Modeling the Behavior of Hair Follicle Receptors as Technical Sensors using Adaptive Control

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Abstract: Based on the paradigm biological receptor and its fundamental feature to filter signals and transduce them, we set up a mechanical sensor system to find hints to establish a measurement or monitoring system. These technical systems have to offer high sensitivity to signals from the environment. To mimic the complex behavior of the biological system, adaptive controllers have to be applied to a mechanical sensor system to compensate and filter unknown ground excitations (uncertainties of the system). Before doing this we summarize previous work on controlling such mechanical systems. We expose the need of improvements of already existing strategies from literature, the corresponding problems are formulated. Improved adaptive controllers are presented. Their working principle is illustrated in various numerical simulations and experiments.

1 INTRODUCTION

In nature there are various senses that allow animals to perceive their environment. Depending on the distance of objects to be sensed from the system boundaries of the animal (usually the skin), sensors are distinguished between “far field” (e.g., vision) and “near field”, of which the sense of vibrations is a special case. Here, we focus on the sensing of vibrations for purposes of exteroception (outside the body), not ignoring the phylogenetic relation to interoception mechanisms like proprioception. Vibrations are an important piece of environmental information that insects rely on, especially arachnids, such as spiders and scorpions. To perceive vibrations, they have different types of sensilla (or tactile hairs, (Barth, 2004)). Vertebrates, such as cats, rats and sea lions, also possess the sense of vibration. They can perceive vibrations with the help of their vibrissae (whiskers).
Although these biological vibration receptors have a different physiology – (Iwasaki et al., 1999) or (Smith, 2008) for classifications, they share common properties: When in touch with an oscillating object, they are moved and stimulate various (pressure-sensitive) receptors which have to analyze the stimulus and to transduce their gained adequate information to the central nervous system (CNS).
Mechanoreceptors of this kind are present throughout the integument of insects (cuticula) and mammals (fur on skin).

2 HAIR FOLLICLE RECEPTORS

Let us focus on mammalian receptors. The vibrissae serve mainly as levers for force transmission. If the hair is deflected due to some excitations, e.g., wind, this mechanical (oscillation) energy is then transmitted to the various receptors, which respond to any movement of the hairs, see Fig. 1. A receptor has only one function: to transduce a (mechanical) stimulus to neural impulses (Soderquist, 2002). However, a receptor never continues to respond to a non-changing stimulus in transducing information to the CNS as long as the stimulus is present. It rather depends on the type of stimulus. If some impulses stimulate a receptor then there is a rapid and brief response of the receptor to it. This response declines if the stimulus is unchanging. Due to permanently changing environments the receptors have to be in a **permanent state of adaptation** to adjust their behavior. The rate or time needed to adapt or stop responding to an unchanging stimulus is the main characteristic to distinguish two different types of tactile receptors, see Fig. 2. The classification is, (Soderquist, 2002) and (Smith, 2008):

- *fast adapting (FA) receptors* encompass hair follicle sense endings: as mentioned previously, FA receptors react to applied movements or pressures with a fast (rapid) response of activity, which is succeeded by a decrease of it even though the stimulus is still present. This means, that, if a

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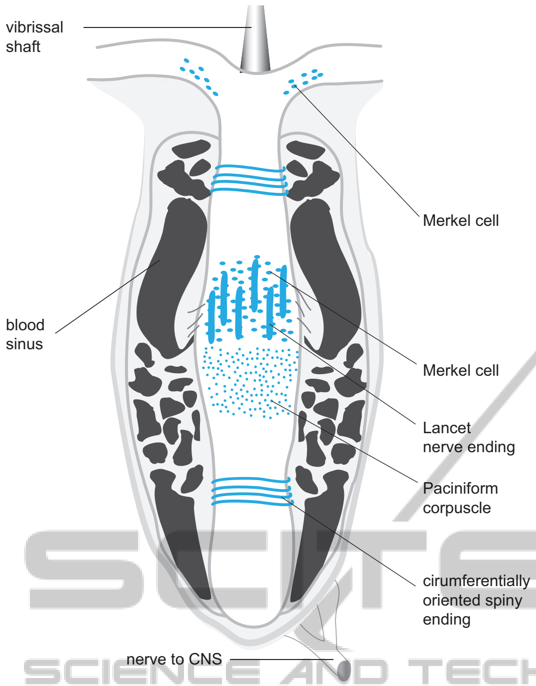

Figure 1: Follicle-sinus complex (FSC) of a vibrissa with various types of receptors (blue); adapted from (Ebara et al., 2002) and (Rice et al., 1986).

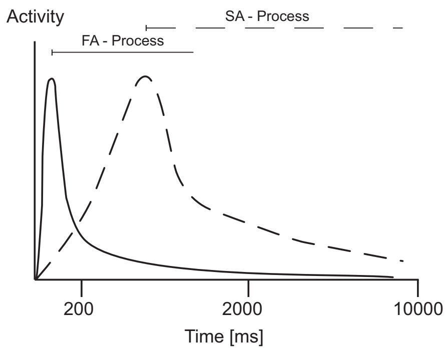

Figure 2: Adaptation processes of FA and SA receptors, modified from (Soderquist, 2002).

mechanical pressure via an unchanging force is applied the FA receptor responds quickly with a steep increase of activity and then decreases this activity and waits for a further stimulus, it adapts its activity in order to notice changes in the stimulus;

- the counterpart are *slow adapting (SA) receptors*, e.g., Merkel cells — these receptors work in a similar way as FA receptors but offer two ways of operation: first, as usual to receptors, a rapid response is followed by a decrease of activity. But, there is also a long duration of time of activity of these receptor cells from the beginning of the stimulus. This is in contrast to FA receptors.

The described behavior is shown in Fig. 3.

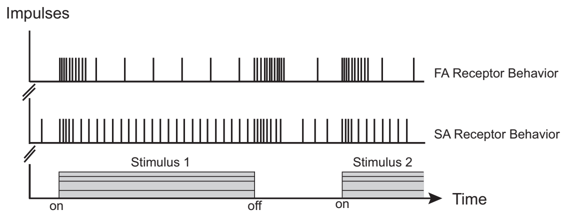

Figure 3: Activity behavior of FA and SA receptors, modified from (Soderquist, 2002).

