Adaptive Deployment of a Mobile Sensors Network to Optimize the Monitoring of a Phenomenon Governed by Partial Differential Equations

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Abstract: This project is intended to develop a comprehensive methodology (theory and numerical methods) in order to achieve an optimal design of experiments in the context of nonlinear ill posed problems related to the evaluation of parameters in systems described by partial differential equations (PDE). An experimental prototype will be developed in order to validate the performance of different strategies to identify location of one (or more) heating source using a set of mobile sensors.

1 STAGE OF THE RESEARCH

This project is intended to develop a comprehensive methodology (theory and numerical methods) in order to achieve an optimal design of experiments in the context of nonlinear ill posed problems related to the evaluation of parameters in systems described by partial differential equations (PDE). An experimental prototype will be developed in order to validate the performance of different strategies to identify location of one (or more) heating source using a set of mobile sensors.

2 OUTLINE OF OBJECTIVES

The protection of the environment and people requires the use of sensors to monitor the movement of mobile phenomena to predict and act on their evolution (ex: polluting cloud, fires, oil slick). These physical phenomena are often modeled by non-linear partial differential equations. Development of a predictive tool for the decision support requires the assessment of some input parameters.

In these cases, the sensors are generally expensive and in limited numbers. However, recent technological advances for communication systems and miniaturization will result in a cost reduction. Thus, it becomes possible to develop low-cost mobile systems and deploy a group of networked vehicles in a number of environments at risk. Our aim is to develop and validate optimal strategies to move a set of sensors for the parametric identification of PDE systems.

Three major objectives emerge from this research project:

- Define a set of methods to propose optimal strategies of mobile sensors movement dealing with conflicts of trajectories, the environmental constraints (no-go areas), duplication of information.
- Design, construction and validation of an experimental device to validate the deployments strategies using sensors embedded on mobile robots.
- Development and implementation of controls distributed to mobile robots so as to have a set of autonomous and intelligent vehicles without centralized control

3 RESEARCH PROBLEM

Disciplinary fields needed for the success of this project are varied: analysis of dynamical systems, robotics, and identification in thermal engineering. The trajectories of mobile sensors will be selected considering a set of points whose interest will be quantified online considering sensitivity functions. It

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comes to send sensors on the most relevant areas collecting information on target phenomenon. Strategies are defined by solving systems of partial differential equations modeling the dynamics of the phenomenon being studied. This resolution must be fast enough given the movement speed of the target (pollutants...), the time of acquisition of the sensors and the speed of the robots mobile media sensors.

4 STATE OF THE ART

The determination of models of dynamic systems is an essential step for the optimization of complex processes. Such problems typically involve systems of differential equations and are commonly used in chemical processes, robotics, electrical engineering, mechanical engineering, etc. However, the complex process control frequently requires models more accurate in which both the spatial dynamic and the temporal dynamic must be taken into account. Such systems are often called distributed parameters systems (DPS) and they are described by PDE (often non-linear and involving different phenomena). They are common for example in air quality control systems, management of groundwater resources, calibration of models in meteorology, oceanography or thermal engineering.

One of the fundamental questions in the study of the DPS is the determination of unknown parameters of the model from observed data of the real system. In such an aim, it is usual to develop a mathematical model and a numerical tool so that the predicted theoretical responses are closest as possible of those of the real system collected by appropriate sensors. A major difficulty is that it is difficult to observe the variables of interest of the process on the whole space. The question then arises of the optimal placement of sensors which allow a reconstruction as relevant as possible to the state of the process. In addition, most of the possible locations for the sensors is rarely specified in the design. Finally, observations are tainted with inaccuracy due to the acquisition chain as well as the noisy environment. All the above-mentioned points make this issue particularly attractive. The location of sensors is not necessarily dictated by physical considerations or by intuition and, therefore, systematic approaches should be developed to reduce the cost of instrumentation and increase the efficiency of estimators.

