Robust Sliding Mode Control for a Roll-to-Roll Machine

Kuo-Ming Chang[1] and Yen-Yeu Lin[2]

[1]*Department of Mechanical Engineering, National Kaohsiung University of Applied Sciences, 415 Chien-Kung Rd., Kaohsiung, Taiwan*

[2]*Institute of Mechanical and Precision Engineering, National Kaohsiung University of Applied Sciences, 415 Chien-Kung Rd., Kaohsiung, Taiwan*

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Abstract: This paper proposes a robust sliding mode controller, which is derived based on the extended state observer and the sliding mode control techniques for a roll-to-roll machine to deal with the system uncertainty problem of unknown system nonlinear functions, and external disturbances. It is worth noting that the proposed sliding mode control scheme can be implemented without the condition that the system nonlinear functions, and the upper bounds of external disturbances must be known in advance and it can achieve the web transmitting speed and tension control goals satisfactorily, which are validated by numerical simulation results.

1 INTRODUCTION

Roll-to-roll (R2R) processing is the process of creating electronic devices on a roll of flexible plastic or metal foil. R2R processing has the advantages in fast and mass replication of microstructures and it is a good fit for fabricating devices based on flexible substrates. In recent years, there has been much progress in the R2R processing (Liang, 2003); (Makela et al., 2007); (Lo et al., 2009).

To obtain a higher imprinting quality of the microstructures in the R2R processing, the moving web (PET substrate) should be under the conditions that the web should be kept at a steady and fixed speed and the web tension should be very small. A moving web under insufficient tension cannot track properly and may wrinkle the web, while excess tension may result in web deformation or even a web break. At the same time, unacceptable levels of speed variation can cause detrimental variation in tension. In view of the importance of both speed and tension controls, a number of control schemes have been proposed. Chang and Weng applied a traditional sliding mode control method to control the web speed and tension. Chen et al., (2004) proposed a sliding mode control with an estimator based on a recurrent neural network, which is used to estimate system uncertainties. Dou and Wang (2010) presented a robust H_∞ control strategy to attenuate tension fluctuations when the system is subject to disturbances and variations in speed or other operating conditions. In this paper, a robust sliding mode control is designed based on the extended state observer and the sliding mode control techniques for R2R machine to achieve the PET web transmitting speed and tension control objectives in the presence of unknown system uncertainties.

2 ROLL-TO-ROLL MACHINE

The configuration of the R2R equipment established for conducting research into the tension and speed controls of PET web is shown in Figure 1. Assume that the web deformation in the thickness and width directions are small compared to the length direction. As a pull force is imposed on the web, the linear density of web, υ_1 can be expressed by

$$\upsilon_1 = \frac{L\upsilon}{L+\Delta L} = \frac{\upsilon}{1+\varepsilon} \tag{1}$$

where L and υ are the length and linear density of web, respectively before the web is elongated, ΔL is the length deformation, ε is the web strain in the length direction.

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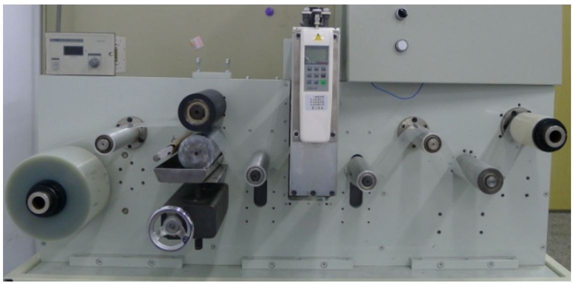


Figure 1: Photograph of the R2R machine.

Assume that the web strain is quite small. It yields that

$$\upsilon_1 \approx (1-\varepsilon)\upsilon \tag{2}$$

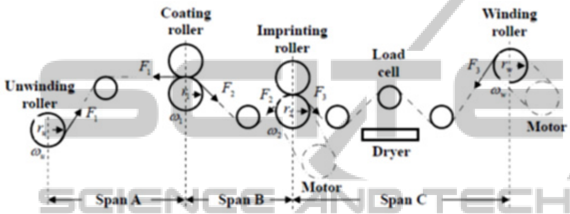


Figure 2: Layout of the R2R machine.