Here, we want to focus on the **fast adapting receptors**. The sensibility of these cells is continuously adjusted so that the receptor system converges to the rest position despite the continued excitation (Dudel et al., 1996). Hence, the perception of the continuous unchanging excitation is damped. Therefore, the excitation is considered irrelevant, once it has been perceived. If however a different excitation, such as a sudden deviation of the vibrissa sensor, occurs, this information is relevant and the sensor has to be sensitive to perceive it. If, for example, a cat is exposed to wind, the recognition of the resulting excitation of the whiskers will be damped and ignored. If the cat encounters an obstacle, the receptors should still be sensitive enough to perceive the sudden deviation of the whiskers while the wind excitation persists. Therefore, the adaption process has to ensure enduring sensitivity.

In the following, we set up a simple mechanical model to map all important features of fast adapting receptors via adaptive control strategies applied to the mechanical system.

3 MECHANICAL MODELING

Motivated by the biological observations in the foregoing section we consider a simple model of a receptor in form of a spring-mass-damper-system within a rigid frame, which is forced by an unknown time-dependent displacement $a(\cdot)$. Moreover, the mass is under the action of an internal control force $u(\cdot)$ to compensate the unknown ground excitations, see Fig. 4, where x is the absolute coordinate. The parameters of this sensory system are m (the forced seismic point mass), the damping factor d and the spring stiffness c.

We derive the differential equation of motion by using Newton's second law:

$$\left.\begin{aligned} m\ddot{x}(t) &= -d\left(\dot{x}(t)-\dot{a}(t)\right) \\ &\qquad -c\left(x(t)-a(t)\right)+u(t)\,, \\ x(0) &= x_0\,, \quad \dot{x}(0)=x_1\,. \end{aligned}\right\} \tag{1}$$

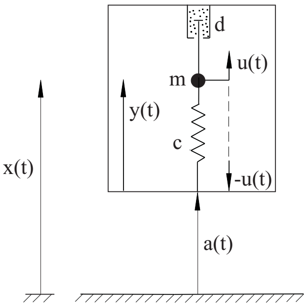

Figure 4: Mechanical model of a sensor system (receptor model) (Behn and Steigenberger, 2010).

With $y = x - a$ as the relative coordinate of the point mass, we arrive at the following differential equation of the relative motion with respect to the frame

$$\left.\begin{aligned} &m\ddot{y}(t) + d\,\dot{y}(t) + c\,y(t) = -m\,\ddot{a}(t) + u(t)\,, \\ &y(0) = x_0 - a(0)\,, \quad \dot{y}(0) = x_1 - \dot{a}(0)\,. \end{aligned}\right\} \qquad (2)$$

If $y(\cdot)$ is the measured output of the system, (2) is presented in normalized form

$$\left.\begin{aligned} \begin{pmatrix} y(t) \\ \dot{y}(t) \end{pmatrix}^{\bullet} &= \begin{bmatrix} 0 & 1 \\ -\frac{c}{m} & -\frac{d}{m} \end{bmatrix} \begin{pmatrix} y(t) \\ \dot{y}(t) \end{pmatrix} \\ &\qquad + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ -\ddot{a}(t) \end{bmatrix} \\ y(0) &= x_0 - a(0)\,, \quad \dot{y}(0) = x_1 - \dot{a}(0)\,. \end{aligned}\right\} \qquad (3)$$

4 SCOPE, PROBLEM AND GOAL

Scope: The goal is to achieve a predefined movement of the receptor mass m of the sensor system in Fig. 4 such as stabilization of the sensor system or tracking of a reference trajectory. It is obvious that the sole possibility of influencing this system lies in the (control) force $u(\cdot)$. Hence, we have to design and implement a controller which ensures a desired system output behavior. Therefore, the scope/object is to find a suitable control strategy that reproduces the specialities of the biological system receptor.
This system is similar to seismic sensor systems to detect (unknown) ground excitations due to the principle of passive oscillation perception.

Problem: In general, one cannot expect to have complete information about a mechanical or biological system, but instead only structural properties are known. It is important to point out that all system parameters are supposed to be unknown because of the sophisticated nature of the biological system. The external excitation $a(\cdot)$ (biological disturbance to the receptor) is unknown and the mass, spring and damping factors are uncertain, e.g., vary in time due to thermic influences. Here, uncertainty of the factors mean that they have a positive value, but they are not known exactly, only a valid range, e.g., $c \in [\underline{c}, \overline{c}]$. Summarizing, we have to deal with a **highly uncertain (control) system** of known structure. This is why traditional control methods fail, as they rely on the knowledge of those parameters. The consideration of uncertain systems leads us to the use of adaptive control. By the above mentioned adjustment of the receptor we are given the task to adaptively compensate the unknown ground excitation: we have to design an adaptive controller, which learns from the behavior of the system, so automatically adjusts its parameters in such a way that the (seismic) point mass tends to the rest position in spite of the continuing excitation.

Goal: We choose the λ-tracking control objective (Behn and Zimmermann, 2006) due to the high-gain property of the sensor system presented in (3). λ-tracking allows for simple feedback laws and does not focus on exact tracking since we deal with an uncertain system. Therefore, the goal is to act on the system in such a way that the system output $y(\cdot)$ is λ-tracked, i.e., the system output is forced into an error neighborhood λ around a set point trajectory $y_{\mathrm{ref}}(\cdot)$. If $y_{\mathrm{ref}}(\cdot) \equiv 0$, the problem is known as λ-stabilization. In this case, the receptor is supposed to remain in its equilibrium state (rest position).

This design of controllers depends tremendously on the system properties. The adaptive control strategies should meet the following *requirements*:

- ability to apply the controllers without any knowledge about system parameters;
- simple feedback structure;
- optimal control performance regarding
  - short settling time: a desired quality of adaptive controllers is finite time behavior. Since all system parameters are unknown and the level of the necessary control gain cannot be anticipated, it is also unknown at which point in time the control objective will be achieved. Controllers with finite time behavior enable the user to specify a time at which the control objective will be achieved at the latest.
  - simple structure of controller equations;
  - small level of gain parameters, level of error inside the λ-tube;
  - ability to quickly adapt to parameter changes. It is imperative to keep the sensitivity of the system high. If, for example, a recurring excitation signal $a(\cdot)$ acts on the system, it is supposed that its influence is to be damped by the controller. Once the sensor/receptor has noticed this excitation it has to fade it out to wait for

further new information. It has to adaptively adjust its parameters. If however the excitation subsides or is replaced be one with a much lower amplitude, the system is supposed to remain sensitive and quickly adjust the control parameters.

