# Fuzzy Optimization Models for Seaside Port Logistics

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Keywords: Fuzzy Optimization Model, Seaside Operations, Maritime Container Terminal, Variable Neighbourhood Search.

Abstract: The main goal of maritime container terminals is to serve the container vessels arriving at port. This means that they must be berthed in a position along the quay, a subset of quay cranes must be assigned to them and work schedules have to be planned for unloading the import containers and loading the export containers onto each container vessel. This work addresses the Tactical Berth Allocation Problem, in which the vessels are assigned to a given berth, and the Quay Crane Scheduling Problem, for which the work schedules of the quay cranes are determined. Due to the fact that the nature of this environment gives rise to inaccurate knowledge about the information related to the incoming vessels, the aforementioned optimization problems are tackled considering fuzzy arrival times for the vessels and fuzzy processing times for the loading/unloading operations.

## 1 INTRODUCTION

The global container trade has grown over the decades[1]. The maritime container terminals are highlighted infrastructures built with the goal of facing the technical requirements arising from the increasing volume of containers in the international sea freight trade. They are aimed at transferring and storing containers within multimodal transportation networks. The main transport modes found at a maritime container terminal are container vessels, trucks and trains. In this regard, a maritime container terminal can be considered as an open system that brings together different container flows, those stemming from freight sources to destinations (Stahlbock and Voβ, 2008).

The layout of a maritime container terminal is usually split into three different functional areas: seaside, yard and landside (Petering, 2011). Firstly, the seaside is the area of the terminal where the container vessels arriving at port are berthed in order to be loaded or unloaded. An exhaustive analysis concerning the seaside operations planning problems is provided in the book by (Meisel, 2010). The yard is the part of the terminal in which the containers are temporarily stored until their later retrieval (Kim et al., 2008). Finally, the landside is the area which connects the container terminal with the land transport modes, that is, trains and trucks (Froyland et al., 2008).

The main goal of a maritime container terminal is to serve appropriately those container vessels that arrive at port. In this regard, the service of a container vessel can be modeled through a well-defined sequence of steps. Firstly, it is required to provide a specific berthing position along the quay and berthing time for each container vessel according to its particular characteristics (dimensions, expected service time, draft, arrival time, etc.) and contractual agreements (Lalla-Ruiz et al., 2012). Afterwards, a subset of the available quay cranes at the terminal is allocated to each container vessel for performing the loading and unloading tasks established by its stowage plan (Bierwirth and Meisel, 2010). Finally, the work schedules associated with the allocated quay cranes are determined (Bierwirth and Meisel, 2009).

The aforementioned planning decisions can be modeled by means of several optimization problems in maritime container terminals. The Tactical Berth Allocation Problem (TBAP) pursues to define the berthing position, berthing time and quay cranes allocated to each container vessel over a given planning horizon. On the other hand, the Quay Crane Scheduling Problem (QCSP) is aimed at determining the work schedules for the quay cranes allocated to a container vessel. It is worth mentioning that solving the TBAP

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Melián-Batista B., Expósito-Izquierdo C., Lalla-Ruiz E., Teresa Lamata M. and Moreno-Vega J..
Fuzzy Optimization Models for Seaside Port Logistics.
DOI: 10.5220/0004652202890299
In *Proceedings of the 5th International Joint Conference on Computational Intelligence* (SCA-2013), pages 289-299
ISBN: 978-989-8565-77-8

and the QCSP for each container vessel provides an overall service planning for a container terminal.

Algorithms and model formulations to solve the TBAP and QCSP have usually assumed that the data are known accurately. However, this is not true in real-world applications and it is particularly problematic for data representing the arrival times of vessels and processing times that cannot be precisely estimated, but that need to be taken into account in order to provide the decision makers with real solutions.

The inherent imprecision that appears in the data involved in real-world problems can have different natures: randomness, subjectivity, vagueness, etc. If the goal is to tackle the problems at hand without altering their nature, what seems more desirable is to consider the approach that fits the best to the origin of the imprecision, either stochastic, interval-based or fuzzy. Among these possibilities, the imprecision considered in this work is not random but linguistical and therefore vague (for instance, "the processing time will be large"). Although vague information appears in these real-world applications, to the best of our knowledge, this is the first work that considers this nature of imprecision in maritime container terminals when solving the TBAP and QCSP. Other imprecision natures are out of the scope of this paper.

In those scenarios in which subjectivity in the interpretation of the data is related to randomness, the fuzzy sets provide us a theoretical framework to solve a wide range of problems in different research areas (Tiwari et al., 2006) (Verdegay, 2003) with a high degree of efficacy and efficiency. In this paper, we are particularly interested in solving the TBAP with fuzzy arrival times for the vessels and the QCSP with fuzzy processing times for the loading/unloading operations. With the purpose of solving these problems, we propose mathematical models in which some coefficients in the constraints are not known accurately. In both cases, in order to solve the optimization problems derived from these situations, we will make use of models which are well known in the area of Fuzzy Mathematical Programming (Cadenas and Verdegay, 2006) (Herrera and Verdegay, 1995) (Sancho-Royo et al., 2006).

The remainder of this paper is structured as follows. Section 2 describes the main logistic problems arising in the seaside. Section 3 describes the fuzzy coefficients used in the mathematical formulations of the TBAP and QCSP. Section 4 proposes two Variable Neighbourhood Searches for the TBAP and QCSP. Section 5 shows the computational experiments performed in this work. Finally, Section 6 presents the main conclusions extracted from the work and indicates several directions for further research.

# 2 SEASIDE OPERATIONS

The seaside operations are those concerning the service of container vessels that arrive at port. As indicated by (Wiegmans et al., 2001), the turnaround time of container vessels constitutes the main indicator of the competitiveness of maritime container terminals. With this fact in mind, terminal managers are particularly interested in reducing the service times and maximizing the usage of the available resources: berths and quay cranes.

## 2.1 Tactical Berth Allocation Problem

The Tactical Berth Allocation Problem (TBAP) pursues to determine the berthing position, berthing time and allocation of quay cranes for the container vessels arriving at port over a well-defined time horizon.

In the TBAP we are given a set of incoming vessels $N$, a set of berths $M$, and a set of quay crane profiles $p \in P_i$ per each vessel $i \in N$. Each container vessel $i \in N$ must be assigned to an available berth $k \in M$ within the vessel and berth time window $[a_i, b_i]$ and $[a^k, b^k]$, respectively. The berthing position of a vessel should be close to the departure position of its containers. In this regard, the housekeeping cost represents the cost derived from moving a given container among different berthing positions of the quay. Moreover, for each vessel $i \in N$, a quay crane profile, $p \in P_i$, determines the distribution of quay cranes used for serving it. $Q$ denotes the number of quay cranes at the terminal. The service time of a vessel depends on the quay crane profile associated with it. Each profile, $p \in P_i$, has an associated value $v_i^p$, which reflects the usage of quay cranes.

