EM WAVE PROPAGATION AND SCATTERING IN SPATIOTEMPORALLY VARYING MOVING MEDIA
The Exponential Model

Dan Censor
Department of Electrical and Computer Engineering, Ben–Gurion University of the Negev, 84105 Beer–Sheva, Israel
censor@ee.bgu.ac.il

Keywords: Electromagnetic Wave Propagation; Electromagnetic Wave Scattering; Special Relativity; Moving Media.

Abstract: An approximate method for analyzing EM wave propagation and scattering in the presence of temporally and spatially varying media is investigated. The method is quasi-relativistic in the sense that for constant velocity it reduces to Einstein's Special Relativity theory to the first order in the normalized speed v/c. The present exponential model was previously used for temporally invariant velocity only. The motion must be irrotational and the characteristic wavelength and period scales of the mechanical motion must be much larger compared to those of the EM field ones. For simple periodic motion it is shown that the EM field is modulated by the motion, and a spectrum of discrete sidebands is created, with frequencies separated by the mechanical frequencies. The results suggest new approaches to the celebrated Fizeau experiment. Rather than using an interferometer setup as in the traditional experiment, the equivalent phase velocity in a periodically moving medium can be deduced from the measured. Simple examples are computed: the effect of the motion on an initially plane harmonic wave, and scattering by perfectly conducting and refractive planes and cylinders.

1 INTRODUCTION AND ABBREVIATIONS

Scattering of EM waves in the presence of moving media and scatterers is of interest for theoretical and engineering applications, see (Van Bladel, 1984) for a comprehensive introduction to the relevant literature. Einstein's SR; Minkowski, 1908; Sommerfeld, 1964; Pauli, 1958) facilitates the analysis for problems involving constant velocities. Historically this is related to the FE and the associated Fresnel drag phenomenon (Einstein, 1905; Pauli, 1958). Heuristic approximations are required for varying velocities, and it stands to reason that they will adequately apply to cases involving the normalized speed v/c to the FO only.

Historically, the present exponential model seems to have originated with Collier and Tai (1965), and later considered for general temporally invariant velocities (Nathan and Censor, 1968; Censor, 1969a, 1972).

1.1 Glossary of Abbreviations

BC=Boundary Condition/s
AKA=Also Known As
BFS= Bessel-Fourier Series
EM=Electromagnetic
EX=Exponential Model/s
FE=Fizeau Experiment/s
FO=First Order in v/c
FT=Field Transformation/s
GT=Galilean Transformation/s
IT=Inverse Transformation/s
LT=Lorentz Transformation/s
MCR= Minkowski Constitutive Relations
ME=Maxwell Equation/s
MM=Mechanical Medium
RE=Relativistic Electrodynamics
RF=Reference Frame/s
SC=Scattering Coefficient/s
SP=Scattering Problem/s
SR=Special Relativity
ZO=Zero Order in v/c

Censor D.
EM WAVE PROPAGATION AND SCATTERING IN SPATIOTEMPORALLY VARYING MOVING MEDIAThe Exponential Model.
DOI: 10.5220/0004785400620068
In *Proceedings of the Second International Conference on Telecommunications and Remote Sensing (ICTRS 2013)*, pages 62-68
ISBN: 978-989-8565-57-0

2 FIRST ORDER RELATIVISTIC ELECTRODYNAMICS

Concepts and notation are introduced via a short recapitulation of the FO RE. The source-free ME in a RF $\boldsymbol{\Gamma}'$ are

$$\begin{aligned}&\partial_{\mathbf{r}'} \times \mathbf{E}' = -\partial_{t'}\mathbf{B}', \partial_{\mathbf{r}'} \times \mathbf{H}' = \partial_{t'}\mathbf{D}' \\ &\partial_{\mathbf{r}'} \cdot \mathbf{D}' = 0, \partial_{\mathbf{r}'} \cdot \mathbf{B}' = 0\end{aligned} \quad (1)$$

Fields are functions of native coordinates, e.g., $\mathbf{E}'(\mathbf{r}', t')$. According to SR, in an inertial RF $\boldsymbol{\Gamma}$ we have the form-invariant ME, i.e., (1) without apostrophes

$$\begin{aligned}&\partial_{\mathbf{r}} \times \mathbf{E} = -\partial_{t}\mathbf{B}, \partial_{\mathbf{r}} \times \mathbf{H} = \partial_{t}\mathbf{D} \\ &\partial_{\mathbf{r}} \cdot \mathbf{D} = 0, \partial_{\mathbf{r}} \cdot \mathbf{B} = 0\end{aligned} \quad (2)$$

where the fields are functions of native coordinates, e.g., $\mathbf{E}(\mathbf{r}, t)$.

The FO LT and its IT are given by

$$\mathbf{r}' = \mathbf{r} - \mathbf{v}t, t' = t - c^{-2}\mathbf{v} \cdot \mathbf{r} \quad (3)$$

$$\mathbf{r} = \mathbf{r}' + \mathbf{v}t', t = t' + c^{-2}\mathbf{v} \cdot \mathbf{r}' \quad (\underline{3})$$

respectively. Henceforth the underline notation for IT will be understood even without explicitly writing out the expressions. Formally, all we have to do is exchange primed and unprimed quantities and replace $\mathbf{v}$ by $-\mathbf{v}$. Effecting the limit $c \to \infty$ in (3), ($\underline{3}$), yields the GT $t' = t$.