Consider Span A in the R2R system, as shown in Figure 2. Based on the mass conservation principle, we have

$$\frac{d}{dt}\left[\int_0^{L_1}(1-\varepsilon_1)\upsilon\, dl\right] = (1-\varepsilon_0)\upsilon v_u(t) - (1-\varepsilon_1)\upsilon v_1(t) \tag{3}$$

where $v_u(t)$ and $v_1(t)$ are the web speeds in the unwinding roller and the coating roller, respectively, ε_0 and ε_1 are the web strains in the unwinding roller and the coating roller, respectively. From Eq. (3), it can be obtained that

$$-L_1\dot{\varepsilon}_1 = (1-\varepsilon_0)v_u(t) - (1-\varepsilon_1)v_1(t) \tag{4}$$

where L_1 is the web length between the unwinding roller and the coating roller.

Since $E = F/A \big/ \varepsilon$, the web tension variation can be derived in the following form from Eq. (4).

$$\dot{F}_1 = -\frac{AE}{L_1}v_u + \frac{1}{L_1}F_0 v_u + \frac{AE}{L_1}v_1 - \frac{1}{L_1}F_1 v_1 \tag{5}$$

where A, E, and F are the cross sectional area, Young's modulus, and the tension of web, respectively. Similarly to the above derivation, we have web tension variations in Spans B and C.

$$\dot{F}_2 = -\frac{AE}{L_2}v_1 + \frac{1}{L_2}F_1 v_1 + \frac{AE}{L_2}v_2 - \frac{1}{L_2}F_2 v_2 \tag{6}$$

$$\dot{F}_3 = -\frac{AE}{L_3}v_2 + \frac{1}{L_3}F_2 v_2 + \frac{AE}{L_3}v_w - \frac{1}{L_3}F_3 v_w \tag{7}$$

Assume that there is no slippage between the web material and rollers. According to Newton's second law, the web speed can be derived as

$$\dot{v}_u = \frac{r_u^2}{J_u}F_1 - \frac{r_u}{J_u}T_{f0}(t) \tag{8}$$

$$\dot{v}_1 = \frac{r_1^2}{J_1}F_2 - \frac{r_1^2}{J_1}F_1 - \frac{r_1}{J_1}T_{f1}(t) \tag{9}$$

$$\dot{v}_2 = \frac{r_2}{J_2}K_{t1}i_1 + \frac{r_2^2}{J_2}F_3 - \frac{r_2^2}{J_2}F_2 - \frac{r_2}{J_2}T_{f2}(t) \tag{10}$$

$$\dot{v}_w = \frac{r_w}{J_w}K_{t2}i_2 - \frac{r_w^2}{J_w}F_3 - \frac{r_w}{J_w}T_{f3}(t) \tag{11}$$

where J, θ, r, and T_f represent the inertia, the angular displacement, the roller radius, and the friction torque, respectively, T_m, K_t, and i are the torque, the torque constant, and the current input of motor, respectively.

Hence, the system dynamics of the R2R machine are represented by Eqs. (5)-(11). In this paper, a robust control system is designed for keeping a steady fixed web speed v_2 and web tension F_3 by controlling motor current inputs i_1 and i_2 under the unknown system uncertainties.

3 SLIDING MODE CONTROL

Define some system states as $x_1 = v_2$, $x_2 = F_3$, and $x_3 = \dot{F}_3$ and control input as $u_1 = i_1$ and $u_2 = i_2$. Eqs. (7) and (10) can be written in the following state-space representation form:

$$\dot{x}_1 = a_{12}x_2 + b_{11}u_1 - d_1(t) \tag{12}$$

$$\dot{x}_2 = a_{21}x_1 - a_{22}x_2 + d_2(t) \tag{13}$$

$$\begin{aligned}\dot{x}_3 &= a_{31}x_1 + a_{32}x_2 - a_{33}x_3 + b_{31}u_1 + b_{32}u_2 \\ &\quad + c_{31}x_1^2 + c_{32}x_2^2 + d_3(t)\end{aligned} \tag{14}$$

where $a_{12} = \frac{r_2^2}{J_2}$, $b_{11} = \frac{r_2}{J_2}K_{t1}$, $d_1(t) = \frac{r_2^2}{J_2}F_2 + \frac{r_2}{J_2}T_{f2}(t)$, $a_{21} = -\frac{AE}{L_3} + \frac{1}{L_3}F_2$, $a_{22} = \frac{1}{L_3}F_3$, and $d_2(t) = \frac{AE}{L_3}v_w$, $a_{31} = \frac{1}{L_3L_2}F_1 v_1 - \frac{AE}{L_3L_2}v_1$, $a_{32} = -\frac{AEr_2^2}{L_3J_2} + \frac{r_2^2}{L_3J_2}F_2 - \frac{AEr_w^2}{L_3J_w} + \frac{r_w}{L_3J_w}T_{f3}$, $a_{33} = \frac{1}{L_3}v_w$,