5 SYSTEM CLASSES

The equations of motion (3) fall into the category of quadratic, finite-dimensional, nonlinearly perturbed, m-input $u(\cdot)$, m-output $y(\cdot)$ systems (**MIMO**-systems) of relative degree two, for short $\mathcal{S}_{2,nonlin1}$, of the form

$$\left.\begin{aligned} \ddot{y}(t) &= A_2\dot{y}(t) + f_1\big(s_1(t),y(t),z(t)\big) + Gu(t)\,,\\ \dot{z}(t) &= A_5 z(t) + A_0\dot{y}(t) + f_2\big(s_2(t),y(t)\big)\,,\\ y(t_0) &= y_0\,,\quad \dot{y}(t_0) = y_1\,,\quad z(t_0) = z_0\,, \end{aligned}\right\} \quad (4)$$

with $y(t)$, y_0, y_1, $u(t) \in \mathbb{R}^m$, $z(t)$, $z_0 \in \mathbb{R}^{n-2m}$, A_2, $G \in \mathbb{R}^{m\times m}$, $A_5 \in \mathbb{R}^{(n-2m)\times(n-2m)}$, $A_0 \in \mathbb{R}^{(n-2m)\times m}$, $n \geq 2m$, and, for natural number q_1 and q_2 it holds

(i) $\operatorname{spec}(G) \subset \mathbb{C}_+$, i.e., the spectrum of the "high-frequency gain" lies in the open right-half complex plane;

(ii) $s_1(\cdot) \in \mathcal{L}^\infty\big(\mathbb{R}_{\geq 0};\mathbb{R}^{q_1}\big)$, $s_2(\cdot) \in \mathcal{L}^\infty\big(\mathbb{R}_{\geq 0};\mathbb{R}^{q_2}\big)$ may be thought of as (bounded) disturbance terms, where $s_i(t) = \psi_i\big(t, y(t), \dot{y}(t), z(t)\big)$ is also possible with $\psi_i(\cdot,\cdot,\cdot,\cdot) \in \mathcal{L}^\infty\big(\mathbb{R}_{\geq 0}\times\mathbb{R}^m\times\mathbb{R}^m\times\mathbb{R}^{n-2m};\mathbb{R}^{q_i}\big)$;

(iii) the functions $f_1 : \mathbb{R}^{q_1}\times\mathbb{R}^m\times\mathbb{R}^{n-2m} \to \mathbb{R}^m$ and $f_2 : \mathbb{R}^{q_2}\times\mathbb{R}^m \to \mathbb{R}^{n-2m}$ are continuous functions and, for compact sets $C_1 \subset \mathbb{R}^{q_1}$ and $C_2 \subset \mathbb{R}^{q_2}$, there exist c_1, $c_2 \geq 0$ such that for all $(s,y,z) \in C_1\times\mathbb{R}^m\times\mathbb{R}^{n-2m}$

$$\big\|f_1(s,y,z)\big\| \leq c_1\left[1 + \|y\| + \|z\|\right],$$

and for all $(s,y) \in C_2\times\mathbb{R}^m$

$$\big\|f_2(s,y)\big\| \leq c_2\left[1 + \|y\|\right].$$

(iv) $\operatorname{spec}(A_5) \subset \mathbb{C}_-$, i.e., the system is minimum phase, provided $f_1 = 0$, $f_2 = 0$.

It is easy to prove that every system of this system class has strict relative degree two. Therefore, relative degree two means that the control $u(\cdot)$ directly influences the second derivative of each output component. The term $A_0\dot{y}$ appears in connection with underactuated systems (Behn and Zimmermann, 2006).

If we inspect system (3) in more detail and take the physical meaning of the parameters into account, i.e., the mass of the forced (seismic) point mass m, the damping factor d and the spring stiffness c represent positive real values, then we can simplify system class (4) to a very special one:

- we restrict to **single**-input $u(\cdot)$, **single**-output $y(\cdot)$ systems (**SISO**-system $m = 1$),
- then, we claim $A_2 < 0$,
- and we neglect the coupling term, $A_0 := 0$.

Hence, we arrive at a subclass of finite-dimensional, nonlinearly perturbed SISO-system with strict relative degree two, $\mathcal{S}_{2,nonlin2}$ for short, of the form

$$\left.\begin{aligned} \ddot{y}(t) &= A_2\dot{y}(t) + f_1\big(s_1(t),y(t),z(t)\big) + Gu(t)\,,\\ \dot{z}(t) &= A_5 z(t) + f_2\big(s_2(t),y(t)\big)\,,\\ y(t_0) &= y_0\,,\quad \dot{y}(t_0) = y_1\,,\quad z(t_0) = z_0\,, \end{aligned}\right\} \quad (5)$$

with $y(t), y_0, y_1, u(t), G, A_2 \in \mathbb{R}$, $z(t), z_0 \in \mathbb{R}^{n-2}$, $A_5 \in \mathbb{R}^{(n-2)\times(n-2)}$, $n \geq 2$, and for $q_1, q_2 \in \mathbb{N}$ it holds

(i) $G > 0$, i.e., a positive input gain ("high-frequency gain");

(ii) $s_1(\cdot) \in \mathcal{L}^\infty\big(\mathbb{R}_{\geq 0};\mathbb{R}^{q_1}\big)$ and $s_2(\cdot) \in \mathcal{L}^\infty\big(\mathbb{R}_{\geq 0};\mathbb{R}^{q_2}\big)$, i.e., they may be thought of as (bounded) disturbance terms;

(iii) the functions $f_1 : \mathbb{R}^{q_1}\times\mathbb{R}\times\mathbb{R}^{n-2} \to \mathbb{R}$ and $f_2 : \mathbb{R}^{q_2}\times\mathbb{R} \to \mathbb{R}^{n-2}$ are continuous ones and, for compact sets $C_1 \subset \mathbb{R}^{q_1}$ and $C_2 \subset \mathbb{R}^{q_2}$, there exist c_1, $c_2 \geq 0$ such that for all $(s,y,z) \in C_1\times\mathbb{R}\times\mathbb{R}^{n-2}$

$$\big|f_1(s,y,z)\big| \leq c_1\left[1 + |y| + \|z\|\right],$$

and for all $(s,y) \in C_2\times\mathbb{R}$

$$\big\|f_2(s,y)\big\| \leq c_2\left[1 + |y|\right];$$

(iv) $\operatorname{spec}(A_5) \subset \mathbb{C}_-$, i.e., the system is minimum phase, provided $f_1 = 0$, $f_2 = 0$.

