A Lagrangian Relaxation based Heuristic for the Static Berth Allocation Problem using the Cutting Plane Method

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Abstract: One of the important seaside operations problems that received a lot of attention in the literature is the assignment of quay space and service time to vessels that have to be unloaded and loaded at a terminal. This problem is commonly referred to as the Berth Allocation Problem (BAP). Different approaches exist in the literature for the berth allocation problem (BAP). Some of those approaches consider static arrival of vessels, so called the static berth allocation problem (SBAP), while other approaches consider dynamic arrival of vessels, called the dynamic berth allocation problem (DBAP). Approaches also differ in the layout used for the quay. In this paper we study one of the SBAP models presented in literature. Since the SBAP is a non-deterministic polynomial-time (NP) problem, we applied a Lagrangian Relaxation heuristic technique with the application of cutting plane method on our problem. We coded the cutting plane method in Matlab, and ran it on different instances of the problem. In most of the cases that we studied, our solution technique converged to an optimal solution.

1 INTRODUCTION

We consider the problem of allocating berth space for vessels in container terminals, which is known as *the berth allocation problem* (BAP). The vital need for efficient berth scheduling is stimulated by the fact that the cost of constructing a berth is considered very high compared to the costs of other facilities in container terminals. Hence, berth is considered as the most critical source for determining the capacity of container terminals. Planners in container terminals usually construct a Berth schedule which shows the berthing position, the arrival time, as well as the handling time of each vessel to be serviced at that berth.

In berth scheduling problem, vessels arrive over time at a port and the terminal operator assigns them to berths for unloading and loading of containers based on several factors and considerations: (i) the discrete, continuous and hybrid berthing space, and (ii) the static versus dynamic vessel arrivals. The Static Berth Allocation Problem (SBAP) is the kind of problems when it is assumed that vessels arrive before berth allocation is planned, while in the Dynamic Berth Allocation Problem (DBAP) vessels can arrive before or after allocation plan is made.

In discrete layout, the quay is divided into separated berths, and a berth can be assigned to only one vessel at a time, while in continuous layout, it's assumed that the quay is not divided, and vessels can berth at any location within the boundary of the quay. Finally in the hybrid layout, the quay is divided into berths as in discrete, but the difference between the two is that small vessels can share one berth and large vessels may be assigned more than one berth.

Imai et al. (1997) studied the discrete SBAP (Imai et al., 1997). Berth allocation was planned with respect to minimum waiting and handling time of the vessels in addition to the deviation between the arrival order of vessels and the service order. It was also assumed that the handling time of a vessel depends on the berth i.e. a vessel has different handling times on different berths. The problem is then reduced to a classical assignment problem. Imai et al. (2001) (Imai et al., 2001) presented another formulation of discrete SBAP where planning was done only with respect to waiting and handling time of vessels. They presented a Lagrangian relaxation based heuristic to solve the problem.

Hansen and Oguz (2003) (Hansen and Oguz, 2003) presented a more compact Mixed Integer

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Program (MIP) formulation for the SBAP. In 2006, Lee et al. (Lee et al., 2006) presented a model for a discrete SBAP considering the minimization of waiting and handling time of vessels only. They assumed that the handling time of a vessel depends on its berthing position. Moreover, they assumed that the handling time depends on the Quay Crane (QC) operation schedule.

A Genetic Algorithm (GA) based heuristic was also used by Imai et al. (2008b) (Imai et al., 2008) for the minimization of the weighted number of vessel rejections. A vessel is rejected if it cannot be serviced without exceeding the due date, represented by the maximum acceptable waiting time.

In this paper we are proposing Lagrangian relaxation using the cutting plane method to the SBAP model that was presented by Imai et al. (2001). In the following section we present Imai et al. (2001) formulation, and in section 3, we discuss the solution methodology used. Numerical results are presented in section 4 and we finally finish with conclusion in section 5.

2 FORMULATION

Imai et al. assumed that all berthed vessels are ready to be serviced in a port when a berth schedule is constructed. This means that any vessel-berth-order assignment combination is considered feasible. However, since we are considering discrete SBAP, only one vessel can be serviced at a berth at a time.

Handling time of a vessel was assumed to be deterministic and dependent on the berth, i.e. a vessel may have different handling times if serviced on different berths. This assumption is valid and reasonable because in public berthing, the schedule is usually determined before the arrival of the vessels at the berth. At the same time, the containers that are to be loaded onto the vessels may arrive after the berth schedule is decided. Therefore, the time it takes to load a container onto a vessel depends on the berthing position of that vessel. Consequently, the handling time of a vessel at a specific berth, which is the total handling time of all the containers, depends on the location of the berth in the quay.

In addition, no technical, physical or draft restrictions were considered in their study. In other words, it is assumed that any vessel can be serviced at any berth, without considering the length, width or height of the vessel or other factors like depth of water and location of a berth in the quay.

