||\bar{C}(sI-\bar{A})^{-1}\bar{B}||_{\infty}$ have been defined, and both the value of f and its gradients with respect to $\bar{A}$, $\bar{B}$, $\bar{C}$ and $\bar{D}$ are provided. Also there, the function $\alpha(\bar{A})$ and its gradient with respect to $\bar{A}$ are provided.

The first part in the cost function (13), namely the H_{∞} component, can therefore be computed by just using the above formulae for the H_{∞}-norm and its gradient which are programmed in the script function *hinfty.m* in (Burke et al., 2006). The second part in the cost function (13), which corresponds to the damping component via $\alpha_R(\bar{A}_W(K))$, is computed using Lemma 1 above. Note that it requires the computation of all the eigenvalues of $\bar{A}$. For the gradient of the second component, we need to derive a formula for $\partial\alpha_R(\bar{A}_W(K))/\partial\bar{A}$. We recall from (Yaesh and Shaked, 2012) that for the case where ω_R tends to infinity (namely all the poles are classified as rigid body poles) $\alpha_R()$ is replaced by $\alpha()$ and one may denote $H := \partial\alpha(\bar{A}_W)/\partial\bar{A}$ and partition the gradient of $\alpha(\bar{A}_W)$ with respect to $\bar{A}_W$ conformally with the partition of $\bar{A}_W$ in (11) as

$$\frac{\partial\alpha(\bar{A}_W)}{\partial\bar{A}_W} = \left[\begin{array}{cc} G_{11} & G_{12} \\ G_{21} & G_{22} \end{array} \right] \tag{14}$$

and obtain that

$$H = G_{11}W_{11} + G_{12}W_{12} + G_{21}W_{21} + G_{22}W_{22}$$

However for finite ω_R one needs an explicit (and unfortunately CPU consuming) numerical calculation of H. Namely, use the definition,

$$H = \{H_{ij}\}, i = 1,2,...n, j = 1,2,...,n \tag{15}$$

where

$$H_{ij} = \partial\alpha_R(W \bigotimes \bar{A})/\partial\bar{A}_{ij} \tag{16}$$

$$= lim_{\delta\rightarrow 0}[\alpha_R(W \bigotimes (\bar{A} + e_i e_j^T \delta)) - \alpha_R(W \bigotimes \bar{A})]/\delta$$

and where $e_i \in \mathcal{R}^n$ is the i'th unit column vector. Since G of (14) can then be computed using the script file *specabsc.m* in (Burke et al., 2006) we can compute

$$\begin{array}{l} f(\bar{A},\bar{B},\bar{C}) = ||\bar{C}(K)(sI-\bar{A})^{-1}\bar{B}||_{\infty} + \rho\beta\alpha(\bar{A}_W) \\ := f_1(\bar{A},\bar{B},\bar{C}) + f_2(\bar{A}) \end{array} \tag{17}$$

where $f_1(\bar{A},\bar{B},\bar{C})$ and $f_2(\bar{A})$ are respectively computed using the script files *hinfty.m* and *specabsc.m* in (Burke et al., 2006). As f_2 does not depend on $\bar{B}$ and $\bar{C}$, the gradients of f with respect to $\bar{B}$,$\bar{C}$ are still computed using *hinfty.m* in (Burke et al., 2006) whereas the gradient of f with respect to $\bar{A}$ is derived using

$$\frac{\partial f}{\partial\bar{A}} = \frac{\partial f_1}{\partial\bar{A}} + \frac{\partial f_2}{\partial\bar{A}}$$

where the second term is computed using (15) and (16).

Remark 1: It may be seen at first sight, that the above closed-loop damping requirements can be always satisfied. Obviously, such a conclusion is wrong due to the following reasons:

- If the plant is uncontrollable, its uncontrollable poles with natural frequency smaller than ω_R, must posses the minimum required damping ratio.
- Even if the latter condition is satisfied, there is no guarantee that static output-feedback suffices to place the poles according to the requirements. In such a case, one may apply either a full-state feedback or under an appropriate observability assumption, a full-order controller. Note that in some cases, where static output-feedback does not suffice, reduced-order controllers maybe adequate.

Remark 2: The cost function of (13) together with (4) involve a tradeoff between the disturbance attenuation γ and the required damping coefficient ζ_{min}. Since for large enough ρ, the suggested solution scheme involves minimization of γ subject to the damping coefficient constraint, one may explore the tradeoff by varying ζ_{min}.