(v) $A_2 < 0$, i.e., this system has a zero-center in the open left-half complex plane (a "stable zero-center"), see (Ogata, 1997).

It is easy to check that $\mathcal{S}_{2,nonlin2} \subset \mathcal{S}_{2,nonlin1}$ holds. In order to capture more relevant SISO-systems we introduce a generalized system class of $\mathcal{S}_{2,nonlin2}$ in the following — system class $\mathcal{S}_{2,nonlin3}$:

$$\left.\begin{aligned} \ddot{y}(t) &= f_0\big(s_0(t),y(t),z(t)\big)\,\dot{y}(t)\\ &\qquad + f_1\big(s_1(t),y(t),z(t)\big) + Gu(t)\,,\\ \dot{z}(t) &= A_5 z(t) + f_2\big(s_2(t),y(t)\big)\,,\\ y(t_0) &= y_0\,,\quad \dot{y}(t_0) = y_1\,,\quad z(t_0) = z_0\,, \end{aligned}\right\} \quad (6)$$

with $y(t), y_0, y_1, u(t), G \in \mathbb{R}$, $z(t), z_0 \in \mathbb{R}^{n-2}$, $A_5 \in \mathbb{R}^{(n-2)\times(n-2)}$, $n \geq 2$, and for $q_0, q_1, q_2 \in \mathbb{N}$ we have to claim that the following will hold additionally:

(ii) $s_0(\cdot) \in \mathcal{L}^\infty\big(\mathbb{R}_{\geq 0};\mathbb{R}^{q_0}\big)$;

(iii) $f_0 : \mathbb{R}^{q_0} \times \mathbb{R} \times \mathbb{R}^{n-2} \to \mathbb{R}$ is a continuous function, and for a compact set $C_0 \subset \mathbb{R}^{q_0}$, there exist c_0, $\tilde{c}_0 > 0$ with $\tilde{c}_0 > c_0$, such that for all $(s,y,z) \in C_0 \times \mathbb{R} \times \mathbb{R}^{n-2}$

$$-\tilde{c}_0 < f_0(s,y,z) < -c_0 ;$$

It follows that $\mathcal{S}_{2,nonlin2} \subset \mathcal{S}_{2,nonlin3}$. For the following control systems, theorems and proofs we then focus on class $\mathcal{S}_{2,nonlin3}$ instead of $\mathcal{S}_{2,nonlin2}$.

6 CONTROLLERS

Since we deal with uncertain, nonlinearly perturbed (ground excitation, see the continuous functions f_i) MIMO-systems, which are not necessarily autonomous, particular attention is paid to the adaptive λ-*tracking control* objective (Behn and Zimmermann, 2006). This is to determine an *online control strategy* that achieves approximate tracking of a given, favored reference signal in the following sense:

(i) every solution of the closed-loop system is defined and bounded for $t \geq 0$, and

(ii) the output $y(\cdot)$ tracks $y_{\text{ref}}(\cdot)$ with asymptotic accuracy $\lambda > 0$ in the sense that

$$\max\left\{0, \left\|y(t) - y_{\text{ref}}(t)\right\| - \lambda\right\} \overset{t\to+\infty}{\to} 0, \qquad (7)$$

i.e., we tolerate a feasible error of prescribed size λ (accuracy). Visually, this means that the output $y(t)$ tends to a tube of radius λ around $y_{ref}(t)$, see Fig. 5.

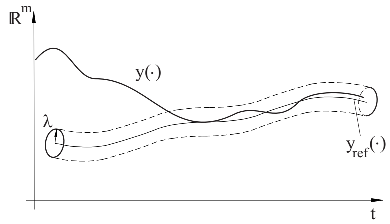

Figure 5: Reference signal and λ-tube.

Classical adaptive high-gain *lambda*-trackers from literature are:

1. The first one is a modification of the preferred stabilizer from literature, see (Ilchmann, 1991). The modified control strategy, which is also presented in (Behn and Zimmermann, 2006), is:

$$\left.\begin{aligned} e(t) &:= y(t) - y_{\text{ref}}(t), \\ u(t) &= -\Big(k(t)e(t) + \tfrac{d}{dt}\big(k(t)e(t)\big)\Big), \\ \dot{k}(t) &= \gamma\Big(\max\left\{0, \left\|e(t)\right\| - \lambda\right\}\Big)^2, \end{aligned}\right\} \qquad (8)$$

with $k(0) = k_0 \in \mathbb{R}$, $\lambda > 0$, $y_{\text{ref}}(\cdot) \in \mathcal{R}$, $u(t)$, $e(t) \in \mathcal{R}^{\text{m}}$, $k(t) \in \mathcal{R}$, and $\gamma \gg 1$.
Due to the presented λ-tracking control objective (tolerating a tracking error of size λ, no exact tracking) this controller consists of a very simple feedback mechanism and adaptation law, and is only based on the output of the system and its time derivative - no knowledge about the system parameters is required. However, the adaptive controller (8) uses the derivative of the output. The following two feedback controls avoid the usage of the derivative of the system output.