Substituting (3) in the chain rule of calculus

$$\begin{aligned}&\partial_{\mathbf{r}} = (\partial_{\mathbf{r}}\mathbf{r}') \cdot \partial_{\mathbf{r}'} + (\partial_{\mathbf{r}}t')\partial_{t'} \\ &\partial_{t} = (\partial_{t}t')\partial_{t'} + (\partial_{t}\mathbf{r}') \cdot \partial_{\mathbf{r}'}\end{aligned} \quad (4)$$

and its IT ($\underline{4}$) leads to the FO differential LT and its associated IT

$$\partial_{\mathbf{r}'} = \partial_{\mathbf{r}} + c^{-2}\mathbf{v}\partial_{t}, \partial_{t'} = \partial_{t} + \mathbf{v} \cdot \partial_{\mathbf{r}} \quad (5)$$

$$\partial_{\mathbf{r}} = \partial_{\mathbf{r}'} - c^{-2}\mathbf{v}\partial_{t'}, \partial_{t} = \partial_{t'} - \mathbf{v} \cdot \partial_{\mathbf{r}'} \quad (\underline{5})$$

respectively. Note that the second formula (5) is in fact the “material derivative” or the “moving derivate” as referred to in continuum mechanics. In the limit $c \to \infty$ the first equations in (5), ($\underline{5}$) become the GT $\partial_{\mathbf{r}'} = \partial_{\mathbf{r}}$, as usually used in continuum mechanics.

Substituting ($\underline{5}$) into (1) and regrouping terms yields (2), subject to the FO FT

$$\begin{aligned}&\mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B}, \mathbf{B}' = \mathbf{B} - c^{-2}\mathbf{v} \times \mathbf{E} \\ &\mathbf{D}' = \mathbf{D} + c^{-2}\mathbf{v} \times \mathbf{H}, \mathbf{H}' = \mathbf{H} - \mathbf{v} \times \mathbf{D}\end{aligned} \quad (6)$$

and similarly for corresponding IT FT

$$\begin{aligned}&\mathbf{E} = \mathbf{E}' - \mathbf{v} \times \mathbf{B}', \mathbf{B} = \mathbf{B}' + c^{-2}\mathbf{v} \times \mathbf{E}' \\ &\mathbf{D} = \mathbf{D}' - c^{-2}\mathbf{v} \times \mathbf{H}', \mathbf{H} = \mathbf{H}' + \mathbf{v} \times \mathbf{D}'\end{aligned} \quad (\underline{6})$$

Assuming in $\boldsymbol{\Gamma}'$ simple linear constitutive relations

$$\mathbf{D}' = \varepsilon\mathbf{E}', \mathbf{B}' = \mu\mathbf{H}' \quad (7)$$

and substituting from (6) into (7) leads to the FO MCR (Minkowski, 1908)

$$\begin{aligned}&\mathbf{D} + \mathbf{v} \times \mathbf{H} / c^{2} = \varepsilon(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \\ &\mathbf{B} - \mathbf{v} \times \mathbf{E} / c^{2} = \mu(\mathbf{H} - \mathbf{v} \times \mathbf{D})\end{aligned} \quad (8)$$

Although applicable to constant $\mathbf{v}$ only (2), when arbitrarily stipulated to hold for constant local $\mathbf{v}$, it provides the basis for many scattering problems involving rotating spheres and cylinders (see Van Bladel, 1984, pp. 392-3, for relevant articles by D. De Zutter and others).

3 FIRST ORDER RELATIVISTIC ELECTRODYNAMICS, VARYING VELOCITIES

Inasmuch as SR deals with constant $\mathbf{v}$ only, there exists no exact transition to varying velocity. Consequently an heuristically extension of the above FO model must be stipulated, e.g., by generalizing (5), ($\underline{5}$) to

$$\partial_{\mathbf{r}'} = \partial_{\mathbf{r}} + c^{-2}\mathbf{v}(\mathbf{r}, t)\partial_{t}, \partial_{t'} = \partial_{t} + \mathbf{v}(\mathbf{r}, t) \cdot \partial_{\mathbf{r}} \quad (9)$$

$$\partial_{\mathbf{r}} = \partial_{\mathbf{r}'} - c^{-2}\mathbf{v}(\mathbf{r}', t')\partial_{t'}, \partial_{t} = \partial_{t'} - \mathbf{v}(\mathbf{r}', t') \cdot \partial_{\mathbf{r}'} \quad (\underline{9})$$

The correspondence makes (9), ($\underline{9}$), plausible. Note that terms involving the velocity are already of FO, hence ZO coordinates can be dropped or added, e.g., $\mathbf{v}(\mathbf{r}', t') = \mathbf{v}(\mathbf{r}, t)$. Once again note that (9) tallies with the material derivative concept.