Although the requirement for systematic methods has been widely recognized, most of the techniques available in the literature are based on a comprehensive search from a set of pre-determined points. This approach is possible when the number of measurements is relatively low, but becomes quickly inadequate to more complex situations. Adopted optimization criteria are generally based on the Fisher Information Matrix (FIM) associated to the unknown considered parameters. The idea is to express the validity of the estimated parameters considering the covariance matrix of the evaluations. To identify optimal sensor placements, it is assumed that an unbiased estimator is implemented. This leads to a great simplification since Cramér-Rao limit of the covariance matrix is the inverse of the FIM, which can be calculated relatively easily, although the exact covariance of a given estimator matrix is difficult to obtain. Fedorov has directed works based on this approach in the early 1970s. This methodology has been considerably developed to extend it to various application fields. An comprehensive treatment of both theoretical and numerical aspects of the resulting sensor placement strategies is presented in (Ucinski, 2005).

To evaluate the parameters, the maximum likelihood (ML) estimator can be used. Due to the nonlinear nature inherent in this optimization, specific numerical techniques should be used. In addition, when the number of parameters to evaluate is important, the evaluation problem is ill-posed in the sense that measurement noise can cause significant variations in the estimated parameters and does not ensure the uniqueness. In this context, known techniques have been developed such regularization methods (Tikhonov-Phillips). While ill posed character of this type of problem is common in many industrial processes, systematic design of experimental conditions ensuring an optimal observation has received very little attention so far. Generally, existing approaches adopt an ideal perspective ignoring the ill-posed nature. Then, they could provide reasonable designs in some situations. However, they lead in general to non-optimal experimental solutions that can in some cases prove to be false qualitatively. This gap between theory and practice for the optimum placement of sensors is the main motivation of this research project

Different works allowed to propose paths of sensors (ensuring a continuous spatial scan for example). In the latter case even if the complexity of the resulting optimization problem is larger, it may be interesting that sensors are able to track the points that provide the most relevant information at any given time. Therefore, by reconfiguring in real time a sensor system (moving) we can expect to obtain an optimality criterion better than that of the stationary case.

5 METHODOLOGY

The thermal context that allows studying PDE's parabolic types possibly non-linear has been selected for the study of this research project. Numerous studies have been performed in one dimensional geometry (Silva Neto and Özisik, 1994),(Yi and Murio, 2002),(Hasanov, 2012) as well as in two dimensional domain (Khachfe and Jarny, 2000), (Ling et al., 2006) and (Yang, 2006).

5.1 Direct Problem Formulation

The studied domain is a thin metallic square plate. Let us consider that thickness e is quite small and that temperature gradients versus the thickness are neglectable. The studied geometry is denoted by $\Omega=\left\{(x,y)\in[0,L]^2\right\}$ and time variable is denoted by $t\in T=\left[0,T_f\right]$, T_f is the final time. Several heat sources S_j $\left(j=1,\cdots,N_s\right)$ move on the surface of the plate. For each source, the density flux $\phi_j(t)$ is assumed to be uniform on a disk $D_j\left(I_j(t),r\right)$ with (centre $I_j(t)=\left(x_j(t),y_j(t)\right)$ and radius of a few centimeters. The total heating flux can be expressed by:

$$\Phi(x,y;t)=\begin{cases}\sum_{j=1}^{N_s}\phi_j(t) & \text{if } (x,y)\in D_j\\ 0 & \text{otherwise}\end{cases} \tag{1}$$

To describe the heat flux in continuous and differentiable manner, spatial regularization is considered:

$$\Phi(.)=\sum_j\frac{\phi_j(t)}{\pi}\left(-\text{atan}\left(\mu\left(\chi_j(.)-r\right)\right)+\frac{\pi}{2}\right) \tag{2}$$

where $\chi_j(x,y,t)=\sqrt{\left(x-x_j(t)\right)^2+\left(y-y_j(t)\right)^2}$.