The goal of the TBAP is to maximize the value of the quay crane profiles used to serve the vessels and minimize the housekeeping costs derived from the transshipment of containers among vessels.

An example of the TBAP is depicted in Figure 1. The example shows 3 container vessels, 3 berths and a maximum number of 8 quay cranes. For each container vessel $i \in N$, the number of QC hours required to perform its loading and unloading operations is termed as $m_i$. The profiles assigned to the vessels determine the distribution of quay cranes and their service time. For instance, the vessel 3 is served by 2, 3, 3, and 2 quay cranes and its service time is 4 hours. It is worth pointing out that, despite vessels 1 and 3 require the same number of quay crane hours, their service times are different because they have assigned different profiles. In this regard, the quay crane profile assigned to vessel 1 is more expensive than that assigned to vessel 3.

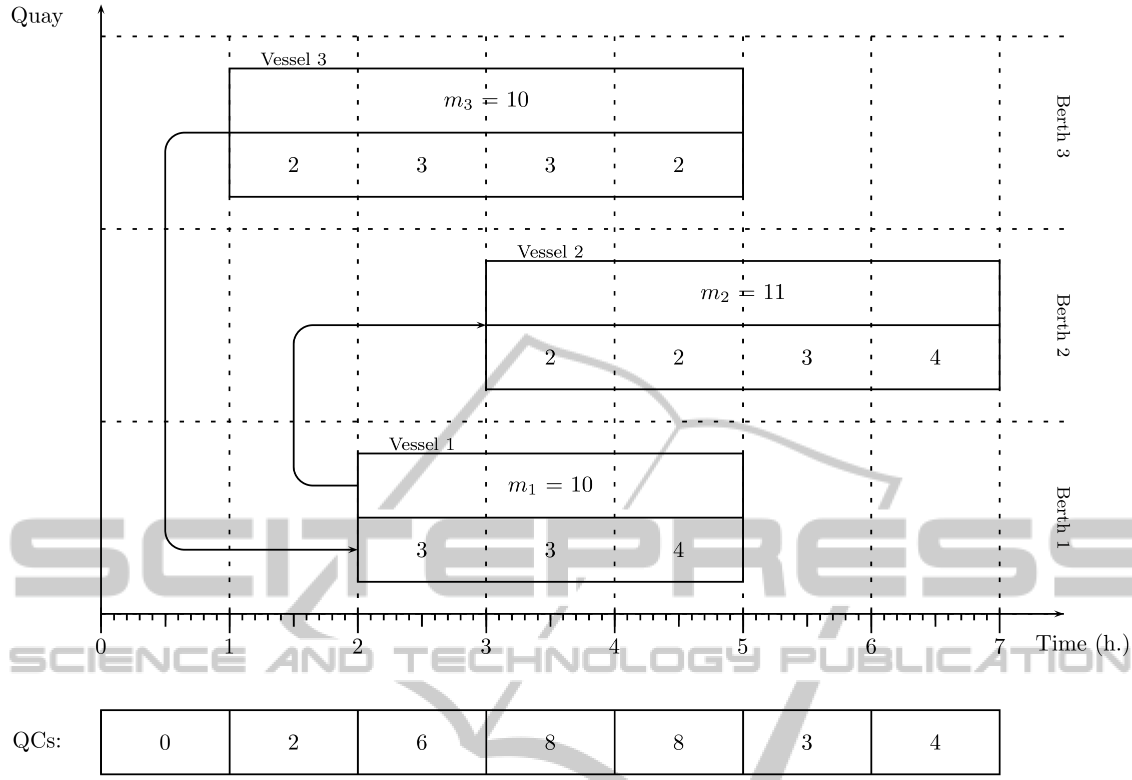



Figure 1: Example of TBAP with 3 container vessels and 3 berths.

In order to make this work self-contained, this subsection describes the mixed integer linear program formulation for the TBAP proposed by (Giallombardo et al., 2010).

The following notations are used in the model:

- $N$ Set of container vessels
- $M$ Set of berths
- $H$ Set of time steps
- $P_i$ Set of feasible quay crane profiles for the container vessel $i \in N$
- $t_i^p$ Service time of container vessel $i \in N$ under QC profile $p \in P_i$
- $v_i^p$ The value of serving the container vessel $i \in N$ with the QC profile $p \in P_i$
- $Q^h$ Maximum number of quay cranes available at the time step $h \in H$
- $q_i^{pu}$ Number of quay cranes assigned to the container vessel $i \in N$ in profile $p \in P_i$ at the time step $u \in H$
- $f_{ij}$ Flow of containers exchanged between container vessels $i, j \in N$
- $d_{kw}$ Housekeeping cost per unit of container between yard slots in berths $k, w \in M$
- $[a_i, b_i]$ [earliest, latest] arrival time of container vessel $i \in N$
- $[a^k, b^k]$ [start, end] of the availability time of the berth $k \in M$
- $[a^h, b^h]$ [start, end] of the time step $h \in H$

A graph $G^k = (V^k, A^k)\ \forall\ k \in M$, where $V^k = N \cup \{o(k), d(k)\}$, with $o(k)$ and $d(k)$ additional vertices representing berth $k$ and $A^k \subseteq V^k \times V^k$ is generated. The decision variables are shown below.

- $x_{ij}^k \in \{0,1\}, \forall\ k \in M, \forall\ (i,j) \in A^k$, set to 1 if container vessel $j$ is scheduled after container vessel $i$ in berth $k$, and 0 otherwise.
- $y_i^k \in \{0,1\}, \forall\ k \in M, \forall\ i \in N$, set to 1 if container vessel $i$ is assigned to berth $k$, and 0 otherwise.
- $z_{ij}^{kw} \in \{0,1\}, \forall\ k,w \in M, \forall\ i,j \in N$, set to 1 if $y_i^k = y_j^w$, and 0 otherwise.
- $\gamma_i^h \in \{0,1\}, \forall\ h \in H, \forall\ i \in N$, set to 1 if the container vessel $i$ arrives in time step $h$, and 0 otherwise.

- $\lambda_p^i \in \{0,1\}$, $\forall\, p \in P_i$, $\forall$ i$\in N$, set to 1 if container vessel $i$ is served under profile $p$, and 0 otherwise.
- $\rho_i^{ph} \in \{0,1\}$, $\forall\, p \in P_i$, $\forall\, h \in H$, $\forall$ i$\in N$, set to 1 if container vessel $i$ is served under profile $p$ and arrives at time step $h$, and 0 otherwise.
- $T_i^k \geq 0$, $\forall\, k \in M$, $\forall$ i$\in N$, berthing time of container vessel $i$ at berth $k$.
- $T_{o(k)}^k \geq 0$, $\forall\, k \in M$, $\forall$ i$\in N$, starting operation time of berth $k$.
- $T_{d(k)}^k \geq 0$, $\forall\, k \in M$, $\forall$ i$\in N$, ending operation time of berth $k$.