2. This one includes a dynamic compensator due to a controller in (Miller and Davison, 1991). This controller avoids the possible drawback of using the derivative of the output.

$$\left.\begin{aligned} e(t) &:= y(t) - y_{\text{ref}}(t), \\ u(t) &= -k(t)\,\theta(t) - \tfrac{d}{dt}\big(k(t)\,\theta(t)\big), \\ \dot{\theta}(t) &= -k(t)^2\,\theta(t) + k(t)^2\,e(t), \\ \dot{k}(t) &= \gamma\max\left\{0, \left\|e(t)\right\| - \lambda\right\}^2, \end{aligned}\right\} \qquad (9)$$

with $\theta(t_0) = \theta_0$, $k(t_0) = k_0 > 0$, $\lambda > 0$, $y_{\text{ref}}(\cdot) \in \mathcal{R}$, $u(t)$, $e(t) \in \mathbb{R}^m$, $k(t) \in \mathbb{R}$, $\gamma \gg 1$ and arbitrary initial data $k_0 > 0$, $\theta_0 \in \mathbb{R}^m$.
We stress that the feedback in (9) does not invoke any derivatives of observables.

3. If $\mathcal{S}_{2,nonlin1}$ is restricted to single-input, single-output systems of class $\mathcal{S}_{2,nonlin3}$, then the following simple feedback control is considered, which reduces in dimension (the number of used variables calculated by internal differential equations):

$$\left.\begin{aligned} u(t) &= -k(t)\big[y(t) - y_{\text{ref}}(t)\big], \\ \dot{k}(t) &= \gamma\max\left\{0, \left|y(t)\right| - \lambda\right\}^2. \end{aligned}\right\} \qquad (10)$$

with $k(0) = k_0 \in \mathbb{R}$. Therefore, we have a controller of order 1 whereas (9) is a controller of order 2. We stress that this feedback in (10) does not invoke any derivatives, too.
The feedback law has a P-structure, a D-term is not necessary for controlling systems of the class $\mathcal{S}_{2,nonlin3}$. Naturally we need a P- and D-term to control systems with strict relative degree two, see (Sontag, 1998).

To summarize, these controllers are simple in their design, rely only on structural properties of the system (and not on the system's data) and do not invoke any estimation or identification mechanism. They only consist of a feedback strategy and a simple parameter adaptation law, and, moreover, do not have to depend on the derivative of the output of the system.

All three controllers achieve λ-tracking (and, of course, λ-stabilization using $y_{\text{ref}}(\cdot) \equiv 0$, as well) in applying all three controllers (8), (9) and (10) to the system classes:

- Theorem I: controller (8) applied to systems of class $\mathcal{S}_{2,nonlin1}$ — Proof in (Behn and Zimmermann, 2006);
- Theorem II: controller (9) applied to systems of class $\mathcal{S}_{2,nonlin1}$ — Proof in (Behn, 2011);
- Theorem III: controller (10) applied to systems of class $\mathcal{S}_{2,nonlin3}$ — proven and submitted.

The parameter γ strongly determines the growth of the gain parameter $k(\cdot)$. In (Behn and Zimmermann, 2006) the case $\gamma = 1$ was dealt with. With small γ (e.g., $\gamma = 1$ as formerly) $k(\cdot)$ often grows too slowly as to achieve a good tracking behavior. Therefore, a sufficiently large $\gamma \gg 1$ should be used. But, if we choose γ too large, we arrive at high feedback values. Furthermore, these high values keep the sensor not really to be sensitive to extraordinary impulses, generally speaking, the receptor is "blind" if the signal is forced once into the tube, because it cannot detect the peak in observing the output. The last requirement to the controllers is not fulfilled: the closed-loop sensor system has to be sensitive to recurring excitation signals — fade it out and wait for further new information. This is not realized yet. We are able to dominate the system, but we are not able to get information on the environment in observing the output. This is addressed in the next section – design of new adaptation laws – to identify the (whole) ground excitation or only some basic characteristics of it.

7 ADAPTORS

The drawback of the **'Classical' Adaptor:**

$$\boxed{\dot{k}(t) = \gamma\Big(\max\Big\{0, \|e(t)\| - \lambda\Big\}\Big)^2,}$$

is $\dot{k}(t) \geq 0$, $\forall t \geq 0$, i.e.,

$$t \mapsto k(t) = k(0) + \int_0^t \gamma\Big(\max\Big\{0, \|e(\tau)\| - \lambda\Big\}\Big)^2 d\tau \geq 0$$

thus implies monotonic increase of $k(\cdot)$. Typically, the classical high-gain adaptive controllers (feedback law including adaptation law) yield a non-decreasing gain, which is usual. Now we propose some new adaptation laws, which let $k(\cdot)$ decrease when e is in the tube.

A very simple modification of the adaptation law is the so-called **σ-modification**, $\sigma > 0$. For λ-tracking control including the gain coefficient γ (Georgieva and Ilchmann, 2001), and revisited in (Behn and Steigenberger, 2009) in simplified form we have **Adaptor 1:**

$$\boxed{\dot{k}(t) = -\sigma k(t) + \gamma\Big(\max\Big\{0, \|e(t)\| - \lambda\Big\}\Big)^2,}$$

with $\sigma > 0$, $\gamma \gg 1$. The term $-\sigma k(t)$ decreases $k(\cdot)$ exponentially, while the second term ensures a quadratic increase of $k(\cdot)$ when $\|y(t)\|$ is outside the λ-strip. Therefore, Adaptor 1 offers two terms which are active simultaneously and counteract each other. Depending on the situation, one of the terms overcomes the effect of the other and results in a global decrease or increase of $k(\cdot)$. This law often leads to *oscillatory* behavior (maybe limit cycles) and even *chaotic* one of the system. Hence, this adaptor has to be treated carefully, because the dynamical behavior depends crucially on the parameters $\sigma > 0$. Therefore, we will not focus on this adaptor type in sequel.

The idea is now to split the part of increase and decrease of the gain as follows in **Adaptor 2** (Behn and Steigenberger, 2009)**:**

$$\boxed{\dot{k}(t) = \begin{cases} \gamma\Big(\|e(t)\| - \lambda\Big)^2, & \|e(t)\| \geq \lambda, \\ -\sigma k(t), & \|e(t)\| < \lambda, \end{cases}}$$

with $\sigma > 0$, $\gamma \gg 1$. This adaptor shows alternating increase and exponential decrease of $k(\cdot)$.