It is easily seen that the form invariance of the ME subject to (9), ($\underline{9}$) is not preserved here, since by substitution of ($\underline{9}$) into the ME (1) we encounter terms like

$$\mathbf{v}\partial_{t'} \times \mathbf{E} = \partial_{t'}(\mathbf{v} \times \mathbf{E}) - \partial_{t'}(\mathbf{v}) \times \mathbf{E}, \mathbf{v} = \mathbf{v}(\mathbf{r}', t') \quad (10)$$

where the term $\partial_{t'}((\mathbf{v}(\mathbf{r}', t') \times \mathbf{E})$ obviates the extension of the FT (6) to varying velocity.

However, it is noted that field time derivatives as in $\partial_{t'} \times \mathbf{E}$ involve wave frequencies, say ω, while $\partial_{t'} \mathbf{v}(\mathbf{r}', t')$ involves MM frequencies. Similarly, space derivatives of $\mathbf{v}(\mathbf{r}', t')$, are characterized by a wave number k, while differentiating the velocity involves K, the MM wave number.

We conclude that when the length and time scales characterizing the velocity are larger than the corresponding parameters of the fields, the FT (6), and with them the form invariance of the ME (1), (2), can be assumed. This also implies that the MCR (8) are valid subject to the present restrictions.

4 THE EXPONENTIAL MODEL

Previously the EX (Collier and Tai, 1965; Nathan and Censor, 1968; Censor, 1969a, 1972) was based on the stipulation that (6) remains valid for local time independent $\mathbf{v}$, although $\mathbf{v}(\mathbf{r})$ is spatially varying. Since the MCR (8) are already of ZO, they can be recast in a simplifies form

$$\mathbf{D} = \varepsilon\mathbf{E} + \mathbf{\Lambda} \times \mathbf{H},\ \mathbf{B} = \mu\mathbf{H} - \mathbf{\Lambda} \times \mathbf{E}$$
$$\mathbf{\Lambda} = \mathbf{v}(\mathbf{r}, t)(C^{-2} - c^{-2}),\ C = 1/\sqrt{\mu\varepsilon} \quad (11)$$

Substituting (11) into (2) yields the relevant FO ME for moving media (Tai, 1964; Nathan and Censor, 1968)

$$\partial_{\mathbf{r}} \times \mathbf{E} = -\mu\partial_t\mathbf{H} + \mathbf{\Lambda} \times \partial_t\mathbf{E},\ \varepsilon\partial_{\mathbf{r}} \cdot \mathbf{E} + \partial_{\mathbf{r}} \cdot \mathbf{\Lambda} \times \mathbf{H} = 0$$
$$\partial_{\mathbf{r}} \times \mathbf{H} = \varepsilon\partial_t\mathbf{E} + \mathbf{\Lambda} \times \partial_t\mathbf{H},\ \mu\partial_{\mathbf{r}} \cdot \mathbf{H} - \partial_{\mathbf{r}} \cdot \mathbf{\Lambda} \times \mathbf{E} = 0 \quad (12)$$

Note that by interchanging $\mathbf{H} \leftrightarrow \mathbf{E}, \varepsilon \leftrightarrow -\mu$ in (12), we switch between the two equations. For irrotational $\mathbf{v}(\mathbf{r}, t)$ we have $\partial_{\mathbf{r}} \times \mathbf{\Lambda} = 0$, entailing a conservative field, associated with the scalar potential Φ

$$\mathbf{\Lambda} = \partial_{\mathbf{r}}\Phi,\ \Phi = \int \mathbf{\Lambda} \cdot d\mathbf{l} \quad (13)$$

therefore the path integral (13) depends on limits only. Accordingly (12) can be recast as

$$\partial_{\mathbf{r}}{*} \times \mathbf{E} = -\mu\partial_t\mathbf{H},\ \partial_{\mathbf{r}}{*} \times \mathbf{H} = \varepsilon\partial_t\mathbf{E}$$
$$\partial_{\mathbf{r}}{*} \cdot \mathbf{E} = 0,\ \partial_{\mathbf{r}}{*} \mathbf{H} = 0,\ \partial_{\mathbf{r}}{*} = \partial_{\mathbf{r}} - \partial_t\mathbf{\Lambda} \times \quad (14)$$

Incorporating the time scales argument as in (10), whereby the velocity's time-derivative is neglected, solutions of (12), (14), can be constructed in the form

$$\mathbf{E} = e^{\Phi\partial_t}\mathbf{E}_1,\ \mathbf{H} = e^{\Phi\partial_t}\mathbf{H}_1$$
$$\Phi = \int \mathbf{\Lambda} \cdot d\mathbf{l},\ \partial_{\mathbf{r}} \times \mathbf{\Lambda} = 0,\ \mathbf{\Lambda} = \partial_{\mathbf{r}}\Phi \quad (15)$$

The operator exponential is understood as a symbolic Taylor series $e^{\Phi\partial_t} \equiv 1 + \Phi\partial_t + \dots$. The ZO fields $\mathbf{E}_1, \mathbf{H}_1$ satisfy the ME

$$\partial_{\mathbf{r}} \times \mathbf{E}_1 = -\mu\partial_t\mathbf{H}_1,\ \partial_{\mathbf{r}} \times \mathbf{H}_1 = \varepsilon\partial_t\mathbf{E}_1$$
$$\partial_{\mathbf{r}} \cdot \mathbf{E}_1 = 0,\ \partial_{\mathbf{r}} \cdot \mathbf{H}_1 = 0 \quad (16)$$