The regularization parameter μ was chosen so as to accurately describe the discontinuity at each heating disk boundary. The time interval is divided into segments as follows: $\left[0,t_f\right]=\bigcup_{i=0}^{N_t}\left[t_i,t_{i+1}\right]$, with time $t_i=\tau i$ and step time $\tau=\frac{t_f}{N_t+1}$. Discretization of the heating flux is also proposed according to continuous linear piecewise function:

$$\xi_i(t)=\begin{cases}\frac{t}{\tau}-i+1 & \text{if } t\in\left[t_{i-1},t_i\right]\\ -\frac{t}{\tau}+i+1 & \text{if } t\in\left[t_i,t_{i+1}\right]\\ 0 & \text{otherwise}\end{cases}$$

Trajectories and heat fluxes are thus parameterized as follows:

$$x_j(t)=\sum_{i=1}^{N_t}x_i^j\xi_i(t)=\overline{x_j}\;\overline{\xi(t)}^{tr}$$

$$y_j(t)=\sum_{i=1}^{N_t}y_i^j\xi_i(t)=\overline{y_j}\;\overline{\xi(t)}^{tr}$$

$$\phi_j(t)=\sum_{i=1}^{N_t}\phi_i^j\xi_i(t)=\overline{\phi_j}\;\overline{\xi(t)}^{tr}$$

The spatio-temporal distribution of temperature $\theta(x,y;t)$ within the domain Ω is solution of the following system of partial derivatives equations:

$$\begin{gathered}\forall(x,y;t)\in\Omega\times\left[0,T_f\right]\\ \rho c\frac{\partial\theta(.)}{\partial t}-\lambda\Delta\theta(.)=\frac{\Phi(.)-2h\left(\theta(.)-\theta_0\right)}{e}\\ \forall(x,y)\in\Omega\\ \theta(x,y;0)=\theta_0\\ \forall(x,y;t)\in\partial\Omega\times\left[0,T_f\right]\qquad -\lambda\frac{\partial\theta(.)}{\partial\vec{n}}=0\end{gathered} \tag{3}$$

where $\rho c\left(\text{J.m}^{-3}.\text{K}^{-1}\right)$ is the volumetric heat capacity, $\lambda\left(\text{W.m}^{-1}.\text{K}^{-1}\right)$ the thermal conductivity, $h\left(\text{W.m}^{-2}.\text{K}^{-1}\right)$ the convective exchange coefficient, $\theta_0\ (\text{K})$ the initial temperature equal to the ambient one. When all parameters are known, temperature evolutions in the plate are predicted considering the numerical resolution of the previous direct problem using the Comsol ® interfaced Matlab © software.

5.2 Inverse Problem

To identify the successive positions of the centers $\bar{I}=\left\{\bar{x}_j,\bar{y}_j\right\}_{j=1,\cdots,N_s}$ as well as the heating flux $\bar{\phi}=\left\{\bar{\phi}_j\right\}_{j=1,\cdots,N_s}$ of each of j sources from the observations provided by N_c mobile sensors C_p, an inverse problem is solved by minimizing the quadratic criterion between calculated $\theta\left(C_p,t;\overline{I},\overline{\phi}\right)$ and measured temperatures: $\hat{\theta}_p(t)$

$$J\left(\theta,\overline{I},\overline{\phi}\right)=\frac{1}{2}\sum_p\int_0^{T_f}\left(\theta\left(C_p,t;\overline{I},\overline{\phi}\right)-\hat{\theta}_p(t)\right)^2 dt \qquad (4)$$

Considering that initial positions of the sources are known: $\left\{x_j(0),y_j(0)\right\}$, a method of iterative regularization based on Conjugate gradient algorithm is implemented (Perez et al., 2008). The resolution algorithm requires iterative resolution of three well problems in the Hadamard's sense (Alifanov et al., 1995) and (Tarantola, 2005):

- The direct problem to calculate the test and judge the quality of the estimate.
- The adjoint problem to calculate the gradient of the test and thus to define the next direction of descent.
- The sensitivity problem to calculate the depth of descent (in the direction of descent).