The objective function of the model is to minimize the summation of waiting and handling time of the vessels that are to be serviced at a port. They assumed that waiting time of a vessel is the time between the arrival of a vessel at the terminal and the vessel service starting time. Handling time was assumed to be the time a vessel spends at a berth to be serviced.

Below is the formulation that was proposed by Imai et al. for SBAP.

Objective function is:

$$\mathrm{Min} \sum_{i \in B} \sum_{j \in V} \sum_{k \in O} \{(T - k + 1)C_{ij} + S_i - A_j\} x_{ijk} \qquad (1)$$

Subject to

$$\textstyle\sum_{i \in B} \sum_{k \in O} x_{ijk} = 1 \quad \forall j \in V \qquad (2)$$

$$\sum_{j \in V} x_{ijk} \leq 1 \quad \forall i \in B,\ k \in O, \qquad (3)$$

$$x_{ijk} \in \{0,1\} \quad \forall i \in B,\ \forall j \in V, \forall k \in O, \qquad (4)$$

Where

$i\ (=1,\ldots,I) \in B$	Set of berths
$j\ (=1,\ldots,T) \in V$	Set of vessels
$k\ (=1,\ldots,T) \in O$	Set of service orders
S_i	Time when berth i becomes idle for berth allocation planning
A_j	Arrival time of vessel j
C_{ij}	Handling time spent by vessel j at berth i
x_{ijk}	1 if vessel j is serviced as the kth vessel at berth i, and 0 otherwise

The objective (1) minimizes the sum of waiting and handling time for every vessel. Constraint (2) ensures that every vessel must be serviced. Constraint (3) states that at each berth, at most one vessel can be serviced at any time.

3 SOLUTION METHODOLOGY

3.1 Lagrangian Relaxation

Lagrangian relaxation heuristic techniques have recently emerged as a practical approach for complex assignment problems, as it can obtain near optimal assignments with quantifiable quality in a reasonable computation time for practical assignment problems. The idea behind relaxation is to relax "difficult constraints" and penalize them in the objective function in order to get a problem that can be solved 'easily'. The Lagrangian problem provides us with a lower bound (LB) (for a minimization problem) of the optimal value of the original problem.

In this paper we are applying Lagrangian relaxation to the problem proposed by Imai which is classified as a nondeterministic polynomial time (NP) problem to the set of constraints (3). This leads to the following formulation:

$$\text{Min} \sum_{i\in B}\sum_{j\in V}\sum_{k\in O}\{(T-k+1)C_{ij}+S_i-A_j\}x_{ijk} + \sum_{i\in B}\sum_{k\in O}\mu_{ik}(\sum_{j\in V}x_{ijk}-1) \quad (5)$$

Subject to the following constraints

$$\sum_{i\in B}\sum_{k\in O}x_{ijk}=1 \quad \forall j\in V, \quad (6)$$

$$x_{ijk}\in\{0,1\} \quad \forall i\in B, j\in V, k\in O \quad (7)$$

Where μ_{ik} is the Lagrangian multiplier for every berth i in the order k. The above objective function after Lagrangian relaxation can be written as:

$$\text{Min}\sum_{i\in B}\sum_{j\in V}\sum_{k\in O}\{(T\text{-}k+1)C_{ij}+S_i\text{-}A_j+\mu_{ik}\}x_{ijk} - \sum_{i\in B}\sum_{k\in O}\mu_{ik} \quad (8)$$

Excluding the term $\sum_{i\in B}\sum_{k\in O}\mu_{ik}$ from the objective function, we get the following sub-problem which we denote SP:

$$[\text{SP}]\quad \text{Min}\sum_{i\in B}\sum_{j\in V}\sum_{k\in O}\{(T\text{-}k+1)C_{ij}+S_i\text{-}A_j+\mu_{ik}\}x_{ijk}$$

Subject to the constraints:

$$\sum_{i\in B}\sum_{k\in O}x_{ijk}=1 \quad \forall j\in V, \quad (9)$$

$$x_{ijk}\in\{0,1\} \quad \forall i\in B, j\in V, k\in O \quad (10)$$

Substituting values for $\mu_{ik}, \forall i\in B, j\in V, k\in O$, and solving the previous SP, we get a solution and an objective function value of the SP which we call Zsp. Thus, the lower bound of any problem is:

$$\text{LB} = \text{Zsp} - \sum_{i\in B}\sum_{k\in O}\mu_{ik}$$

3.2 Cutting Plane Method

Our goal now is to find the sharpest bound which can be obtained by solving the following problem:

$$\text{Max LB} = \text{Zsp} - \sum_{i\in B}\sum_{k\in O}\mu_{ik}$$

Subject to

$$\mu_{ik}\geq 0, \qquad \forall i\in B, \forall k\in O$$

This problem is called the Lagrangian Dual. In order to solve the Lagrangian Dual, we should compute good multipliers (μ_{ik}). Thus, we use the cutting plane method which is based on the idea that the SP can be solved by generating all solutions that satisfy constraints (5) and (6) in the form of $[x_{111}^{h=1}, x_{112}^{h=1}, \ldots x_{ijk}^{h=1}]$, where $h \in H$ and H is the set of all feasible solutions satisfying constraint (2) and (4). In order to calculate good Lagrangian multipliers, a general purpose procedure called the sub-gradient method is sometimes used because it is easy to implement. However, it does not prove optimality (Fisher, 1985).