Inasmuch as the operator $e^{\Phi\partial_t}$ acts on the ZO fields, for time harmonic fields possessing the factor $e^{-i\omega_1 t}$ with frequency ω_1, we identify

$$\partial_t \leftrightarrow -i\omega_1,\ \partial_{\mathbf{r}}{*} = \partial_{\mathbf{r}} + i\omega_1\mathbf{\Lambda} \times \quad (17)$$

Thus the EX is a perturbation scheme whereby we start with well-known solutions of the ME in media at rest (16), and with the exponential operator (15) as a factor, a FO solution of the ME in moving media is created. Of course, BC, where applicable, must be taken on the complete fields (15). For simplicity, incompressible media are considered here, therefore Φ satisfies the Laplace equation $\partial_r^2\Phi = 0$.

Once (11)-(16) are accepted as our working formalism, everything takes place in the "laboratory" unprimed RF.

5 PLANE WAVE PROPAGATION IN OSCILLATING MEDIA

Consider a plane harmonic wave satisfying (16)

$$\mathbf{E}_1 = \hat{\mathbf{E}}_1 E_1 e^{i\theta_1},\ \mathbf{H}_1 = \hat{\mathbf{H}}_1 H_1 e^{i\theta_1},\ \theta_1 = \mathbf{k}_1 \cdot \mathbf{r} - \omega_1 t \quad (18)$$

with mutually perpendicular $\mathbf{k}_1, \mathbf{E}_1, \mathbf{H}_1$, launched into the moving medium. The medium time-dependent velocity is given according to (11), (13), as

$$\mathbf{\Lambda}(t) = \mathbf{\Lambda}_0 \cos\Omega t,\ \Phi = \int \mathbf{\Lambda} \cdot d\mathbf{l} = \mathbf{\Lambda} \cdot \mathbf{r} \quad (19)$$

It follows that the solution of the ME (15) is given by

$$\mathbf{E} = \hat{\mathbf{E}}_1 E_1 e^{i\theta},\ \mathbf{H} = \hat{\mathbf{H}}_1 H_1 e^{i\theta}$$
$$\theta = \theta_1 - \omega_1 \mathbf{\Lambda}_0 \cdot \mathbf{r} \cos\Omega t = \theta_1 - \omega_1 \Lambda_0 \xi \cos\Omega t \tag{20}$$

where ξ is the coordinate in the direction of $\mathbf{\Lambda}$. Note that in (19), (20), $\mathbf{r}$ can assume any value, therefore $\mathbf{\Lambda}\cdot\mathbf{r}$ is not necessarily small, in spite of $\mathbf{\Lambda}$ being FO. This is a consequence of choosing a time-dependent velocity as in (19) (cf. (23) below). Recasting $e^{i\theta}$ in terms of a BFS (e.g., see Stratton, 1969) yields

$$e^{i\theta} = \Sigma_n F_n e^{i\mathbf{k}_1\cdot\mathbf{r} - i\nu_n t},\ F_n = (-i)^n J_n(\omega_1 \Lambda_0 \xi)$$
$$\nu_n = \omega_1 - n\Omega,\ \Sigma_n = \Sigma_{n=-\infty}^{n=\infty} \tag{21}$$

revealing the spectral structure the plane wave assumes in the moving medium, with the initial carrier frequency ω_1 for $n = 0$ and additional discrete sidebands ν_n for integers n. Throughout J_q denotes the non-singular Bessel function of order q. Thus (20) can be recast as

$$\mathbf{E} = \hat{\mathbf{E}}_1 \Sigma_n E_n e^{i\mathbf{k}_1\cdot\mathbf{r} - i\nu_n t},\ \mathbf{H} = \hat{\mathbf{H}}_1 \Sigma_n H_n e^{i\mathbf{k}_1\cdot\mathbf{r} - i\nu_n t}$$
$$E_n / E_1 = H_n / H_1 = F_n \tag{22}$$

The time periodic velocity (19) can be generalized
to a MM space and time harmonic plane velocity wave

$$\mathbf{\Lambda}(\mathbf{r}, t) = \mathbf{\Lambda}_0 \cos(\mathbf{K}\cdot\mathbf{r} - \Omega t),\ |\mathbf{K}| = 2\pi / \lambda_m \tag{23}$$

with λ_m denoting the MM wavelength. For longitudinal compression waves $\mathbf{K}, \mathbf{\Lambda}_0$ are parallel, hence we have

$$\partial_{\mathbf{r}} \times \mathbf{\Lambda} = \partial_{\mathbf{r}} \times (\mathbf{\Lambda}_0 \cos(\mathbf{K}\cdot\mathbf{r} - \Omega t))$$
$$= \mathbf{\Lambda}_0 \times \mathbf{K} \sin(\mathbf{K}\cdot\mathbf{r} - \Omega t) = 0 \tag{24}$$