The corresponding MILP formulation for the TBAP is stated as follows:

$$\max \sum_{i\in N}\sum_{p\in P_i} \lambda_i^p \nu_i^p - \frac{1}{2}\sum_{i\in N}\sum_{j\in N}\sum_{k\in M}\sum_{w\in M} f_{ij} d_{kw} z_{ij}^{kw} \quad (1)$$

$$\sum_{k\in M} y_i^k = 1 \quad \forall i \in N \quad (2)$$

$$\sum_{j\in N\cup\{d(k)\}} x_{o(k)j}^k = 1 \quad \forall k \in M \quad (3)$$

$$\sum_{i\in N\cup\{o(k)\}} x_{id(k)}^k = 1 \quad \forall k \in M \quad (4)$$

$$\sum_{j\in N\cup\{d(k)\}} x_{ij}^k - \sum_{j\in N\cup\{o(k)\}} x_{ji}^k = 0 \quad \forall k \in M,\ \forall i \in N \quad (5)$$

$$\sum_{j\in N\cup\{d(k)\}} x_{ij}^k = y_i^k \quad \forall k \in M,\ \forall i \in N \quad (6)$$

$$T_i^k + \sum_{p\in P_i} t_i^p \lambda_i^p - T_j^k \leq (1 - x_{ij}^k)M1 \quad \forall k \in M \forall i \in N,\ \forall j \in N \cup \{d(k)\} \quad (7)$$

$$T_{o(k)}^k - T_j^k \leq (1 - x_{o(k)j}^k)M2 \quad \forall k \in M,\ \forall j \in N \quad (8)$$

$$a_i y_i^k \leq T_i^k \quad \forall k \in M, \forall i \in N \quad (9)$$

$$T_i^k \leq b_i y_i^k \quad \forall k \in M, \forall i \in N \quad (10)$$

$$a^k \leq T_{o(k)}^k \quad \forall k \in M \quad (11)$$

$$T_{d(k)}^k \leq b^k \quad \forall k \in M \quad (12)$$

$$\sum_{p\in P_i} \lambda_i^p = 1 \quad \forall i \in N \quad (13)$$

$$\sum_{h\in H^s} \gamma_h^i = \sum_{p\in P_i^s} \lambda_i^p \quad \forall i \in N,\ \forall s \in S \quad (14)$$

$$\sum_{k\in M} T_i^k - b^h \leq (1 - \gamma_i^h)M3 \quad \forall h \in H,\ \forall i \in N \quad (15)$$

$$a^h - \sum_{k\in M} T_i^k \leq (1 - \gamma_i^h)M4 \quad \forall h \in H,\ \forall i \in N \quad (16)$$

$$\rho_i^{ph} \geq \lambda_i^p + \gamma_i^h - 1 \quad \forall h \in H,\ \forall i \in N,\ \forall p \in P_i \quad (17)$$

$$\sum_{i\in N}\sum_{p\in P_i}\sum_{u=max(h-t_i^p+1;1)}^{h} \rho_i^{pu} q_i^{p(h-u+1)} \leq Q^h \quad \forall h \in H^{\bar{s}} \quad (18)$$

$$\sum_{k\in M}\sum_{w\in M} z_{ij}^{kw} = g_{ij} \quad \forall i,j \in N \quad (19)$$

$$z_{ij}^{kw} \leq y_i^k \quad \forall i,j \in N \ \ \forall k,w \in M \quad (20)$$

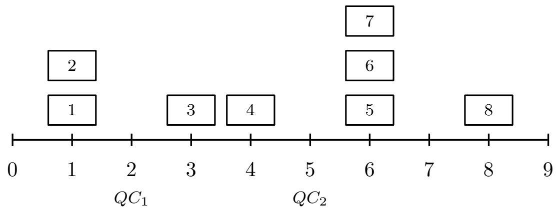
$$z_{ij}^{kw} \leq y_j^w \quad \forall i,j \in N \ \ \forall k,w \in M \quad (21)$$

In this model $M1$, $M2$, $M3$ and $M4$ represent sufficiently large constants. The objective function (1) maximizes the sum of the values of the chosen quay crane assignment profiles over all the container vessels and, at the same time, minimizes the housekeeping cost generated by the flows of containers exchanged between container vessels. Constraints (2) establish that every container vessel must be assigned to one and only one berth. Constraints (3) and (4) define the outcoming and incoming flows to the berths, whereas flow conservation for the remaining vertices is ensured by constraints (5). Constraints (6) establish the link between variables $x_{ij}^k$, whereas precedences in every sequence are ensured by constraints (7) and (8). The time windows of the container vessels are defined by the constraints (9) and (10), whereas berths time windows are defined by constraints (11) and (12). Constraints (13) ensure that one and only one QC profile is assigned to every container vessel. Constraints (14) define the link between variables $\gamma_i^h$ and $\lambda_i^p$, whereas constraints (15) and (16) link binary variables $\gamma_i^h$ and $T_i^k$. Variables $\rho_i^{ph}$ are linked to variables $\lambda_i^p$ and $\gamma_i^h$ by constraints (17). Finally, constraints (18) ensure that, at every time step, the total number of assigned QCs does not exceed the number of maximum QCs available in the terminal. Constraints (19), (20) and (21) are included to linearize the quadratic objective function.

## 2.2 Quay Crane Scheduling Problem

The QCSP pursues to define the sequences of transshipment operations performed by a set of quay cranes in order to load and unload the containers associated with a given vessel berthed at the container terminal.

The input data for the QCSP are composed by the set of tasks $\Omega = \{1,...,n\}$ and the set of quay cranes $Q = \{1,...,m\}$ allocated to the vessel. Each task $t \in \Omega$ represents a set of containers with similar characteristics (weight, dimensions, destination port, etc.) located adjacent to each other in the same bay, $l_t$. The processing time of the task $t \in \Omega$ is denoted by $p_t$. Two dummy tasks 0 and $T$ with $p_0 = p_T = 0$ are considered with the goal of representing the beginning




| Task, $t$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Position, $l_t$ | 1 | 1 | 3 | 4 | 6 | 6 | 6 | 8 |
| Processing Time, $p_t$ | 10 | 8 | 10 | 15 | 7 | 6 | 5 | 10 |

Figure 2: Example of QCSP instance composed of 8 tasks.

and the ending of the vessel service, respectively. In addition, we define the set $\bar{\Omega} = \Omega \cup \{0, T\}$. The structure of the vessel imposes limitations in the transshipment operations order (Kim and Park, 2004). For instance, unloading operations have to be performed before loading operations. The precedence relationships among tasks located in the same bay are defined by the set $\Phi$, in such a way that, $(i, j) \in \Phi$ if and only if task $i$ has to be finished before the starting of task $j$. On the other hand, each quay crane $q \in Q$ is located in the bay $l_0^q$ and it is available after time $r^q$. The time required by the quay crane $q \in Q$ to move between the bays in which the tasks $i, j \in \Omega$ are currently located is denoted by $t_{ij}^q$. For safety reasons, the quay cranes must keep a minimum distance between them, $\delta$, and measured in bay units. The safety distance gives rise to that same pairs of tasks cannot be performed simultaneously due to the fact that they are close. These pairs of tasks are gathered into the set $\Psi$.