4 FLEXIBLE AIR VEHICLE - CONTROLLER DESIGN ON NOTCH FILTERED PLANT

This example deals with a flexible air vehicle, where the suggested controller includes a $6th$ order bending-modes-filter (BMF) consisting of a cascade of $4th$ order notch filter and a $2nd$ order low-pass filter, to attenuate the effect of the bending modes, and a simple PID (Proportional + Integral + Derivative) controller which operates on the filtered plant outputs.

The PI controller gains are then tuned using the method of the present paper, where the original plant is replaced by the augmented plant which includes the BMF in cascade.

The suggested method avoids tuning of a higher order controller which would be of order 7 including the BMF (order 6) and the tracking error integrator. If such a $7th$ order controller were designed for different flight conditions, the resulting controller would be expected to possess an intricate dependence on the flight condition parameters (e.g. Mach number, dynamic pressure and so on). In the suggested control method, the central frequency of the notch filter, simply depends on the $1st$ order flexible mode natural frequency (which in turn depends on the fuel mass in the vehicle, its take off configuration etc.). The tuned parameters are then the PID gains only, leaving 3 parameters only for gain scheduling.

We consider a single flight condition (Mach no. 0.62) of the vehicle where the airframe $G_a(s) = C_a(sI-A_a)^{-1}B_a + D_a$ state-space representation is :

$$A_a = \begin{bmatrix} -0.2064 & -702.6 & 0.01184 & 0 & 0 \\ 0.00737 & -0.54 & 0 & 0 & 0 \\ 0 & 1.473 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -3.93 & 0 & 0 & -64337 & -3.551 \end{bmatrix}$$

$$B_a = col\{8.0128, 17.66, 0, 0, 2543.3\}$$

$$C_a = \begin{bmatrix} 0 & 1 & 0 & 0 & 0.037165 \\ 0 & 0 & 1 & 0.037165 & 0 \end{bmatrix}$$

and

$$D_a = col\{0, 0\}$$

The states in this representation are $x = col\{v, r, \psi, q_1, q_2\}$ where v is the lateral component of the airspeed, r is the yaw-rate, ψ is the azimuth and q_1, q_2 are the states of the 1st order bending mode. The input in this representation is the rudder angle δ_r and the outputs are the versions $\bar{r}$ and $\bar{\psi}$ of respectively r and ψ which are affected by the flexible dynamics.

The servo model $G_S(s)$ which generates the rudder angle δ_r from the corresponding command u is described by a second order system with unit DC gain, a natural frequency of $113 rad/sec$ and damping coefficient of 0.7.

The BMF is given by $G_{BMF}(s) = \frac{1}{(1+s\tau)^2} G^2_{Notch}(s)$:

$$G_{Notch}(s) = \frac{s^2 + 2\zeta_1 \omega s + \omega^2}{s^2 + 2\zeta_2 \omega s + \omega^2}$$

where $\zeta_1 = 0.005$, $\zeta_1 = 0.2, \omega = 2\pi \times 40.4 rad/sec$ and $\tau = \frac{1}{2\pi\times 60} sec$..

An integral weight on the azimuth angle error $\psi - \psi_c$ is defined, as well as weights on the output ψ and and the rudder command u. The minimized output is:

$$z = col\{\frac{5}{s}(\psi - \psi_c), \psi, 0.3u\}$$

We define the exogenous disturbance to be $w := \psi_c$. We note that the first component in the transference T_{zw} relating w and z, corresponds to a weighted (via $5/s$) version of the sensitivity $S(s) = (1 + KP(s))^{-1}$. The second term there corresponds to the complementary sensitivity $T(s) = 1 - S(s)$ whereas the third term is just the control effort transference relating u and $w = \psi_c$.

The overall plant consists of the airframe $G_a(s)$ cascaded with the BMF, the 2nd order model $G_R(s)$ of the rate sensor (natural frequency of $90 rad/sec$ and damping coefficient of 0.125) and a pure delay of $2.5 msec$ represented by 2nd order Pade' approximation $G_D(s)$. It is given by :

$$P(s) = G_R(s) G_D(s) G_S(s) G_{BMF}(s) G_a(s)$$

The measured outputs vector is chosen as :

$$y = col\{\ \bar{r}, \quad \bar{\psi}, \quad \tfrac{5}{s}(\bar{\psi} - \psi_c)\}\ \}$$

A couple of the PID-like controller designs are compared :