Instead, there is a smarter method to calculate good Lagrangian multipliers called the cutting plane method, which is more problem-specific. In this paper, we are using the cutting plane method as shown below.

Zsp can be written as:

$$\text{Zsp} = \min\{\sum_{i\in B}\sum_{j\in V}\sum_{k\in O}\{(T\text{-}k+1)C_{ij}+S_i\text{-}A_j+\mu_{ik}\}\, x_{ijk}^{h}$$

$, \forall h\in H$

Hence, the best lower bound is:

$$\text{Max}\{\text{Min}\{\sum_{i\in B}\sum_{j\in V}\sum_{k\in O}\{(T\text{-}k+1)C_{ij}+S_i\text{-}A_j+\mu_{ik}\}\, x_{ijk}^{h} - \sum_{i\in B}\sum_{k\in O}\mu_{ik}\}, \qquad \forall h\in H$$

Subject to constraint (8)

Let us define a variable θ where

$$\theta = \min\{\sum_{i\in B}\sum_{j\in V}\sum_{k\in O}\{(T\text{-}k+1)C_{ij}+S_i\text{-}A_j+\mu_{ik}\}\, x_{ijk}^{h}\ \forall h\in H$$

This implies:

$$\theta \leq \sum_{i\in B}\sum_{j\in V}\sum_{k\in O}\{(T\text{-}k+1)C_{ij}+S_i\text{-}A_j+\mu_{ik}\}\, x_{ijk}^{h} \qquad \forall h\in H$$

This can be written as:

$$\theta - \sum_{i\in B}\sum_{k\in O}\mu_{ik}\sum_{j\in V}x_{ijk}^{h} \leq \sum_{i\in B}\sum_{j\in V}\sum_{k\in O}\{(T\text{-}k+1)C_{ij}+S_i\text{-}A_j\}\, x_{ijk}^{h}$$

$\forall h\in H$

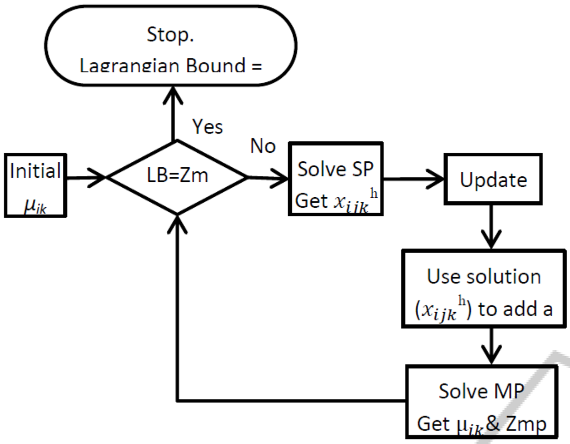


Figure 1: Cutting Plane Method Flowchart.

Accordingly, the Lagrangian Dual problem is:
[MP]

$$\text{Max } \theta - \sum_{i\in B}\sum_{k\in O}\mu_{ik}$$

Subject to

$$\theta - \sum_{i\in B}\sum_{k\in O}\mu_{ik}\sum_{j\in V}x_{ijk}^{h} \leq \sum_{i\in B}\sum_{j\in V}\sum_{k\in O}\{(T-k+1)C_{ij}+S_i-A_j\}\ x_{ijk}^{h}$$

$\forall h\in H$

The above problem is called the master problem, denoted MP, and the solution of the problem is denoted Zmp. The cutting plane method works as the following: we start with values of $\mu_{ik}\geq 0$. As long as the lower bound does not equal Zmp, we solve [SP] to get a solution ($x_{ijk}{}^{h}$) and a lower bound, the new LB is the minimum of the previous LB and the new one. Then we use this solution ($x_{ijk}{}^{h}$) to generate a constraint to [MP]. Next, we solve the [MP] to get solution of $\mu_{ik}$and a Zmp. After that, we return back to [SP] with the resulting μ_{ik} and solve it again. The iteration between the [SP] and [MP] continues till LB becomes equal to Zmp. This value is called the Lagrangian bound which is the best lower bound.