The optimization criterion of the QCSP is to minimize the makespan of the schedule, that is, the finishing time of the last task performed by the quay cranes, $c_T$. The QCSP is already known to be an NP-hard problem (Sammarra et al., 2007).

An example of the QCSP is depicted in Figure 2. The example represents a vessel berthed at the quay with 10 bays, ranged from the bay 0 up to the bay 9, for which the bays 1, 3, 4, 6 and 8 have at least one task to perform by a quay crane. For each bay, the tasks are sorted according to their precedence relationships. For instance, the task 1 has to be performed before the starting of task 2 in the bay 1. The location and processing time of each task are reported in the associated table. A schedule with 2 quay cranes for this example is shown in Figure 3. The quay crane 1 performs the tasks 1, 2, 3, 5, 6 and 7, whereas the quay crane 2 performs the tasks 4 and 8. As can be seen, the quay cranes keep a safety distance of at least 2 bays and move with a speed of 1 bay per time unit. In this case, the makespan is 52 time units.

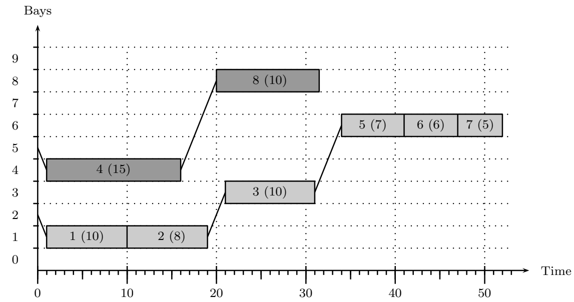



Figure 3: Schedule with 2 quay cranes for the example depicted in Figure 2.

In the following, we present the mathematical formulation proposed by (Bierwirth and Meisel, 2009) for the QCSP. The following notation is used by the formulation:

- $\Delta_{ij}^{vw}$. Minimum temporal span to elapse between the processing of the tasks $i$ and $j$ if they are processed by the quay cranes $v$ and $w$, respectively.
- $\Theta$. Set of all combinations of tasks and quay cranes that potentially lead to quay crane interference.

The decision variables of the model are:

- $x_{ij}^k \in \{0, 1\}$, set to 1 if tasks $i$ and $j$ are processed consecutively by quay crane $k$, 0 otherwise.
- $z_{ij} \in \{0, 1\}$, set to 1 if task $j$ starts after the completion of task $i$, 0 otherwise.

The mathematical formulation is as follows:

$$\min c_T \tag{22}$$

Subject to:

$$\sum_{j \in \Omega^T} x_{0j}^k = 1, \forall k \in Q \tag{23}$$

$$\sum_{j \in \Omega^0} x_{jT}^k = 1, \forall k \in Q \tag{24}$$

$$\sum_{k \in Q} \sum_{j \in \Omega^T} x_{ij}^k = 1, \forall i \in \Omega \tag{25}$$

$$\sum_{j \in \Omega^0} x_{ji}^k - \sum_{i \in \Omega^T} x_{ij}^k = 0, \forall i \in \Omega, \forall \in Q \tag{26}$$

$$c_i + t_{ij} + p_j - c_j \leq M(1 - x_{ij}^k), \forall i, j \in \bar{\Omega}, \forall k \in Q \tag{27}$$

$$c_i + p_j - c_j \leq 0, \forall (i, j) \in \Phi \tag{28}$$

$$c_i + p_j - c_j \leq M(1 - z_{ij}), \forall i, j \in \Omega \tag{29}$$

$$c_j - p_j - c_i \leq M z_{ij}, \forall i, j \in \Omega \tag{30}$$

$$z_{ij} + z_{ji} = 1, \forall (i, j) \in \Psi \tag{31}$$

$$\sum_{u \in \Omega^0} x_{ui}^v + \sum_{u \in \Omega^0} x_{uj}^w \leq 1 + z_{ij} + z_{ji}, \forall (i, j, v, w) \in \Theta \tag{32}$$

$$c_i + \Delta_{ij}^v w + p_j - c_j \leq M(3 - z_{ij} - \sum_{u \in \Omega^0} x_{ui}^v - \sum_{u \in \Omega^0} x_{uj}^w),$$
$$\forall (i,j,v,w) \in \Theta \quad (33)$$

$$c_j + \Delta_{ij}^v w + p_i - c_i \leq M(3 - z_{ji} - \sum_{u \in \Omega^0} x_{ui}^v - \sum_{u \in \Omega^0} x_{uj}^w),$$
$$\forall (i,j,v,w) \in \Theta \quad (34)$$

$$r + t_{0j}^k + p_j - c_j \leq M(1 - x_{0j}^k), \forall j \in \Omega, \forall k \in Q \quad (35)$$

$$c_i \geq 0, \forall i \in \bar{\Omega} \quad (36)$$

The minimization of the makespan is modeled by (22). Each quay crane starts performing the task 0 and finishes performing the task $T$. This is modeled by constraints (23) and (24). All the non-dummy tasks have to be performed exactly once as set by constraints (25) and have a previous task and a succeeding task, constraint (26). Constraints (27) set the finishing time of each task. Constraints (28) ensure the precedence relationships among tasks. The values of variables $z_{ij}$ are established by constraints (29) and (30). Constraints (31) ensure the non-simultaneity of tasks. The interference between quay cranes are avoided by constraints (32)-(34). The ready times of the quay cranes are handled by constraints (35). Finally, the domain of the finishing times of each task is defined by constraints (36).

# 3 FUZZY COEFFICIENTS IN THE CONSTRAINTS FOR THE TBAP AND QCSP

In the TBAP, the arrival times of the vessels $a_i, i \in N$, are considered to be uncertain values. In the QCSP, the processing times of the loading/unloading operations, $p_t$, of the different tasks $t \in \Omega$ are also considered to be uncertain values. Even though the number of quay cranes assigned to carry out the loading and unloading operations of a container vessel is fixed, the real service time, will depend on several factors, such as interferences or breaks of the quay cranes assigned to that container vessel.