- H_∞ control without pole-placement (Burke et al., 2006). The results of this attempt are illustrated in Figures 1 - 4. We see in Fig. 1 a satisfying step response, but somewhat low stability margins when loop is cut at control (about 7 db and 32 degrees phase margin (see Fig. 2). The stability margins when loop is cut at the feedback (Fig. 3) are higher (about 13 db and 64 degrees phase margin). Note that the low overshoot in the step response to ψ_c is associated with the high stability margins when the loop is cut at the feedback. This is since the transference from ψ_c to ψ is $L/(1+L)$ where L is just the loop transfer function obtained when cutting the loop in the feedback. The control gains are $K = \begin{bmatrix} -1.1823 & -11.802 & -14.8 \end{bmatrix}$.
- H_∞ / SPP control with an attempt to place only the poles with natural frequency smaller than $\omega_R = 150 rad/sec$ to have damping ratio of 0.4 or larger : With the method of the present paper, all poles within $150 rad/sec$ possess damping ratios greater than or equal to 0.4. The closed-loop poles complying with the design requirements are shown in Fig. 8 (see also Fig. 4 to compare to the case where no pole-placement requirements are imposed). The closed-loop step response is depicted in Fig. 5, whereas the corresponding Nichols chart when loop is cut in the feedback is depicted in Fig. 7. The somewhat larger overshoot in the step response is due to the lower margins at the feedback cut. However, when loop is open at the control signal (see Fig. 6) one notices that the gain and phase margin are much improved ($16 db$ and 62 degrees) with respect to

the corresponding margins in design using the original method of (Burke et al., 2006) without pole placement requirements . Since larger uncertainties are expected at the plant input (aerodynamic and flexible mode dynamics uncertainties), where no significant uncertainties are expected in the feedback, one may conclude that design with the H_∞ / SPP method of the present paper, has improved robust stability with respect the design with (Burke et al., 2006) with no modifications. The improved stability margins are achieved at the cost of some degradation in the step response to command. One should, however, keep in mind that the command response can be readily improved with a shaping filter (i.e. 2 degrees of freedom compensator) without any cost regarding stability. The control gains are $K = \begin{bmatrix} -0.47704 & -2.0889 & -1.4118 \end{bmatrix}$.

Remark 3: As noted above, the notch filter G_{Notch} attenuates the 1*st* order flexible mode (which is included in the plant model $G_a(s)$) whereas the second order low-pass filter $\frac{1}{(1+s\tau)^2}$ in $G_{BMF}(s)$, attenuates all other flexible modes which are of higher order and frequency. This low-pass filter and the SPP, which is aimed at avoiding damping high frequency modes, both reduce the risk to spill-over which may result in right-half-plane poles of higher order modes. Nevertheless, to rule out spill-over one needs to perform higher order modes identification and flight tests.

5 CONCLUSIONS

A non-smooth optimization approach for designing static output-feedback controllers for a linear time-invariant systems has been considered. The design is aimed at achieving, for the closed-loop system, a minimization of an H_∞-norm bound together with satisfaction of frequency-selective damping ratio requirements. As in the case of non-selective pole-placement, the design method applies a simple augmentation of the H_∞-norm to include a large penalty whenever the regional pole-placement requirements are violated. The augmented function is expressed in terms of a modified version of the spectral abscissa of the closed-loop transformed matrix. The stability of this transformed matrix, is equivalent to the requirements of the frequency-selective regional pole-placement. The gradient of the resulting augmented function is numerically calculated by defining the appropriate directional derivatives. The new method has been implemented within the *hifoo* software package (Burke et al., 2006) and has been applied to a flexible aircraft control example where the plant is first augmented with bending mode rejection filters, and then a static output-feedback controller is designed. This numerical example demonstrates that the suggested design method is very effective. A more efficient approach to derive the cost function gradient is left for a future research.