$b_{31}=\frac{r_2}{L_3J_2}F_2K_{t1}-\frac{AEr_2}{L_3J_2}K_{t1}$, $b_{32}=\frac{AEr_w}{L_3J_w}K_{t2}-\frac{r_w}{L_3J_w}F_3K_{t2}$,

$c_{31}=\frac{AE}{L_3L_2}-\frac{1}{L_3L_2}F_2$, $c_{32}=\frac{r_w^2}{L_3J_w}$,

$d_3(t)=\frac{AEr_2^2}{L_3J_2}F_2+\frac{AEr_2}{L_3J_2}T_{f2}-\frac{r_2^2}{L_3J_2}F_2^2-\frac{r_2}{L_3J_2}F_2T_{f2}-\frac{AEr_w}{L_3J_w}T_{f3}$.

The system uncertainties arise from unknown nonlinear system functions, system parameter variations, and external disturbances. To obtain a better control performance in this work, the designed controller should be with the available unknown system nonlinear functions and external disturbances for compensating system uncertainties. In this paper, the extended state observer is applied to estimate system uncertainties. Then, a sliding mode control is derived by using the estimated states from the extended state observer. State equation in (12) is firstly extended to be

$$\begin{cases}\dot{x}_1=x_{1e}=a_{12}x_2+b_{11}u_1-d_1(t)=\phi_1(t)\\ \dot{x}_{1e}=\dot{\phi}_1(t)\end{cases} \tag{15}$$

where x_{1e} is an extended system state.

Then, according to the work (Han, 1995), an extended state observer is given in the following form:

$$\begin{cases}\dot{\hat{x}}_1=\hat{x}_{1e}-k_{11}(\hat{x}_1-x_1)\\ \dot{\hat{x}}_{1e}=-k_{12}(\hat{x}_1-x_1)\end{cases} \tag{16}$$

Subtracting Eq. (15) from Eq. (16), it yields that the state error dynamic equation is given by

$$\begin{cases}\Delta\dot{x}_1=\Delta x_{1e}-k_{11}\Delta x_1\\ \Delta\dot{x}_{1e}=-k_{12}\Delta x_1-\dot{\phi}_1(t)\end{cases} \tag{17}$$

where $\Delta x_1=\hat{x}_1-x_1$ and $\Delta x_{1e}=\hat{x}_{1e}-x_{1e}$ are two state errors. Eq. (17) is further represented in the vector-matrix form by

$$\begin{bmatrix}\Delta\dot{x}_1\\ \Delta\dot{x}_{1e}\end{bmatrix}=\begin{bmatrix}-k_{11} & 1\\ -k_{12} & 0\end{bmatrix}\begin{bmatrix}\Delta x_1\\ \Delta x_{1e}\end{bmatrix}+\begin{bmatrix}0\\ -\dot{\phi}_1(t)\end{bmatrix}=A_1\Delta X_1+\Phi_1(t) \tag{18}$$

where $A_1=\begin{bmatrix}-k_{11} & 1\\ -k_{12} & 0\end{bmatrix}$, two designed parameters k_{11} and k_{12} in matrix A_1 should be chosen such that the matrix A_1 is a Hurwitz matrix and then the extended state observer can asymptotically estimate system states. For designing a sliding mode control, a sliding surface is set as

$$s_1=e_1+c_1\int e_1 dt \tag{19}$$

where $e_1=x_{1d}-x_1$, x_{1d} is a desired reference signal. It follows that the equivalent control by setting $\dot{s}_1=0$ is obtained as

$$u_{1eq}=\frac{1}{b_{11}}[\dot{x}_{1d}-a_{12}x_2+d_1(t)+c_1(x_{1d}-x_1)] \tag{20}$$