Modeling the uncertainty in the values of the parameters can be tackled by considering that they are fuzzy numbers. In this sense, independently of the wide range of different models that may be used, the problems above can be addressed by the following model.

$$\max\{cx / A^f x \leq_I b^f, x \geq 0\},$$

where $A^f$ and $b^f$ refer to the fact that we are considering fuzzy numbers in the coefficients that define the restrictions (thereby allowing, as a trivial case, them to also be real numbers when there are no ambiguities), and the symbol $\leq_I$ means that the way of comparing both members in the inequality, due to formal coherence, must be done by using a relationship for ordering the fuzzy numbers. This comparison relation $\leq_I$ may be any one from the extensive list available (Wang and Kerre, 1996), which in turn would also allow the decision-maker to have a greater degree of freedom when it comes to establish preferences. In more specific terms, in order to provide that theoretical model with a way for operating, let us briefly refer back to the different indices for comparing fuzzy numbers that have been described in the literature (Wang and Kerre, 1996). Amongst the different approaches described for comparing them, for the sake of simplicity, in this paper we shall only deal with the one that is derived from the use of indices for comparison. Hence, by denoting as F(R) the set of fuzzy numbers, if

$$I : F(R) \to [0,1]$$

is a comparison index for this kind of numbers, then

$$\forall P^f, Q^f \in F(R), P^f \leq_I Q^f \Leftrightarrow I(P^f) \leq I(Q^f)$$

whereby, according to the index $I$ that is used, different auxiliary models may be obtained for effectively solving the problems described above from the practical point of view. Therefore, in general, the auxiliary models that are used to solve the problems described above from the practical viewpoint, would be approached as follows.

$$\max\{cx / I(A^f x) \leq I(b^f), x \geq 0\}$$

Using an index $I$ or another depends on each decision-maker, and hence what index I to choose to be used is not the matter here. In order to illustrate the approach, and as a trivial example, let us consider two triangular fuzzy numbers $P^f, Q^f \in F(R)$, denoted as usually as $P^f = (P, P_i, P_d)$ and $Q^f = (Q, Q_i, Q_d)$, and as form of comparison, the one given by Yager's First Index (Wang and Kerre, 1996),

$$P^f \leq_I Q^f \Leftrightarrow (1/3)(P + P_i + P_d) \leq (1/3)(Q + Q_i + Q_d)$$

The membership function corresponding to a triangular fuzzy number $A^f = (A, A_i, A_d)$ is stated as follows:

$$\mu_{A^f}(x) = \begin{cases} 0, & x \leq A_i \\ \frac{x - A_i}{A - A_i}, & A_i < x \leq A \\ \frac{A_d - x}{A_d - A}, & A < x \leq A_d \\ 0, & x \geq A_d \end{cases}$$

Then, the previous model takes the following operating form,

$$\max\{cx / (A + A_i + A_d)x \leq (b + b_i + b_d), x \geq 0\}$$

from which we can obtain a solution for the previous models in a straightforward way.

Since in the TBAP model described above, the arrival times of the vessels, $a_i, i \in N$, which cannot be estimated accurately, appear as coefficients in the constraints, the methodology explained in this section can be used to provide the decision makers with adequate solutions. The same fact happens in the QCSP formulation, in which the processing times of the loading/unloading operations, $p_t$, of the different tasks $t \in \Omega$, appear as coefficients in the constraints. When applying this methodology with the Yager's First Index as a simple example, the original fuzzy models can be converted into mixed integer linear models that can be solved using any effective optimization technique from the literature.

# 4 OPTIMIZATION TECHNIQUES

In order to generate suitable solutions for the fuzzy considerations of the TBAP and QCSP, the solution approaches proposed by (Lalla-Ruiz et al., 2013) are considered. These approaches are based on the Variable Neighbourhood Search (VNS) which has demonstrated to be a high competitive metaheuristic when solving combinatorial and global optimization problems (Hansen et al., 2010). In general terms, the foundation of a VNS is to perform a systematic change of neighbourhood structures within a local search algorithm.

With the goal of providing a self-contained paper, in the following subsections we present the VNS approaches used for solving the TBAP and QCSP, respectively. For further details the interested reader is referred to (Lalla-Ruiz et al., 2013).

## 4.1 VNS for Solving the TBAP

Algorithm 1 depicts the pseudocode of the VNS used for solving the TBAP. Given a solution $\omega$, it considers two neighbourhood structures based upon the reinsertion movement, $N_a(\omega, \lambda)$, in which $\lambda$ vessels and their assigned profiles are removed from the berth $b \in B$ and reinserted into another berth $b'$, where $b \neq b'$, and the interchange movement, $N_b(\omega)$, which consists of exchanging a vessel $v \in V$ assigned to berth $b \in B$ with another vessel $v'$ assigned to berth $b'$, where $b \neq b'$.

The starting solution of the VNS, $\omega$, is generated by assigning the profile $p \in P$ with the highest usage cost to each container vessel. The berthing position of each vessel is selected at random, whereas the starting of its service time is selected as the earliest possible within its time window (line 1). The value of

**Algorithm 1:** VNS for the TBAP.

1: $\omega \leftarrow$ Generate initial solution
2: $k \leftarrow 1$
3: **repeat**
4: $\quad \omega' \leftarrow \text{Shake}(\omega, k)$
5: $\quad k_1 \leftarrow 1$
6: $\quad$ **repeat**
7: $\quad\quad \omega'' \leftarrow \text{Local Search}(\omega', k_1)$
8: $\quad\quad$ **if** $f(\omega'') > f(\omega')$ **then**
9: $\quad\quad\quad \omega' \leftarrow \omega''$
10: $\quad\quad\quad k_1 \leftarrow 1$
11: $\quad\quad$ **else**
12: $\quad\quad\quad k_1 \leftarrow k_1 + 1$
13: $\quad\quad$ **end if**
14: $\quad$ **until** $k_1 = k_{1max}$
15: $\quad$ **if** $f(\omega') > f(\omega)$ **then**
16: $\quad\quad \omega \leftarrow \omega'$
17: $\quad\quad k \leftarrow 1$
18: $\quad$ **else**
19: $\quad\quad k \leftarrow k + 1$
20: $\quad$ **end if**
21: **until** $k = k_{max}$

the parameter $k$ is set to 1 (line 2). The shaking process (line 4) allows to escape from those local optima found along the search by using the neighbourhood structure $N_a$. The solution exploitation phase of the VNS is based on a Variable Neighbourhood Descent Search (VND) (lines $6-14$). Given a solution $\omega'$, it explores one neighbourhood at a time until a local optimum with respect to the neighbourhood structures $N_a$ and $N_b$ is found. The application of the neighbourhoods structures in the VND is carried out according to the value of the parameter $k_1$, initially set to 1 (line 5). The first neighbourhood structure explored is $N_a$ and later $N_b$. The best solution found by means of the VND is denoted by $\omega'$. The objective function value of $\omega'$ allows to update the best solution found along the search (denoted by $\omega$) and restart the value of $k$ (lines $15-17$). Otherwise, the value of $k$ is increased (line 19). These steps are carried out until $k = k_{max}$ (line 21).