as prescribed for (13). Furthermore (15) prescribes

$$\Phi = \mathbf{\Lambda}_0 \int \cos(\mathbf{K}\cdot\mathbf{r} - \Omega t)\cdot d\mathbf{r} =$$
$$\Lambda_0 \int \cos(\mathrm{K}\xi - \Omega t) d\xi = \tag{25}$$
$$= (\Lambda_0 / \mathrm{K}) \sin(\mathrm{K}\xi - \Omega t),\ \mathbf{K} = \mathrm{K}\hat{\boldsymbol{\xi}},\ \mathbf{\Lambda}_0 = \Lambda_0 \hat{\boldsymbol{\xi}}$$

with ξ defining the coordinate in the direction of $\mathbf{K}, \mathbf{\Lambda}_0$. Instead of (20) we now have

$$\mathbf{E} = \hat{\mathbf{E}}_1 E_1 e^{i\theta},\ \mathbf{H} = \hat{\mathbf{H}}_1 H_1 e^{i\theta}$$
$$\theta = \theta_1 - \omega_1 \Phi_\xi,\ \Phi_\xi = (\Lambda_0 / \mathrm{K}) \sin(\mathrm{K}\xi - \Omega t) \tag{26}$$

For $\mathrm{K} \to 0$, i.e., for $\lambda_m \to \infty$, the problem reduces to (19)-(22).

The analog of (21) is now

$$e^{i\theta} = \Sigma_n G_n e^{i\theta_n},$$
$$G_n = J_n(\omega_1 \Lambda_0 / \mathrm{K}),\ \theta_n = \boldsymbol{\kappa}_n \cdot \mathbf{r} - \nu_n t \tag{27}$$
$$\boldsymbol{\kappa}_n = \mathbf{k}_1 - n\mathbf{K} = \mathbf{k}_1 - n\mathrm{K}\hat{\boldsymbol{\xi}},\ \nu_n = \omega_1 - n\Omega$$

It is noted that even though the Bessel functions argument (27) is of FO, it involves the ratio of the MM and EM wavelengths $k_1 / \mathrm{K} = \lambda_m / \lambda_1$, which is not necessarily small and must be assessed for each concrete case. The analog of (22) is now

$$\mathbf{E} = \hat{\mathbf{E}}_1 \Sigma_n E_n e^{i\theta_n},\ \mathbf{H} = \hat{\mathbf{H}}_1 \Sigma_n H_n e^{i\theta_n}$$
$$\theta_n = \boldsymbol{\kappa}_n \cdot \mathbf{r} - \nu_n t,\ E_n / E_1 = H_n / H_1 = G_n \tag{28}$$

In (27), (28), in addition to the temporal spectrum ν_n, we have a discrete spatial spectrum of $\boldsymbol{\kappa}_n$. Due to the vector character of $\boldsymbol{\kappa}_n$, when $\mathbf{k}_1$ is not parallel to $\mathbf{K}$, each spectral component propagates in a slightly different direction, possessing a different phase velocity according to

$$C_n = \nu_n / |\boldsymbol{\kappa}_n| = (\omega_1 - n\Omega) / |\mathbf{k}_1 - n\mathbf{K}| \tag{29}$$

6 IMPORT FOR NEW FIZEAU-TYPE EXPERIMENTS

Doppler Effect frequency shifts are usually associated with moving sources or boundaries. It is therefore of interest to note, as shown in (28) that wave-fronts in moving media can also create a spectrum, without involving moving material boundaries. As far as this author is aware, this phenomenon was not documented before (24) in the present EX context. In a sense, it is akin to some acousto-optics experiments involving interaction of sound and EM waves, but rather than having constitutive parameters modulated by sound, here medium velocity is involved. The present results might suggest new approaches to the celebrated FE.

The classical FE (e.g., Van Bladel, 1984, p. 120ff.) measures the EM wave effective phase velocity C_{eff} in a column of a moving medium (water in the original FE), characterized by C in the rest RF. The results tally with the SR velocity addition formula (Pauli, 1958). Consider (18)-(22), or (23)-(28), with $\mathrm{K} \to 0$, for parallel velocity and propagation directions, and $\Omega = 0$. From θ (20) or (26) we then find effective values

$$k_{eff} = k_1 - \omega_1\Lambda_0 = \omega_1 / C - \omega_1\Lambda_0 \tag{30}$$

$$\begin{aligned} C_{eff} &= \omega_1 / k_{eff} = c / n_{eff} = 1/(C^{-1} - \Lambda_0) \\ &\approx C + C^2\Lambda_0 = C + \mathrm{v}_0(1 - 1/n^2) \end{aligned} \tag{31}$$

with index of refraction $n = c/C$ in the rest RF. This is the basis for the classical FE. Essentially, with C known, the quest is for the value of Λ_0 in order to compute C_{eff} or vice-versa.

Exploiting the present theory, rather than using an interferometer setup and measuring the displacement of diffraction fringes, as done in the traditional FE, Λ_0 can be found from measurements of spectral components. Thus by measuring the amplitude of waves, and solving for relevant arguments of F_n, G_n in (22), (28), respectively, the value of Λ_0 can be extracted. Using (30), (31), the effective parameters can be computed. One can envision a medium set into periodic motion as in (19) or (23). The EM wave propagated through the medium will display a spectrum of discrete frequencies ν_n, (21), (27). In both cases the cumbersome interferometer setup involving a moving water column is obviated. This also solves the problem of the irregular flow at the source and sink regions where the fluid is injected and drained, as in the classical FE. Better resolution (AKA selectivity) of sidebands can be attained by electronically down-shifting frequencies after detection (AKA mixing, or heterodyning) employed in radio communications techniques.