The crucial steps that are the resolution of the sensitivity problem and the computation of the gradient functional by the adjoint problem resolution are detailed hereafter (See (Huang and Wang, 1999), (Huang and Chen, 2000), (Autrique et al., 2005) and (Beddiaf, 2012) for examples related to parametric identification.

5.2.1 Sensitivity Problem

The temperature variation caused by variation in centres disks $\delta\overline{I}=\left\{\delta\overline{x}_j,\delta\overline{y}_j\right\}_{j=1,\cdots,N_s}$ and the heat fluxes $\delta\overline{\phi}=\left\{\delta\overline{\phi}_j\right\}_{j=1,\cdots,N_s}$ of each of the j sources is noted $\delta\theta(x,y;t)$ and is solution of the sensitivity problem:

$$\begin{gathered}
\forall(x,y;t)\in\Omega\times\left[0,T_f\right]\\
\rho c\frac{\partial\delta\theta(.)}{\partial t}-\lambda\Delta\delta\theta(.)=\frac{\delta\Phi(.)-2h\delta\theta(.)}{e}\\
\forall(x,y)\in\Omega\\
\delta\theta(x,y;0)=0\\
\forall(x,y;t)\in\partial\Omega\times\left[0,T_f\right]\\
-\lambda\frac{\partial\theta(.)}{\partial\vec{n}}=0
\end{gathered} \qquad (5)$$

with:

$$\begin{aligned}
\delta\Phi(x,y;t)=&\\
&-\frac{1}{\pi}\sum_j\left[\overline{\delta\phi_j\xi(t)}^{tr}\left(-\text{atan}\left(\mu\left(\varepsilon_j(.)-r\right)\right)+\frac{\pi}{2}\right)\right]\\
&-\frac{1}{\pi}\sum_j\left[\frac{\overline{\phi_j\xi(t)}^{tr}\mu\overline{\delta x_j\xi(t)}^{tr}\left(\overline{x_j\xi(t)}^{tr}-x\right)}{1+\left(\mu\left(\varepsilon_j(.)-r\right)\right)^2}\right]\\
&-\frac{1}{\pi}\sum_j\left[\frac{\overline{\phi_j\xi(t)}^{tr}\mu\overline{\delta y_j\xi(t)}^{tr}\left(\overline{y_j\xi(t)}^{tr}-y\right)}{1+\left(\mu\left(\varepsilon_j(.)-r\right)\right)^2}\right]
\end{aligned}$$

where:

$$\varepsilon_j(x,y,t)=\sqrt{\left(x-\overline{x}_j\overline{\xi(t)}^{tr}\right)^2+\left(y-\overline{y}_j\overline{\xi(t)}^{tr}\right)^2}\ .$$

Thus, at iteration $k+1$, the depth of descent γ^{k+1} in the direction of descent d^{k+1} can be expressed by : (Beddiaf, 2012).

$$\gamma^{k+1}=\frac{\int_0^{T_f}\sum_p E_p^k(t)\delta\theta_{d^{k+1}}\left(C_p,t;\overline{I}^k,\overline{\phi}^k\right)dt}{\int_0^{t_f}\sum_p\left(\delta\theta_{d^{k+1}}\left(C_p,t;\overline{I}^k,\overline{\phi}^k\right)\right)^2 dt} \qquad (6)$$

where $E_p^k(t)=\theta\left(C_p,t;\overline{I}^k,\overline{\phi}^k\right)-\hat{\theta}_p(t)$.

The sensitivity problem (5) has to be numerically solved at each iteration $k+1$ in the descent direction d^{k+1} in order to calculate the descent depth γ^{k+1} according to relation (6).