4 NUMERICAL EXPERIMENTS

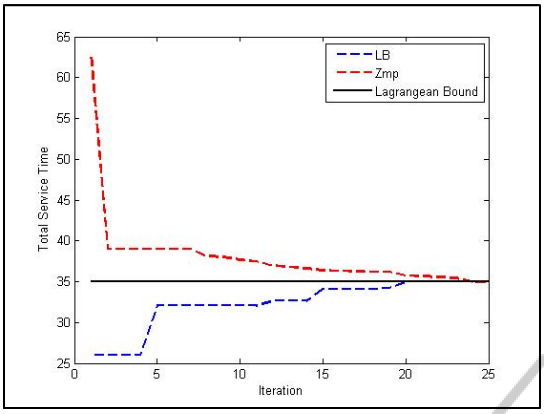
The cutting plane method was coded in Matlab on a Dell Latitude E6420, 2.60 Ghz machine. We applied the method on the small problems (berths x vessels): 3x5, 5x10, 7x20, 10x30, 15x35, 7x40, 13x40, 20x40 and 13x50 with values for the parameters generated randomly yet reasonably. The reason why we selected small problems is because for those problems, solvers of General Algebraic Modeling System (GAMS) could find a solution, and this allows us to test the accuracy of our solution method. However, in big problems, GAMS may not be able to solve, or it may take a very long time to converge, and that's when our solution becomes the most useful. For the previous problems, we recorded the CPU computational time, number of iterations, gap from the optimal value and the Lagrangian bound after termination.

Table 1: Numerical Results of Lagrangian Relaxation.

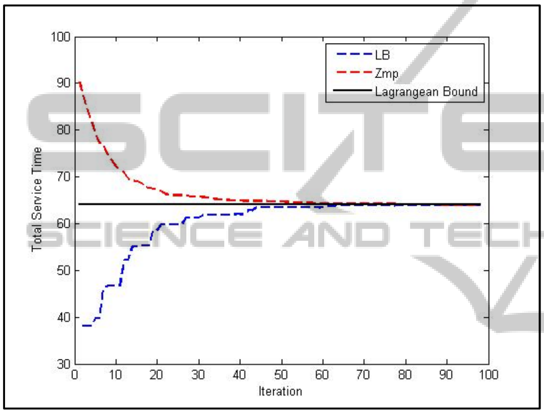
Problem Size(berths x vessels)	Lagrangian Bound	Gap[1] (%)	Iteration	Time(s)
3x5	16	0.0	9	0.7
5x10	35	0.0	25	2.4
7x20	64	0.0	98	34.4
10x30	70	1.4	89	88
7x40	182	0.0	198	284.9
15x35	66	0.0	110	219.4
13x40	97	0.0	246	591.7
20x40	65	0.0	225	219.4
13x50	134	0.0	188	772.7

The numerical experiments show that the Lagrangian relaxation gives precise results. In figures 2.a, 2.b and 2.c we show the behavior of our solution method and the convergence of LB and Zmp for the problems 5-berthx10-vessel, 7-berthx20-vessel and 15-berthx35-vessel respectively. It was found that the optimal solution of the relaxed version for each of these instances is optimal to the original problem.

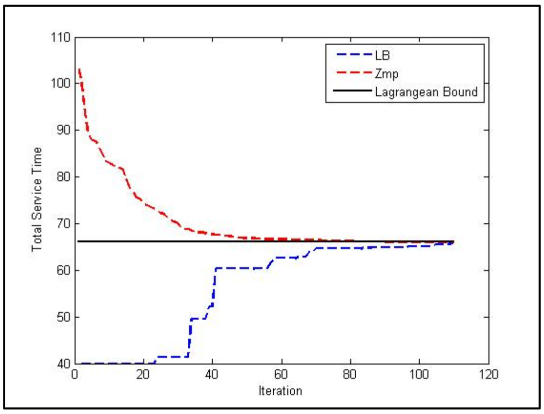
[1] Gap is calculated based on the optimal solution



(a) 5-berth x 10-vessel.



(b) 7-berth x 20-vessel.



(c) 15-berth x 35-vessel.

Figure 2: Convergence of Sub-problems and Master Problem.

The figures above shows the strength of the cutting plane method as it ensures the convergence between the sub-problem (SP) and the master problem (MP).

5 CONCLUSIONS

In this paper, we studied the Static Berth Allocation Problem (SBAP) model proposed by Imai et. al (2001). The SBAP cannot be solved in a polynomial time, therefore we developed a Lagrangian relaxation heuristic with the application of cutting plane method to solve the problem.

The cutting plane method was implemented on 9 instances of the SBAP to solve the relaxed problems, and it was found that in eight of the problems that we tested, the method was able to reach the exact optimal solution, while it gave a near optimal solution with a very small gap in one case only.

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