7 SCATTERING PROBLEMS

SP for the EX with time-independent $\mathbf{v}$ have been discussed before (Collier and Tai, 1965; Censor, 1969; Censor, 1972).

As in the FE and other cases (Censor, 1969b), fluid-dynamics continuity problems of the medium flow in the presence of the scatterer are arbitrarily ignored, assuming that the flow is maintained as if the material scatterer has no effect. Otherwise complicated problems ensue that cannot be tackled with the analytical tools employed here. Realistic flows have been considered in (Censor, 1972).

7.1 Scattering by Plane Interfaces

As the simplest example for a SP consider a perfectly conducting plane at $x = 0$, with a perpendicularly incident wave as in (26) with Φ_x replacing Φ_ξ

$$\begin{aligned} &\mathbf{E}_i = \hat{\mathbf{z}}E_1 e^{i\theta_i},\, \mathbf{H}_i = -\hat{\mathbf{y}}H_1 e^{i\theta_i} \\ &\theta_i = k_1 x - \omega_1 t - \omega_1\Phi_x,\, E_1/H_1 = \sqrt{\mu/\varepsilon} = \eta \end{aligned} \tag{32}$$

the ratio η defining the medium impedance in the region $x < 0$.

The reflected wave $\mathbf{E}_r$ must satisfy the BC $\mathbf{E}_r = -\mathbf{E}_i|_{x=0}$. The BC prescribe identical time variation for all waves at the boundary $x = 0$, hence

$$\begin{aligned} &\mathbf{E}_r = \hat{\mathbf{z}}\rho E_1 e^{i\theta_r},\, \mathbf{H}_r = \hat{\mathbf{y}}\rho H_1 e^{i\theta_r} \\ &\theta_r = -k_1 x - \omega_1 t - \omega_1\Phi_x \end{aligned} \tag{33}$$

with $\rho = -1$ denoting the reflection coefficient.

Consider next a refractive medium in the region $x > 0$, with frequency dependent rest RF constitutive parameters $\bar{\mu}(\nu)$, $\bar{\varepsilon}(\nu)$. Correspondingly the RF phase velocity, impedance, are $\bar{C}(\nu) = 1/\sqrt{\bar{\mu}\bar{\varepsilon}}$, $\bar{\eta}(\nu) = \sqrt{\bar{\mu}/\bar{\varepsilon}}$, respectively. In the region $x < 0$ the parameters $\mu(\omega_1)$, $\varepsilon(\omega_1)$, remain dependent on the excitation frequency ω_1 only, in order to satisfy the ME subject to the EX, (15).

The EX solutions (32), (33), are recast in spectral components as in (28)

$$\begin{aligned} &\mathbf{E}_i = \hat{\mathbf{z}}\Sigma_n E_n e^{i\theta_{n,i}},\, \mathbf{H}_i = -\hat{\mathbf{y}}\Sigma_n H_n e^{i\theta_{n,i}} \\ &\theta_{n,i} = \kappa_n x - \nu_n t \end{aligned} \tag{34}$$

$$\begin{aligned} &\mathbf{E}_r = \hat{\mathbf{z}}\Sigma_n \rho_n E_n e^{i\theta_{n,r}},\, \mathbf{H}_r = \hat{\mathbf{y}}\Sigma_n \rho_n H_n e^{i\theta_{n,r}} \\ &\theta_{n,r} = -\kappa_n x - \nu_n t \end{aligned} \tag{35}$$

and the transmitted wave is given by

$$\begin{aligned} &\mathbf{E}_t = \hat{\mathbf{z}}\Sigma_n \bar{E}_n e^{i\theta_{n,t}},\, \mathbf{H}_t = -\hat{\mathbf{y}}\Sigma_n \bar{H}_n e^{i\theta_{n,t}},\, \bar{E}_n = \tau_n E_n \\ &\theta_{n,t} = \bar{\kappa}_n x - \nu_n t,\, \bar{\kappa}_n = \nu_n/\bar{C}_n,\, \bar{C}_n = \bar{C}(\nu_n) \\ &\bar{E}_n/\bar{H}_n = \bar{\eta}_n,\, \bar{H}_n = H_n\tau_n\eta/\bar{\eta}_n \\ &\bar{\mu}_n = \bar{\mu}(\nu_n),\, \tau_n = \tau_n(\nu_n) \end{aligned} \tag{36}$$

The solution for ρ_n, τ_n, are given by the familiar formulas

$$\rho_n = (\bar{\eta}_n - \eta)/(\bar{\eta}_n + \eta),\, \tau_n = 2\bar{\eta}_n/(\bar{\eta}_n + \eta) \tag{37}$$