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APPENDIX

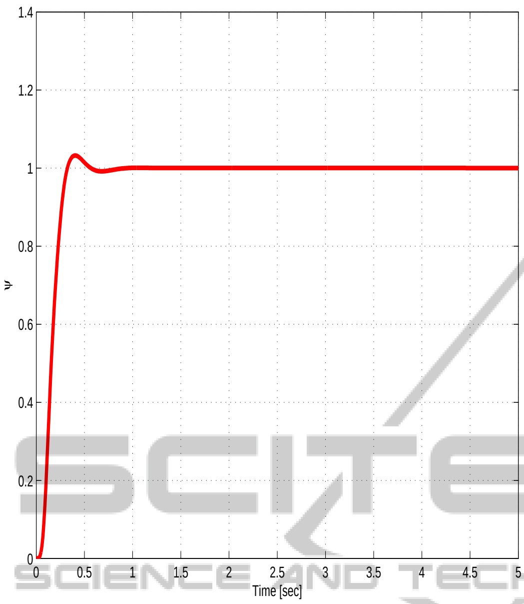

Figure 1: H_∞ Optimization without Pole Placement - Step Response.

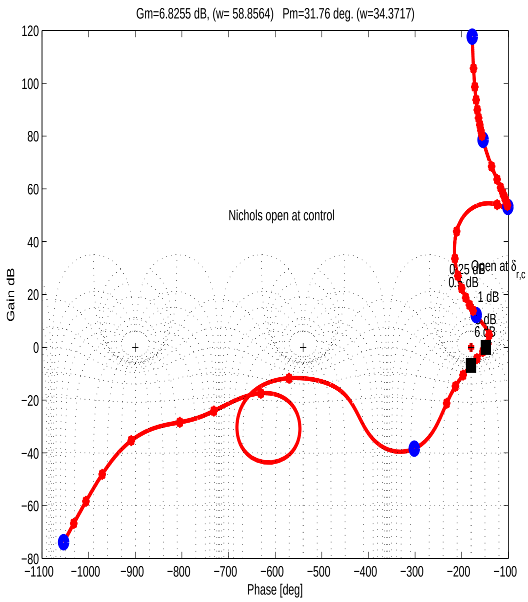

Figure 2: H_∞ Optimization without Pole Placement - Nichols chart - loop open at control.

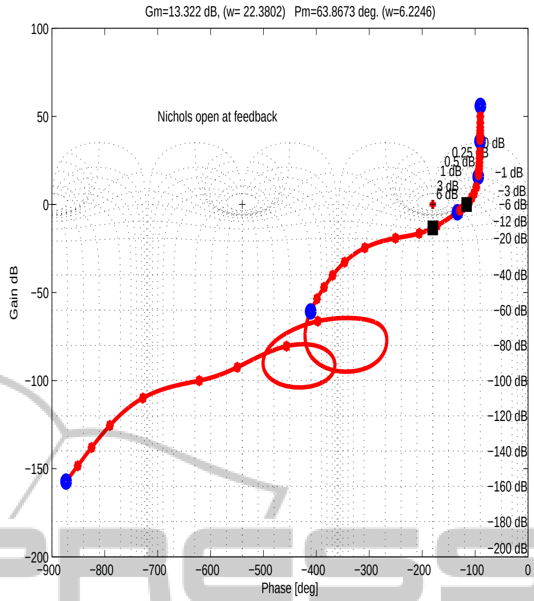

Figure 3: H_∞ Optimization without Pole Placement - Nichols chart - loop open at feedback d) Closed-loop eigenvalues.

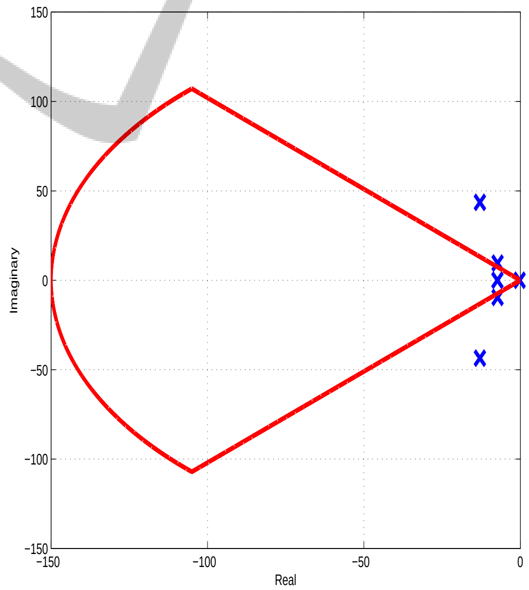

Figure 4: H_∞ Optimization without Pole Placement - Closed-loop eigenvalues.