It could happen that e rapidly traverses the λ-tube. Then it would be inadequate to immediately decrease $k(\cdot)$ after e entered the tube. Rather we should distinguish three cases:

1. increasing $k(\cdot)$ while e is outside the tube,
2. constant $k(\cdot)$ after e entered the tube - no longer than a pre-specified duration t_d of stay, and
3. decreasing $k(\cdot)$ after this duration has been exceeded.

So, a another adaptation law of this kind is **Adaptor 3** (Behn and Steigenberger, 2010)**:**

$$\boxed{\dot{k}(t) = \begin{cases} \gamma\Big(\|e(t)\| - \lambda\Big)^2, \|e(t)\| \geq \lambda, \\ 0, \Big(\|e(t)\| < \lambda\Big) \wedge (t - t_E < t_d), \\ -\sigma k(t), \Big(\|e(t)\| < \lambda\Big) \wedge (t - t_E \geq t_d), \end{cases}}$$

with given $\sigma > 0$, $\gamma \gg 1$, and $t_d > 0$, whereas the entry time t_E is an internal time variable.

If the norm of the error value $\|e\|$ is close to the λ-strip, i.e., the system output y is already close to

the λ-tube, and $0 < \|e\| - \lambda < 1$ holds, an exponent of $p = 2$ leads to an even smaller number. This is the main disadvantage in such a way, that, if $\|y\|$ is already close to the λ-strip around the prescribed reference signal, the adaption process, i.e., the increase of $k(\cdot)$, is very slow.
In order to make the attraction of the tube stronger, it would be advantageous to use different exponents p with better performance such as a square root. Hence, a kind of scheduling of $\dot{k}$ is introduced, different exponents for large/small distances from the tube, see (Behn and Steigenberger, 2010) and **Adaptor 4:**

$$\dot{k}(t) = \begin{cases} \gamma\Big(\|e(t)\| - \lambda\Big)^2, \ \|e(t)\| \geq \lambda + 1, \\ \gamma\Big(\|e(t)\| - \lambda\Big)^{0.5}, \ \lambda + 1 > \|e(t)\| \geq \lambda, \\ 0, \ \Big(\|e(t)\| < \lambda\Big) \wedge (t - t_E < t_d), \\ -\sigma k(t), \ \Big(\|e(t)\| < \lambda\Big) \wedge (t - t_E \geq t_d), \end{cases}$$

with σ, γ, t_d, t_E as before.

Let $\lambda > 0$ be chosen in regard of certain requirements given by the context. To ensure that the system output y stays within that λ-tube along the reference signal (e will not leave the λ-strip after entering the strip) is to track a smaller safety radius $\varepsilon\lambda < \lambda$, suggestions for adaptation laws are now **Adaptor 5:**

$$\dot{k}(t) = \begin{cases} \gamma\Big(\|e(t)\| - \varepsilon\lambda\Big)^2, \|e(t)\| \geq \varepsilon\lambda + 1, \\ \gamma\Big(\|e(t)\| - \varepsilon\lambda\Big)^{\frac{1}{2}}, \varepsilon\lambda + 1 > \|e(t)\| \geq \varepsilon\lambda, \\ 0, \Big(\|e(t)\| < \varepsilon\lambda\Big) \wedge (t - t_E < t_d), \\ -\sigma k(t), \Big(\|e(t)\| < \varepsilon\lambda\Big) \wedge (t - t_E \geq t_d), \end{cases}$$

with σ, γ, t_d, t_E as before.

8 SIMULATION

We point out, that the adaptive nature of the controllers is expressed by the **arbitrary choice** of the system parameters. Obviously numerical simulation needs fixed (and known) system data, but the controllers **adjust** their gain parameter **to each set** of system data. The numerical simulations will demonstrate and illustrate that the adaptive controllers work successfully and effectively.

Choosing the parameters from Table 1 (which are arbitrarily chosen, not measured or identified from the biological paradigm, just for simulation purposes) and $\varepsilon = 0.7$ we get the results shown in Figs. 6 and 7 in applying Adaptor 5.

Table 1: Global simulation parameters (dimensionless).

sensor mass m	1
damping coefficient d	5
spring stiffness c	10
initial values $\big(y(0), \dot{y}(0)\big)$	$\big(-a(0), -\dot{a}(0)\big)$
tolerance λ	0.2
initial gain value k_0	0
ref. signal $t \mapsto y_{\text{ref}}(t)$	0 (rest position)
ground excitation	$t \mapsto a(t) = \sin(2\pi t)$

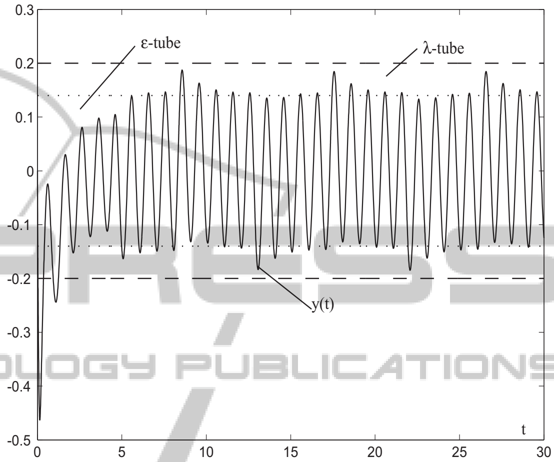

Figure 6: Output $y(\cdot)$ and tubes.

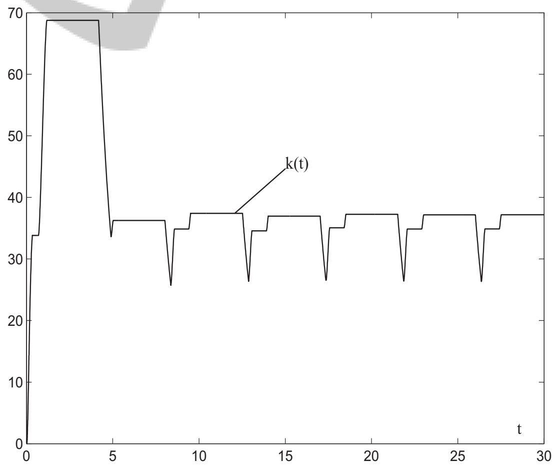

Figure 7: Gain parameter $k(\cdot)$.