## 4.2 VNS for Solving the QCSP

The pseudocode of the proposed VNS for solving the QCSP is depicted in Algorithm 2. It is based upon two neighbourhood structures, the reassignment ($N_1$) and interchange of tasks ($N_2$). The search starts generating an initial schedule, $\sigma$, by assigning each task $t \in \Omega$ to its nearest quay crane (line 1). The value of the parameter $k$ is also set to 1 (line 4). A shaking procedure allows to reach unexplored regions of

**Algorithm 2:** VNS for the QCSP.

1: $\sigma \leftarrow$ Generate initial solution
2: $ES \leftarrow \emptyset$
3: **repeat**
4:   $k \leftarrow 1$
5:   **repeat**
6:     $\sigma' \leftarrow \text{Shake}(\sigma, k)$
7:     $\sigma'' \leftarrow \text{Local Search}(\sigma')$
8:     Update $ES$
9:     **if** $f(\sigma'') < f(\sigma)$ **then**
10:       $\sigma \leftarrow \sigma''$
11:       $k \leftarrow 1$
12:     **else**
13:       $k \leftarrow k + 1$
14:     **end if**
15:   **until** $k = k_{max}$
16:   $\sigma', \sigma'' \leftarrow$ Select schedules from $ES$
17:   $\sigma \leftarrow \text{Combine}(\sigma', \sigma'')$
18: **until** Stopping Criteria

the search space by means of the reassignment of $k$ tasks to another quay crane. The reassigned tasks are selected on the basis of a frequency memory. In this way, at each step, a neighbour schedule, $\sigma'$, is generated at random from $\sigma$ within the neighbourhood structure $N_k$ (line 6). A local optimum, $\sigma''$, is reached through a local search based on the proposed neighbourhood structures (line 7). An improvement in the value of $\sigma''$ allows to update $\sigma$ and restart $k$ (lines 9, 10 and 11). Otherwise, the value of $k$ is increased (line 13). These steps are carried out until $k = k_{max}$ (line 15).

An elite set, $ES$, is included into the VNS with the goal of collecting the promising schedules found during the search process. It is composed of those schedules with the lowest objective function value and those local optima with the highest diversity in the $ES$. The diversity of two schedules is measured as the number of tasks performed by different quay cranes. At each step, $ES$ provides a pair of schedules $\sigma$ and $\sigma'$ selected at random (line 16) in order to be combined (line 17) and restart the search. The combination process keeps those tasks performed by the same quay crane, whereas the remaining ones are randomly assigned to one quay crane on the basis of the objective function values of $\sigma$ and $\sigma'$.

# 5 COMPUTATIONAL EXPERIMENTS

This section is devoted to assess and analyze the performance of the VNSs described in the Section 4 for solving the TBAP and the QCSP under imprecise scenarios. All the computational experiments described along the present section have been carried out on a computer equipped with a CPU Intel 3.16 GHz and 4 GB of RAM.

## 5.1 Computational Experiments for the TBAP

The computational tests aimed at evaluating the behaviour of the VNS introduced in the Subsection 4.1 for the TBAP were conducted by using the problem instances proposed in the work by (Giallombardo et al., 2010). These instances are based upon real data provided by the Medcenter Container Terminal of Gioia Tauro located in Italy. In this case, only a subset of 9 of these instances are used during this computational experiment. The size of the problem instances ranges from 20 up to 40 container vessels that must be located in 5 berths over a time horizon of one week.

The Table 1 reports the computational results obtained by means of the VNS over the group of instances taken up from the benchmark suite described above. The first column (*Instance*) shows the instances to solve. For each instance, the name (*Name*), the number of container vessels (*N*), the number of berths (*M*) and the maximum number of quay crane profiles per each vessel (*P*) are presented. The second column (*CPLEX*) shows the upper bound (*UB*) obtained by the CPLEX Optimizer with a maximum computational time of 2 hours. The column *VNS* shows the results obtained when the arrival time of the vessels is deterministic. Under this heading, it is reported the objective value of the best solution found by the VNS ($f_{VNS}$), the execution time (*t. (s.)*) and the relative error (*(Gap (%)*) regarding the upper bound. Lastly, the next columns ($VNS_{S-I}$ and $VNS_{S-II}$) show the results by considering scenarios with fuzzy numbers to model the arrival times of the container vessels. In this context, we have evaluated the performance of the VNS concerning the following fuzzy numbers:

- S-I = ($a_i$, $a_i - 1$, $a_i + 4$)
- S-II = ($a_i$, $a_i - 2$, $a_i + 8$)

These fuzzy scenarios (S-I and S-II) model, on one hand, the anticipation or delay of the arrival time of the container vessel arrived at port regarding their expected times. These scenarios represent a common problem in maritime container terminals, since the vessels are subject to tidal, traffic or contractual changes. In this regard, these issues are frequently translated into a delay of the expected arrival time of the container vessel, due to that, the fuzzy numbers of both scenarios S-I and S-II consider a higher delay than anticipation of their arrival time. For each scenario, the best solution value found by the VNS ($f_{VNS}$) and its required computational time (*t. (s.)*), measured in seconds, are reported.

In spite of the change of scenario there is not a clear trend in the target values, as it is reported in Table 1. It is expected that the uncertainty in the arrival times of the container vessels has a direct impact on the feasibility of the solutions due to the reduction in the time window constraints and the availability of quay cranes. Moreover, it may implicitly affect to the objective function value if the late arrival of the container vessel forces to allocate it in another berth than the expected one. The reason is found in that this fact would increase the housekeeping cost derived from the transshipment operations. In this regard, the anticipation or delay of the arrival time of the vessels may also impact on the assignment of quay crane profiles, namely, early arrival times would allow to assign longer quay cranes profiles, whereas late arrival times would require shorter quay crane profiles if one is willing to keep the vessels assigned to their initial assigned berth. These facts and further analysis of the structure of the final solutions would be a topic of future work. It is worth mentioning that for the instances from the literature are obtained feasible solutions for fuzzy arrival times considered in this computational experiment.