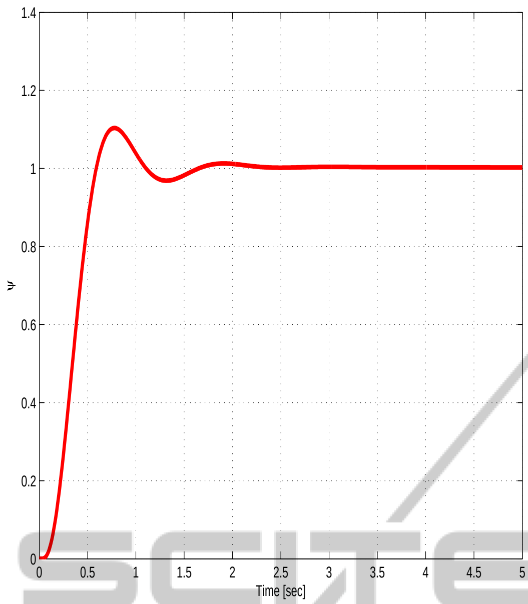

Figure 5: H_∞ Optimization with Pole Placement - Step Response.

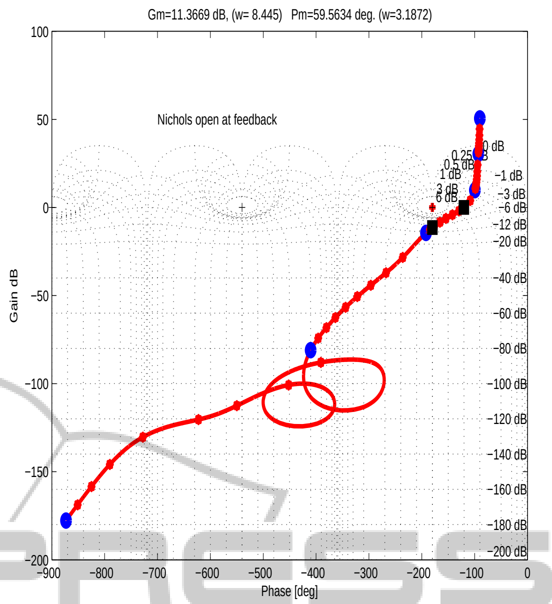

Figure 7: H_∞ Optimization with Pole Placement - Nichols chart - loop open at feedback d) Closed-loop eigenvalues.

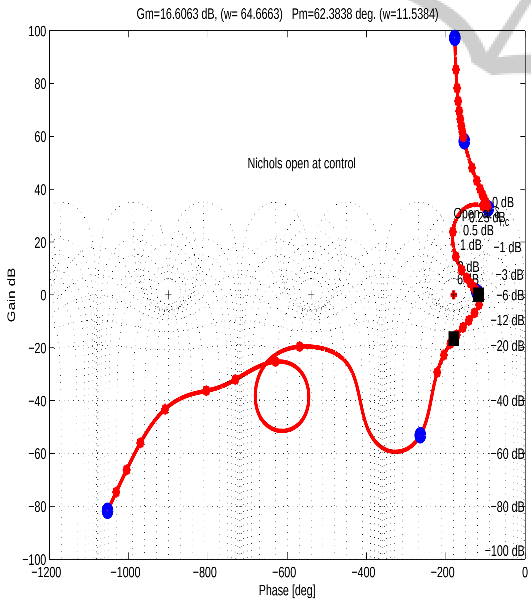

Figure 6: H_∞ Optimization with Pole Placement - Nichols chart - loop open at control.

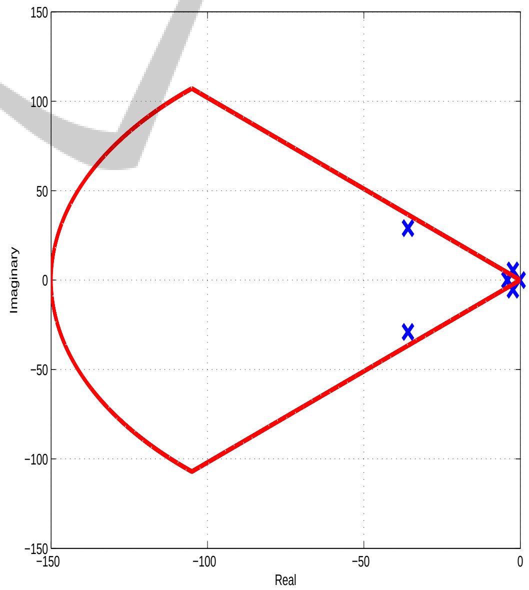

Figure 8: H_∞ Optimization with Pole Placement - Closed-loop eigenvalues.