7.2 Scattering by Circular Cylinders

For the SP of a perfectly conducting circular cylinder of radius a, the incident excitation plane wave is once again given by (32). Leaving the EX factor intact and recasting the ZO solution $e^{ik_1 x}$ in BFS yields, (15)-(17), in cylindrical coordinates

$$\mathbf{E}_i = e^{-i\omega_1 \Phi_x}\mathbf{E}_{i,1} = \hat{\mathbf{z}}E_1 e^{-i\omega_1\Phi_x}\Sigma_m L_m e^{im\psi - i\omega_1 t}$$

$$\begin{aligned}\mathbf{H}_i &= e^{-i\omega_1\Phi_x}\partial_{\mathbf{r}}\times\mathbf{E}_{i,1}/i\omega_1\mu \\ &= i(E_1/\eta)e^{-i\omega_1\Phi_x}\Sigma_m\mathbf{L}_m e^{im\psi - i\omega_1 t}\end{aligned} \quad (38)$$

$$L_m = i^m J_m(k_1 r),\ \mathbf{L}_m = \hat{\mathbf{z}}\times\partial_{k_1\mathbf{r}}L_m$$

Accordingly we construct the scattered wave as

$$\begin{aligned}\mathbf{E}_s &= \hat{\mathbf{z}}E_1 e^{-i\omega_1\Phi_x}\Sigma_m a_m M_m e^{im\psi - i\omega_1 t} \\ \mathbf{H}_s &= i(E_1/\eta)e^{-i\omega_1\Phi_x}\Sigma_m a_m \mathbf{M}_m e^{im\psi - i\omega_1 t} \\ M_m &= i^m H_m^{(1)}(k_1 r),\ \mathbf{M}_m = \hat{\mathbf{z}}\times\partial_{k_1\mathbf{r}}L_m\end{aligned} \quad (39)$$

with $H_m^{(1)}$ denoting the first kind Hankel functions. On application of the BC $\mathbf{E}_r = -\mathbf{E}_i\,|_{r=a}$, the EX factor cancels and we find

$$a_m = -J_m(k_1 r)/H_m^{(1)}(k_1 r) \quad (40)$$

the familiar SC of the ZO problem.

For a material cylinder we start with (38), (39), and recast $e^{-i\omega_1\Phi_x}$ in BFS

$$\begin{aligned}& e^{-i(\omega_1\Lambda_0/\mathrm{K})\sin(\mathrm{K}x - \Omega t)} = \Sigma_n G_n e^{-in(\mathrm{K}r\cos\psi - \Omega t)} \\ &= \Sigma_{n,p} G_n e^{in\Omega t}P_{n,p}e^{ip\psi},\ P_{n,p} = (-i)^p J_p(n\mathrm{K}r)\end{aligned} \quad (41)$$

Hence

$$\begin{aligned}\mathbf{E}_i &= \hat{\mathbf{z}}E_1\Sigma_{n,p,m}Q_{n,p,m}e^{i(p+m)\psi - i\nu_n t} \\ \mathbf{H}_i &= i(E_1/\eta)\Sigma_{n,p,m}\mathbf{Q}_{n,p,m}e^{i(p+m)\psi - i\nu_n t} \\ Q_{n,p,m} &= G_n P_{n,p}L_m,\ \mathbf{Q}_{n,p,m} = G_n P_{n,p}\mathbf{L}_m\end{aligned} \quad (42)$$

Similarly, (39) becomes

$$\begin{aligned}\mathbf{E}_s &= \hat{\mathbf{z}}E_1\Sigma_{n,p,m}a_{n,p,m}S_{n,p,m}e^{i(p+m)\psi - i\nu_n t} \\ \mathbf{H}_s &= i(E_1/\eta)\Sigma_{n,p,m}a_{n,p,m}\mathbf{S}_{n,p,m}e^{i(p+m)\psi - i\nu_n t} \\ S_{n,p,m} &= G_n P_{n,p}M_m,\ \mathbf{S}_{n,p,m} = G_n P_{n,p}\mathbf{M}_m\end{aligned} \quad (43)$$

In the cylinder's interior $r < a$ fields are obtained as a superposition of regular cylindrical waves of modes u, at frequencies ν_n, satisfying the ME (16) with rest RF parameters $\bar{\mu}(\nu_n), \bar{\varepsilon}(\nu_n)$

$$\begin{aligned}\mathbf{E}_t &= \hat{\mathbf{z}}E_1\Sigma_{n,u}b_{n,u}T_{n,u}e^{iu\psi - i\nu_n t} \\ \mathbf{H}_t &= i(E_1/\bar{\eta})\Sigma_{n,u}b_{n,u}\mathbf{T}_{n,u}e^{iu\psi - i\nu_n t} \\ T_{n,u} &= i^u J_u(\bar{\kappa}_n r),\ \mathbf{T}_{n,u} = \hat{\mathbf{z}}\times\partial_{\bar{\kappa}_n\mathbf{r}}T_{n,u}\end{aligned} \quad (44)$$

On application of BC

$$\mathbf{E}_i + \mathbf{E}_s - \mathbf{E}_t = 0\,|_{r=a},\ \hat{\boldsymbol{\psi}}\cdot(\mathbf{H}_i + \mathbf{H}_s - \mathbf{H}_t)\,|_{r=a} \quad (45)$$

prescribing the fields continuity on the interface, the orthogonality of angular modes $e^{iu\psi}$ prescribes nonzero coefficients $b_{n,u} \neq 0$ for $u \neq p+m$. Hence (44) can be recast to include the constraint

$$\begin{aligned}\mathbf{E}_t &= \hat{\mathbf{z}}E_1\Sigma_{n,p,m}\delta(p+m-u)b_{n,u}T_{n,u}e^{iu\psi - i\nu_n t} \\ \mathbf{H}_t &= i(E_1/\bar{\eta})\Sigma_{n,p,m}\delta(p+m-u)b_{n,u}\mathbf{T}_{n,u}e^{iu\psi - i\nu_n t}\end{aligned} \quad (46)$$

where δ denotes the Discrete Kronecker Delta Function. For each spectral component n, and angular mode u the BC lead to an infinite set of equations, which can only solved if properly truncated.