From Eq. (15), we have

$$d_1(t)-a_{12}x_2=b_{11}u_1-x_{1e} \tag{21}$$

Since x_{1e} is unknown and can not be measured, it can be replaced by the estimated state $\hat{x}_{1e}$. Since the control input u_1 is not an available signal, which is obtained from the proposed control law, the value of u_1 in Eq. (21) is replaced by a filter signal $\hat{u}_1$ given from the following equation

$$\dot{\hat{u}}_1=-\delta_1\hat{u}_1+\delta_1u_1 \tag{22}$$

where δ_1 is a sufficiently large positive constant and the filter can let $\hat{u}_1$ have the property, $\lim_{t\to\infty}\lim_{\delta_1\to\infty}\hat{u}_1=u_1$

Therefore, the equivalent control (20) is rewritten as

$$u_{1eq}=\frac{1}{b_{11}}[\dot{x}_{1d}+b_{11}\hat{u}_1-\hat{x}_{1e}+c_1(x_{1d}-x_1)] \tag{23}$$

In addition to the equivalent control input, a traditional nonlinear switching control input is given as

$$u_{1s}=\mu_1\sin(s_1)$$

To reduce the chattering in the control input signal, in this paper, the nonlinear switching control input is given by

$$u_{1s}=-\eta_1s_1-\mu_1 sat\left(\frac{s_1}{\varepsilon_1}\right) \tag{24}$$

where ε_1 is a sufficiently small positive constant, η_1 and μ_1 are two designed positive constants.

Hence, in this paper, the control input is finally designed as

$$u_1=u_{1eq}+u_{1s}$$

$$=\frac{1}{b_{11}}[\dot{x}_{1d}+b_{11}\hat{u}_1-\hat{x}_{1e}+c_1(x_{1d}-x_1)]-\eta_1s_1-\mu_1 sat\left(\frac{s_1}{\varepsilon_1}\right) \tag{25}$$

Similarly to the above derivation of control input u_1, state equations Eqs. (13) and (14) are extended as:

$$\begin{cases}\dot{x}_2=x_3\\ \dot{x}_3=x_{3e}=f(x,u_1)+b_{32}u_2+d_3(t)=\phi_2(t)\\ \dot{x}_{3e}=\dot{\phi}_2(t)\end{cases} \tag{26}$$

where $f(x,u_1)=a_{31}x_1+a_{32}x_2-a_{33}x_3+b_{31}u_1+c_{31}x_1^2+c_{32}x_2^2$ is an unknown system nonlinear function. It yields that from Eq. (26) the extended state observer is designed as

$$\begin{cases} \dot{\hat{x}}_2 = \hat{x}_3 - k_{21}(\hat{x}_2 - x_2) \\ \dot{\hat{x}}_3 = \hat{x}_{3e} - k_{22}(\hat{x}_2 - x_2) \\ \dot{\hat{x}}_{3e} = -k_{23}(\hat{x}_2 - x_2) \end{cases} \quad (27)$$

where positive parameters k_{21} , k_{22} , and k_{23} are chosen to satisfy the matrix $A_2 = \begin{bmatrix} -k_{21} & 1 & 0 \\ -k_{22} & 0 & 1 \\ -k_{23} & 0 & 0 \end{bmatrix}$ is a Hurwitz matrix. The switching surface is set as

$$s_2 = 2c_2 e_2 + e_3 + c_2^2 \int e_2 dt \quad (28)$$

where $e_2 = x_{2d} - x_2$, x_{2d} is a desired reference signal. Then, it yields that the designed control input is given by

$$u_2 = u_{2eq} + u_{2s}$$
$$= \frac{1}{b_{32}}[2c_2(\dot{x}_{2d} - x_3) + \dot{x}_{3d} - b_{32}\hat{u}_2 + \hat{x}_{3e} + c_2^2(x_{2d} - x_2)]$$
$$-\eta_2 s_2 - \mu_2 sat\left(\frac{s_2}{\varepsilon_2}\right) \quad (29)$$

$$\dot{\hat{u}}_2 = -\delta_2 \hat{u}_2 + \delta_2 u_2 \quad (30)$$

where ε_1 is a sufficiently small positive constant, η_1 and μ_1 are two designed positive constants, δ_2 is a sufficiently large positive constant.