5.2.2 Adjoint Problem

A Lagrangian formulation $\ell(\theta,I,\psi)$ for the quadratic function minimization based on an adjoint function $\psi(x,y;t)$ is introduced in order to determine the functional gradient for each iteration of the minimization algorithm (Perez et al., 2008; Beddiaf et al., 2012; Rouquette et al., 2007):

$$\ell\left(\theta,\bar{I},\bar{\phi},\psi\right)=J\left(\theta,\bar{I},\bar{\phi}\right)$$
$$+\int_0^{t_f}\int_\Omega\left(\rho c\frac{\partial\theta}{\partial t}-\lambda\Delta\theta-\left(\frac{\Phi-2h\left(\theta-\theta_0\right)}{e}\right)\right)\psi\ dxdydt$$

Let us introduce $\delta_{C_p}(x,y)$ is the Dirac distribution at mobile sensors C_p, then :

$$\ell\left(\theta,\bar{I},\bar{\phi},\psi\right)=$$
$$\frac{1}{2}\int_0^{t_f}\int_\Omega\sum_p\left(\theta\left(.;\bar{I},\bar{\phi}\right)-\hat{\theta}_p(t)\right)^2\delta_{C_p}(x,y)\,dxdydt$$
$$+\int_0^{t_f}\int_\Omega\left(\rho c\frac{\partial\theta}{\partial t}-\lambda\Delta\theta-\frac{\Phi-2h\left(\theta-\theta_0\right)}{e}\right)\psi\,dxdydt$$

Thus,

$$\delta\ell\left(\theta,\bar{I},\bar{\phi},\psi\right)=\frac{\partial\ell}{\partial\theta}\delta\theta+\frac{\partial\ell}{\partial\bar{I}}\delta\bar{I}+\frac{\partial\ell}{\partial\bar{\phi}}\delta\bar{\phi}+\frac{\partial\ell}{\partial\psi}\delta\psi$$

If the temperature $\theta(x,y,t)$ is the solution of the direct problem (3) then $\ell\left(\theta,\bar{I},\bar{\phi},\psi\right)=J\left(\theta,\bar{I},\bar{\phi}\right)$ and $\delta\ell\left(\theta,\bar{I},\bar{\phi},\psi\right)=\delta J\left(\theta,\bar{I},\bar{\phi}\right)$. If the Lagrange multiplier $\psi(x,y,t)$ is fixed then $\frac{\partial\ell}{\partial\psi}\delta\psi=0$. Moreover $\psi(x,y,t)$ is fixed such that $\frac{\partial\ell}{\partial\theta}\delta\theta=0 \quad \forall\delta\theta$. Considering boundary conditions of the sensitivity problem $\psi(x,y,t)$ has to be solution of the adjoint problem:

$$\begin{aligned}
&\forall(x,y;t)\in\Omega\times\left[0,T_f\right]\\
&\rho c\frac{\partial\psi(.)}{\partial t}+\lambda\Delta\psi(.)=E(.)+\frac{2h}{e}\psi(.)\\
&\forall(x,y)\in\Omega\\
&\psi\left(x,y;T_f\right)=0\\
&\forall(x,y;t)\in\partial\Omega\times\left[0,T_f\right]\\
&-\lambda\frac{\partial\psi(.)}{\partial\vec{n}}=0
\end{aligned}\qquad(7)$$

with $E(x,y,t)=\sum_p\left(\theta(x,y;t)-\hat{\theta}_p(t)\right)\delta_{C_p}(x,y)$,

thus: $\delta\ell=-\int_0^{T_f}\int_\Omega\left(\frac{\partial\Phi}{\partial\bar{I}}\delta\bar{I}+\frac{\partial\Phi}{\partial\bar{\phi}}\delta\bar{\phi}\right)\frac{\psi(.)}{e}dxdydt$.

As $\delta\ell\left(\theta,\bar{I},\bar{\phi},\psi\right)=\delta J\left(\theta,\bar{I},\bar{\phi}\right)$ then the gradient of the criterion can be obtained from the resolution of (7) and the next descent direction can be calculated. Both adjoint and sensitivity problems can be solved at each iteration with the same numerical scheme as for the direct problem.