In former simulations, the output apparently periodically leaves the λ-tube. Then $\varepsilon\lambda$-tracking (we will call this kind of tracking **ε-safe λ-tracking**) with $\varepsilon = 0.7$ makes e not to leave the desired λ-tube, see Fig. 6.
The steep increase of $k(\cdot)$ at the beginning is due to the "switching on" of the controller and the small initial value of $k(0) = k_0 = 0$. This could be prevented in choosing a larger k_0. But, it depends on the system data which is unknown a-priori. The constant high level is due to the (only) monotonic increase of the

gain. This problem is addressed in the next section. Comparing the simulation results and the sensor behavior of the gain parameter $k(\cdot)$ in Fig. 7 with the impulse sequences in Fig. 3, one clearly recognizes that we achieved the behavior of the biological paradigm.

9 EXPERIMENT

This section is devoted to the experimental verification of the successful implementation of the controller (feedback including Adaptor 5) developed above. For this purpose, we built up a demonstrator in form of an electrical oscillating circuit, the test rig is presented in Fig. 8.

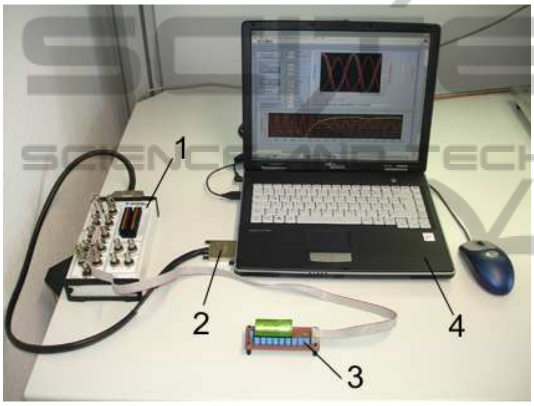

Figure 8: Test rig with electrical oscillating circuit: 1 - I/O-system (BNC-2110), 2 - DAQ-6036-PCMCIA-card, 3 - demonstrator, 4 - PC with LabView.

The demonstrator, see (3) in Fig. 8, is shown in Fig. 9.

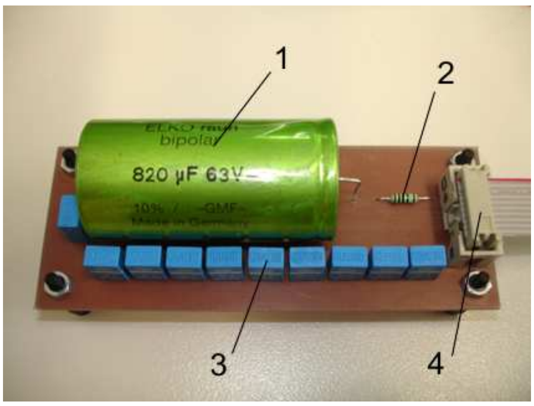

Figure 9: Circuit: 1 - capacitor ($C = 800\,\mu F$) , 2 - resistor ($R = 100\,\Omega$), 3 - one inductor (overall inductance $L_{ges} = 640\,mH$), 4 - communication to PC.

Then, the equations of motion are, using Lagrange's equations of the 2nd kind

$$L\ddot{q}(t) + R\dot{q}(t) + \frac{1}{C}q(t) = U(t) + u(t)\,. \qquad (11)$$

The system output shall be the charge $q(\cdot)$. The goal is to adaptively compensate changes of $U(t)$ by means of the control input $u(t)$ to λ-track $q_{\mathrm{ref}}(\cdot) \equiv 0$. As a rule, the charge is measured due to the voltage at the capacitor in form of

$$q(t) = C\,U_C(t)\,.$$

Due to the small system parameter values the gain $k(\cdot)$ will increase tremendously and we need high computing capacity. To avoid this we will directly control the voltage $U_C(\cdot)$ which depends linearly on the measured $q(\cdot)$, see above.

We apply Adaptor 5 to guarantee that the error will not leave the λ-tube in tracking a tube of smaller radius $\varepsilon\lambda$. We have

- excitation: $t \mapsto U(t) = U_0 \sin(\omega t)$ with amplitude $U_0 = 5\,\mathrm{V}$ and frequency $f = 0.5\,\mathrm{Hz}$;
- Adaptor 5: $\gamma = 1000$, $\lambda = 0.03\,\mathrm{V}$, $\varepsilon\lambda = 0.02\,\mathrm{V}$ (much smaller tolerance), $\sigma = 0.05$, $t_d = 6\,\mathrm{s}$.

We perform this experiment in using LabView to handle and to control the circuit. By means of a programmed LabView control panel, see Fig. 10, we are able to switch on/off the excitation $U(\cdot)$ and the control strategy $u(\cdot)$. Furthermore, several signals are displayed via this panel:

- in the top window (actual measured data): the control input $u(\cdot)$ (orange line), the system output $U_C(\cdot)$ (red line), the excitation signal $U(\cdot)$ (blue line);
- the bottom window (data on time horizon): the depicted curves are capacity voltage $U_C(\cdot)$ (i.e., the output $y(\cdot)$, red line), the λ-strip (blue lines), and gain parameter $k(\cdot)$ (green line), in only one window.

The plots of the exported data-files from LabView are shown in Fig. 11 and 12.

The capacitor voltage (output) never leaves the λ-tube, the adaptor works effectively. Further experiments can be found in (Behn, 2013).

Comparing the simulation results with $\sigma = 0.05$ in Fig. 12 with the impulse sequences in Fig. 3 and the adaptation behavior in Fig. 2, we conclude that this technical device offers the behavior of the biological paradigm.