## 5.2 Computational Experiments for the QCSP

In order to check the suitability of the VNS introduced in the Subsection 4.2 for solving the QCSP, we have considered a representative subset of the problem instances proposed by (Bierwirth and Meisel, 2009). The original set of instances is composed of 90 instances grouped into 9 groups with 10 instances each one. Each group of instances has different number of tasks (from 10 up to 50) and quay cranes (from 2 up to 6) which allows to cover real-world scenarios. In this case, we have selected one instance from each original group in such a way that our benchmark suite is composed of 9 instances. It is worth mentioning that, as done in previous works, we have established in this experiment that the quay cranes are available from the starting of the service time ($r^q, \forall q \in Q$) and they have to keep a safety distance of one bay ($\delta = 1$) among them.

Table 2 shows the computational results we have obtained by means of the proposed VNS when solving the aforementioned instances. The first column (*Instance*) reports the characteristics of the instances used during the experiment: name (*Name*), number of tasks ($n$) and number of quay cranes ($m$). The second column (*Optimal*) shows the objective function value of the optima schedules for the instances at hand reported in the work by (Bierwirth and Meisel, 2009). The third column (*VNS*) show the computational results obtained by means of the VNS described in the Subsection 4.2 under deterministic scenarios. In this case the objective function value of the best schedule found during the search ($f_{VNS}$) and the computational time (*t. (m.)*), measured in minutes, are reported. Finally, as described in the Section 3, we consider fuzzy numbers to model the processing times of the tasks. In this context we have evaluated the performance of the VNS under the following fuzzy numbers:

- S-III = $(p_t,\, p_t - 25\% p_t,\, p_t + 10\% p_t)$
- S-IV = $(p_t,\, p_t - 10\% p_t,\, p_t + 25\% p_t)$
- S-V = $(p_t,\, p_t - 10\% p_t,\, p_t + 50\% p_t)$

The first fuzzy number (S-III) models that scenario in which reductions in the processing time of the tasks associated with a given container vessel are more likely than delays. On the other hand, due to the fact that delays in the processing time of the tasks are the most common in maritime container terminals, we have proposed the numbers S-IV and S-V considering a higher delay than reduction. In this regard, Table 2 has one column ($VNS_{S-III}$, $VNS_{S-IV}$ and $VNS_{S-V}$) with the computational results reported by the VNS when the previous fuzzy numbers are taken into account. In each case, the objective function value of the best schedule found during the search ($f_{VNS}$) and the computational time (*t. (m.)*), measured in minutes, are reported.

In spite of the fact that analyzing the performance of the VNS under deterministic scenarios is not a major goal of this work, with the aim of providing an overall study we firstly focus on this issue. In this regard, the computational results reported in the Table 2 indicate that the proposed VNS is highly effective at finding optimal or near-optimal schedules for the QCSP under deterministic scenarios (column *VNS*). As can be seen, it provides the optima schedules for 7 instances from the benchmark suite at hand, whereas the gap is below 1.2% in the worst case (*k94*). Moreover, below 4.5 minutes have been required for finishing the search process in all the cases. An exhaustive analysis of the performance of the VNS under deterministic scenarios is described in the original work by (Lalla-Ruiz et al., 2013). Additionally, the efficiency of our optimization technique has allowed to apply it successfully to integrated approach as that described in the same work.

Table 1: Computational results obtained by means of the Variable Neighbourhood Search described in the Subsection 4.1 considering different fuzzy numbers.

| *Instance* | | | | *CPLEX* | *VNS* | | | $VNS_{S-I}$ | | $VNS_{S-II}$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *Name* | *N* | *M* | *P* | *UB* | $f_{VNS}$ | t. (s.) | Gap (%) | $f_{VNS}$ | t. (s.) | $f_{VNS}$ | t. (s.) |
| $A_{p10}$ | 20 | 5 | 10 | 1383614 | 1344035 | 8.70 | 2.86 | 1348244 | 16.70 | 1345272 | 71.87 |
| $A_{p20}$ | 20 | 5 | 20 | 1384765 | 1349455 | 18.67 | 2.55 | 1344308 | 11.62 | 1346265 | 24.81 |
| $A_{p30}$ | 20 | 5 | 30 | 1385119 | 1343119 | 13.09 | 3.03 | 1344374 | 10.56 | 1342836 | 13.14 |
| $B_{p10}$ | 30 | 5 | 10 | 1613252 | 1546878 | 64.78 | 4.11 | 1557261 | 80.69 | 1542448 | 21.39 |
| $B_{p20}$ | 30 | 5 | 20 | 1613769 | 1550839 | 26.26 | 3.90 | 1551015 | 27.90 | 1546422 | 28.03 |
| $B_{p30}$ | 30 | 5 | 30 | 1613805 | 1553673 | 36.73 | 3.73 | 1550520 | 13.12 | 1548215 | 27.67 |
| $C_{p10}$ | 40 | 5 | 10 | 2289660 | 2229807 | 195.76 | 2.61 | 2231800 | 170.91 | 2235877 | 176.04 |
| $C_{p20}$ | 40 | 5 | 20 | 2290662 | 2234516 | 330.21 | 2.45 | 2231730 | 263.82 | 2229076 | 310.01 |
| $C_{p30}$ | 40 | 5 | 30 | 2291301 | 2232260 | 228.64 | 2.58 | 2227989 | 221.56 | 2226721 | 134.12 |

Table 2: Computational results obtained by means of the Variable Neighbourhood Search described in the Subsection 4.2 considering different fuzzy numbers.

| *Instance* | | | *Optimal* | *VNS* | | | $VNS_{S-III}$ | | $VNS_{S-IV}$ | | $VNS_{S-V}$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *Name* | *n* | *m* | Obj. | $f_{VNS}$ | t. (m.) | Gap (%) | $f_{VNS}$ | t. (m.) | $f_{VNS}$ | t. (m.) | $f_{VNS}$ | t. (m.) |
| k13 | 10 | 2 | 453 | 453 | $< 0.01$ | 0.00 | 423 | $< 0.01$ | 468 | $< 0.01$ | 504 | $< 0.01$ |
| k27 | 15 | 2 | 657 | 657 | 0.02 | 0.00 | 612 | 0.02 | 681 | 0.01 | 729 | $< 0.01$ |
| k42 | 20 | 3 | 573 | 573 | 0.11 | 0.00 | 540 | 0.08 | 591 | 0.04 | 639 | 0.04 |
| k48 | 25 | 3 | 639 | 639 | 0.23 | 0.00 | 600 | 0.18 | 657 | 0.14 | 708 | 0.13 |
| k53 | 30 | 4 | 717 | 717 | 0.64 | 0.00 | 675 | 0.50 | 741 | 0.48 | 804 | 0.47 |
| k69 | 35 | 4 | 807 | 807 | 0.99 | 0.00 | 756 | 0.85 | 837 | 0.84 | 900 | 0.83 |
| k82 | 40 | 5 | 717 | 723 | 2.22 | 0.84 | 678 | 2.29 | 747 | 2.30 | 807 | 2.46 |
| k89 | 45 | 5 | 843 | 843 | 2.83 | 0.00 | 792 | 2.94 | 867 | 2.67 | 942 | 2.76 |
| k94 | 50 | 6 | 786 | 795 | 4.42 | 1.15 | 768 | 4.05 | 828 | 4.26 | 903 | 4.07 |