5.3 Observation Strategies

The previous paragraph has allowed to define the methodology of identification based on solving iteratively three well-posed problems. The success of this research project partially rely on the observations strategy and more specifically on the movement of mobile sensors. These displacements are planned from the two following approaches:

- Implementation of a *sliding horizon* for the identification based on the iterative regularization of the Conjugate gradient method, in order to update the trajectories of sensors based on estimates of the unknown parameters,
- Analysis of the evolution of sensitivity distributions obtained by iterative resolution of the sensitivity problem in order to define the new areas of interest for the process observations.

An example of such distributions is presented on figure 1: depending on the number of sensors and their previous positions, strategies of displacement can be proposed.

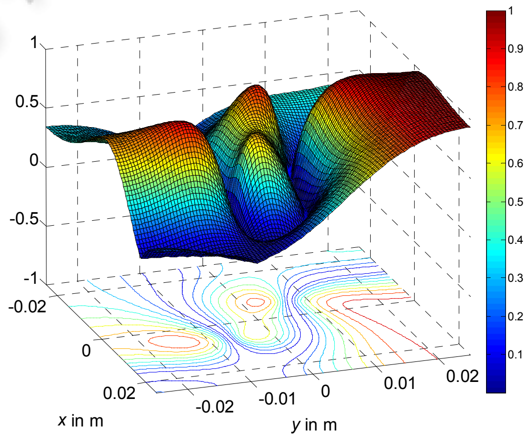

Figure 1: Example of spatial distribution of sensitivity (normalized).

5.4 Experimental Prototype

To assess the different strategies of positioning sensors and before considering the application on a large scale (ex: detection of pollutants), an experimental device is currently under development. In such a way, several heating sources embedded on mobile robots (Khepera III) evolve on a plane surface and provide the heating of a thin plane material (figure 2) on a surface of approximately 4 m². The time dependant heat flux of these heat sources can be controlled in order to reach temperatures for which the thermal properties of the material are thermo-dependent (introducing non-linearity).

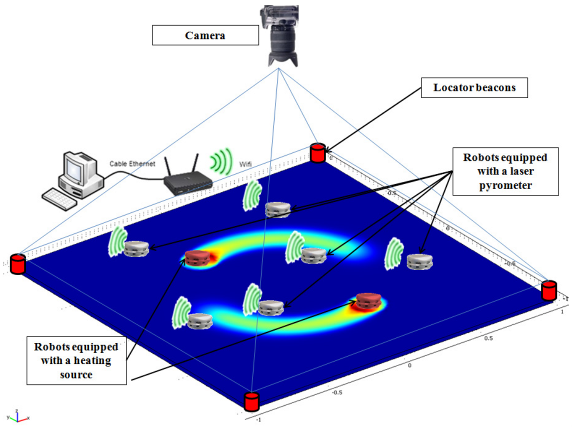

Figure 2: Representation of the experimental prototype.

On this same surface several mobile robots will be equipped with pyrometers laser (figure 3) to measure the temperature of the plate on a small area (a few mm2).Their locations and the measured temperature will be transmitted to a central computer via a wireless (WIFI) technology.

Figure 3: Khepera robot equipped with a Laser pyrometer.

In order to accurately measure the robots location on the material surface, a camera will be placed above the plate and then by image processing (Martinez-Gomez and Weitzenfel, 2004; Zickler et al., 2009; Wang et al., 2001) the positions of different robots will be returned to the computer that will synchronize the received measures of robots with their positions. This positioning by tag system comes in addition to the position data sent from the robots in order to take into account odometry errors that may be encountered while displacements.

6 EXPECTED OUTCOME

In this communication, the DARC-EDP project is presented as a whole. It deals with the deployment of mobile sensors to identify moving heating sources in the context of thermal engineering. Several points have been briefly addressed:

- modeling heat transfers and direct problem formulation
- inverse problem formulation,
- minimization by a descent method : iterative regularization based on conjugate gradient method (sensitivity problem and adjoint problem),
- observations strategies,
- design of the experimental prototype.

The prospects for these works consist of the confrontation of experimental campaigns with the previous numerical studies.

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