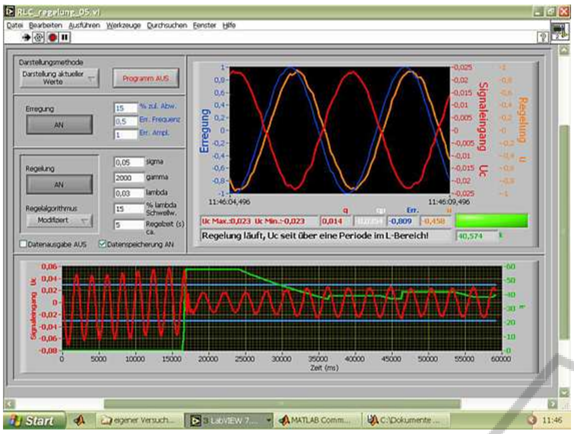

Figure 10: LabView front panel on PC screen, using adaptor (11); depicted curves in bottom window: capacity voltage $U_C(\cdot)$, i.e., new output $y(\cdot)$, (red line), λ-strip (blue lines), and gain parameter $k(\cdot)$ (green line).

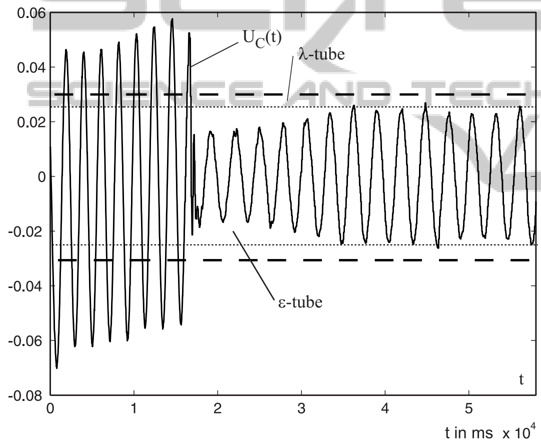

Figure 11: Output $U_C(\cdot)$ and tubes using Adaptor 5.

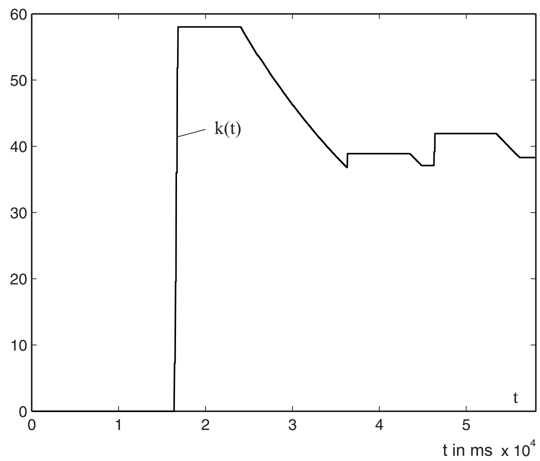

Figure 12: Gain parameter using Adaptor 5.

10 CONCLUSIONS

The development of new control strategies and sensor models was motivated by the open question which occurred during analysis of the functional morphology of vibrissal sensor systems. The vibrissa receptors are in a permanent state of adaption to filter the perception of tactile stimuli. This behavior now may be mimicked by the artificial sensor system. The sensor system was modeled as a spring-mass-damper system with relative degree two and the system parameters are supposed to be unknown, due to the complexity of biological systems. Using a simple linear model of a sensory system, adaptive controllers have been considered which compensate unknown permanent ground excitations. Classical adaptors suffer from a monotonic increase of the control gain parameter, thereby possibly paralyzing the sensor's capability to detect future extraordinary excitations. The existing adaptive controllers from literature were improved with respect to performance, sensitivity and capabilities. Various modifications of existing controllers are made and new controller designs were discussed:

- tuning parameters γ and gain exponent p increase the growth rate of the gain parameter k;
- new adaptors allow for gain parameter decrease that improves the sensor system's sensitivity to further ground excitations;
- a smaller $\varepsilon\lambda$-tube is introduced to prevent the output y from leaving the λ-neighborhood.

These proposed and modified adaptors avoid the drawbacks from literature and do not invoke any estimation or identification techniques. The working principle of the new controller (feedback law including Adaptor 5) is shown in a numerical simulation which proves that this controller in fact works successfully and effectively. This controller is simple in its design: its adaptation law is not complex as current adaptive control strategies in literature. Moreover, a practical implementation of this controller to a demonstrator in form of an electrical oscillating circuit results in a successful experiment which confirms the theoretical results. However, the preceding simulation results shed some light upon the behavior of the sensor system under the governance of various adaptive controllers. It becomes clear that both the choice of controller type and the tuning of the chosen controller (optimize controller data) is a delicate task. Simulation and experiment show that the developed adaptive control strategy applied to the mechanical sensor system achieve the fast adapting behavior of the biological receptor.

There are various aspects of adaptive controllers that can be improved and modified in current and future work on the subject.

- **Avoidance of Error Derivative**
  Since the derivative term is difficult to measure, it might be possible to implement an observer in order to estimate the system state. If this is also not possible, it would be best to omit the derivative term. This would avoid the occurrence of noise in the feedback loop, see (Behn, 2011). However, such a term is imperative to achieve stability. Thus, methods of not having to differentiate the system output can be investigated.
  One possibility to do so is the control with output delay feedback. With delay feedback, the output derivative is approximated by computing a difference quotient with a fixed time span:

$$\dot{y} \approx \frac{y(t) - y(t-h)}{h}$$

  with $h > 0$.
  This method computes a value for $\dot{y}$, which is not exact, but might be sufficiently approximated if h is chosen sufficiently small. However, there remains an error in the derivative feedback term.
  Current investigations on this topic are done.

- **Constrained Control Input**
  In technical realizations of controllers, there usually exists a limit for the control value that cannot be exceeded. This is quite obvious, as there are no actuators that can generate an infinite force, for example. Therefore, some adaptive controllers may not be implemented in certain applications, as they rely on the possibility to increase the control value as high as necessary. In order to cope with this issue, controllers with constrained input values might be investigated.

- **Intelligent Control**
  The fuzzy controller is built upon expert knowledge that is used to form the rule set of the controller. This knowledge is not given a priori and is obtained from experimenting with previous controllers. However, it is possible to generate expert knowledge by using intelligent control methods, such as artificial neural nets. The expert knowledge – or “intelligence” – in a neural net is obtained by training the controller with data generated by the system. However, this training process takes time and therefore diminishes the adaptive capabilities of the controller.

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