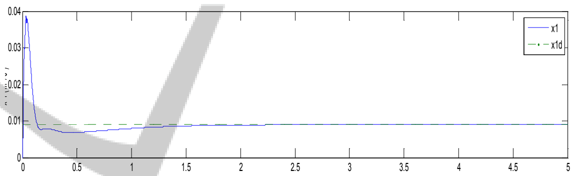
4 SIMULATION RESULTS

In order to show the performance of the proposed control scheme for the R2R machine, some numerical simulation results are given and analyzed. Consider a R2R machine with system parameters as shown in Table 1.

For simulations, the desired speed and tension are set as $x_{1d} = 0.009$ (m/s) and $x_{2d} = 5$ (N) , control parameters are designed by $k_{11} = 100$, $k_{12} = 200$, $k_{21} = 500$, $k_{22} = 600$, $k_{23} = 500$, $\delta_1 = 25$, $\varepsilon_1 = 0.1$, $\eta_1 = 0.0013$, $\mu_1 = 0.08$, $c_1 = 200$, $\delta_2 = 27$, $\varepsilon_2 = 0.1$, $\eta_2 = 0.068$, $\mu_2 = 0.031$, $c_2 = 5$. Figure 3 (1) and (2) show that the web speed and the web tension can achieve the control objective. Figure 3 (3) and (4) show the control input time responses. From simulation results, it is validated that the proposed control scheme can effectively compensate system uncertainties with unknown $d_1(t)$, $d_2(t)$, $\phi_1(t)$, and $\phi_2(t)$ to achieve web speed and tension control

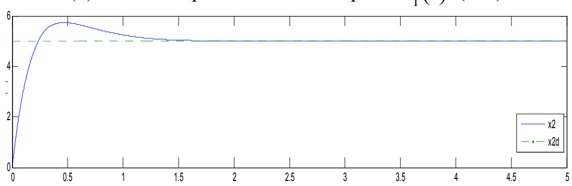
Table 1: PET material and system parameters.

Notation	Data
E	3.150×10^9 N/m^2
A	3.384×10^{-5} m^2
L_1	0.4787 m
L_2	0.3501 m
L_3	0.6263 m
ρ	1390 kg/m^3
h	188 μm
w	0.180 m
r_{u0}	0.015 m
r_1	0.0425 m
r_2	0.0425 m
r_{w0}	0.015 m
J_{u0}	7.805×10^{-4} kg·m^2
J_1	2.19×10^{-3} kg·m^2
J_2	2.19×10^{-3} kg·m^2
J_{w0}	7.805×10^{-4} kg·m^2



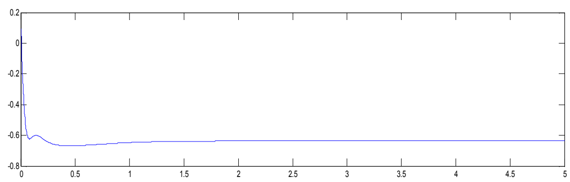
Time (sec)

(1) Time response of the web speed $x_1(t)$ (m/s).



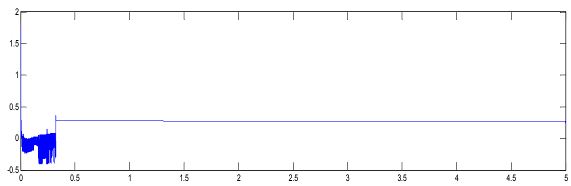
Time (sec)

(2) Time response of the web tension $x_2(t)$ (N).



Time (sec)

(3) Time response of the control input $u_1(t)$.



Time (sec)

(4) Time response of the control input $u_2(t)$.

Figure 3: The performance of the sliding mode control.

objectives satisfactorily.

In Table 1, h and w represent the PET web thickness and width, respectively.

5 CONCLUSIONS

In this paper, a robust sliding mode control is developed based on extended state observer and sliding mode control techniques for a R2R machine to control PET web speed and tension. From simulation results, it is shown that the proposed control scheme can be implemented without the knowledge of system uncertainties and can achieve the control objective satisfactorily under the unknown system uncertainties. In the future work, the proposed control scheme will be used in the roll-to-roll experimental facility to evaluate and validate its control performance.

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