On the other hand, as might be expected, the uncertainty on the processing times of the tasks defined by the stowage plan of a given container vessel has a direct impact on its service time (makespan). This is evidenced in particular by a decrease in the objective function value of the schedules obtained by the VNS over those scenarios in which the processing times tend to be shorter than the original ones. That is, when the fuzzy number S-III is considered. For instance, we have obtained a reduction of 51 time units in the makespan of the schedule reported by the VNS for the instance k89 in comparison with the deterministic version of the problem. A similar reasoning should be applied to the scenarios in which the processing times of the tasks tend to be larger than the original ones, that is, under the fuzzy numbers S-IV and S-V. In these cases, the makespan of the schedules reported by the VNS are larger than those of the schedules found under the deterministic scenarios. For example, an increase of 108 times units in the makespan of the schedule reported by the VNS has been obtained for the instance k94 when the fuzzy number S-V is considered in comparison with the deterministic version of the problem. Finally, it is worth mentioning that the makespan of the schedules found when the fuzzy number S-V is considered are larger that those of the schedules obtained under the fuzzy number S-IV. This fact is derived from the higher increase produced in the former case.

## 6 CONCLUSIONS

The maritime container terminals are large infrastructures aimed at serving the container vessels arrived at port. In this regard, a berthing time and berthing position along the quay is assigned to each vessel. For loading and unloading its containers, a subset of quay cranes is allocated to it. These quay cranes perform the loading and unloading operations associated with the containers of the vessel. In this context, two relevant logistical problems should be highlighted: Tactical Berth Allocation Problem (TBAP) and the Quay Crane Scheduling Problem (QCSP). Unfortunately, due to the inherent imprecision that appears in the data involved in this environment, terminal managers are particularly interested in solving these problems by considering the uncertainty arising in the terminals.

In this paper the TBAP and the QCSP are tackled under imprecise scenarios. For this purpose, two fuzzy models that consider the uncertainty of the arrival time of the vessels and the processing time of the quay cranes are proposed. Moreover, in order to effectively solve these models, two solutions approaches based on the Variable Neighbourhood Search metaheuristic are introduced. Both methods are able to provide feasible solutions with reasonable computational times.

## ACKNOWLEDGEMENTS

This work has been partially funded by the Spanish Ministry of Economy and Competitiveness (project TIN2012-32608). Christopher Expósito Izquierdo and Eduardo Lalla-Ruiz thank CajaCanarias and the Canary Government the financial support they receive through their post-graduate grants, respectively.



## REFERENCES

Bierwirth, C. and Meisel, F. (2009). A fast heuristic for quay crane scheduling with interference constraints. *Journal of Scheduling*, 12(4):345–360.

Bierwirth, C. and Meisel, F. (2010). A survey of berth allocation and quay crane scheduling problems in container terminals. *European Journal of Operational Research*, 202(3):615 – 627.

Cadenas, J. and Verdegay, J. (2006). A primer on fuzzy optimization models and methods. *Iranian Journal of Fuzzy Systems*, 3(1):1–21.

Froyland, G., Koch, T., Megow, N., Duane, E., and Wren, H. (2008). Optimizing the landside operation of a container terminal. *OR Spectrum*, 30(1):53–75.

Giallombardo, G., Moccia, L., Salani, M., and Vacca, I. (2010). Modeling and solving the tactical berth allocation problem. *Transportation Research Part B: Methodological*, 44(2):232–245.

Hansen, P., Mladenović, N., and Pérez, J. A. M. (2010). Variable neighbourhood search: methods and applications. *Annals of Operations Research*, 175(1):367–407.

Herrera, F. and Verdegay, J. (1995). Three models of fuzzy integer linear programming. *European J. of Operational Research*, 83(3):581 – 593.

Kim, K., Park, Y.-M., and Jin, M.-J. (2008). An optimal layout of container yards. *OR Spectrum*, 30(4):675–695.

Kim, K. H. and Park, Y.-M. (2004). A crane scheduling method for port container terminals. *European Journal of Operational Research*, 156(3):752–768.

Lalla-Ruiz, E., Expósito-Izquierdo, C., Melián-Batista, B., and Moreno-Vega, J. M. (2013). A metaheuristic approach for the seaside operations in maritime container terminals. In *IWANN (Part II)*, volume 7903 of *Lecture Notes in Computer Science*. Springer.

Lalla-Ruiz, E., Melián-Batista, B., and Moreno-Vega, J. M. (2012). Artificial intelligence hybrid heuristic based on tabu search for the dynamic berth allocation problem. *Engineering Applications of Artificial Intelligence*, 25(6):1132 – 1141.

Meisel, F. (2010). *Seaside Operations Planning in Container Terminals*. Contributions to Management Science. Physica-Verlag HD.

Petering, M. E. (2011). Decision support for yard capacity, fleet composition, truck substitutability, and scalability issues at seaport container terminals. *Transportation Research Part E: Logistics and Transportation Review*, 47(1):85 – 103.

Sammarra, M., Cordeau, J.-F., Laporte, G., and Monaco, M. F. (2007). A tabu search heuristic for the quay crane scheduling problem. *Journal of Scheduling*, 10(4-5):327–336.

Sancho-Royo, A., Pelta, D., and Verdegay, J. (2006). A proposal of metaheuristics based in the cooperation between operators in combinatorial optimization problems. In *Proceedings of the IEEE 20-th International Parallel and Distributed Processing Symposium*, Rhodes, Greece.

Stahlbock, R. and Voβ, S. (2008). Operations research at container terminals: a literature update. *OR Spectrum*, 30:1–52.

Tiwari, A., Knowles, J., Avineri, E., Dahal, K., and Roy, R. (2006). *Applications of Soft Computing : Recent Trends (Advances in Soft Computing Series)*. Springer.

Verdegay, J. (2003). *Fuzzy Sets based Heuristics for Optimization. Studies in Fuzziness and Soft Computing*. Springer.

Wang, X. and Kerre, E. (1996). On the classification and the dependencies of the ordering methods. In Ruan, D., editor, *Fuzzy Logic Foundation and Industrial Applications. International Series in Intelligent Technologies*, pages 73–90. Kluwer.

Wiegmans, B. W., Rietveld, P., and Nijkamp, P. (2001). Container terminal services and quality. Serie Research Memoranda 0040, VU University Amsterdam, Faculty of Economics, Business Administration and Econometrics.