Consider (41)-(46) for the case of a monopole $u = p = m = 0$. This only works for thin cylinders, hence n must be properly truncated, otherwise higher multipole terms must be included. Accordingly

$$\begin{aligned}\mathbf{E}_i &= \hat{\mathbf{z}}E_1\Sigma_n Q_n e^{-i\nu_n t},\ Q_n = Q_{n,0,0} = G_n P_{n,0}L_0 \\ \mathbf{H}_i &= i(E_1/\eta)\Sigma_n\mathbf{Q}_n e^{-i\nu_n t},\ \mathbf{Q}_n = \mathbf{Q}_{n,0,0} = G_n P_{n,0}\mathbf{L}_0\end{aligned} \quad (47)$$

and similarly

$$\begin{aligned}\mathbf{E}_s &= \hat{\mathbf{z}}E_1\Sigma_n a_n S_n e^{-i\nu_n t} \\ \mathbf{H}_s &= i(E_1/\eta)\Sigma_n a_n\mathbf{S}_n e^{-i\nu_n t} \\ S_n &= S_{n,0,0} = G_n P_{n,0}M_0,\ \mathbf{S}_n = \mathbf{S}_{n,0,0} = G_n P_{n,0}\mathbf{M}_0\end{aligned} \quad (48)$$

$$\begin{aligned}\mathbf{E}_t &= \hat{\mathbf{z}}E_1\Sigma_n b_n T_n e^{-i\nu_n t} \\ \mathbf{H}_t &= i(E_1/\bar{\eta})\Sigma_n b_n\mathbf{T}_n e^{-i\nu_n t} \\ T_n &= T_{n,0} = J_0(\bar{\kappa}_n r),\ \mathbf{T}_n = \mathbf{T}_{n,0} = \hat{\mathbf{z}}\times\partial_{\bar{\kappa}_n\mathbf{r}}T_n\end{aligned} \quad (49)$$

From (45) and (47)-(49), we get explicit equations for the SC

$$\begin{aligned}& J_0(k_1 a) + a_n H_0^{(1)}(k_1 a) - \bar{b}_n J_0(\bar{\kappa}_n a) = 0 \\ & J_0'(k_1 a) + a_n H_0'^{(1)}(k_1 a) - \bar{b}_n(\eta/\bar{\eta})J_0'(\bar{\kappa}_n r) = 0 \\ & \bar{b}_n = b_n/[J_n(\omega_1\Lambda_0/\mathrm{K})J_0(n\mathrm{K}a)]\end{aligned} \quad (50)$$

with the prime denoting differentiation with respect to the argument. In form (50) is similar to the classical SP, but including the present velocity effects, therefore solving for the coefficients a_n, b_n is straightforward.

8 SUMMARY AND CONCLUDING REMARKS

The advent of SR (Einstein, 1905) facilitated the analysis of SP involving moving objects and media. However, SR is founded on the concept of inertial RF moving at constant $\mathbf{v}$.

A multitude of scientific and engineering problem involve varying velocities. Heuristic models that in the case of constant $\mathbf{v}$ merge into exact SR are not unique. Presently the Quasi Lorentz Transformation (Censor, 2005, 2010) (9) is employed. Subject to the constraint of MM and EM space and time scaling, the FO ME and FT (1), (2), (6), apply to varying $\mathbf{v}(\mathbf{r}, t)$.

The EX, originating with Collier and Tai (1965) provides FO SR solutions to ME in moving media. The method is generalized here to time-dependent irrotational velocity fields. Previously (Collier and Tai, 1965) only time-independent velocity systems have been considered.

In periodically moving media the solution for the ZO case of plane waves displays discrete sideband spectra. This provides new approaches to the FE. Unlike the original FE, employing interference experiments, the present results suggest measurements based on analysis of the spectra created by periodical mechanical flows or waves.

Canonical SP examples are given for scattering by plane interfaces and by circular cylinders, in the presence of periodically moving embedding media. It is shown that opaque objects, like the perfectly conducting interfaces above, yield the classical SC for media at rest, involving only the excitation frequency ω_1. On the other hand, refractive scatterers are excited by the frequencies created by the MM motion, (37), (43), displaying SC depending on the sideband frequencies.

The results suggest new methods for remote sensing the material parameters of objects that are not directly accessible. To further investigate the present model, more canonical SP will have to be investigated, with various MM motional modes.

REFERENCES

Censor, D., 1969a. Propagation and scattering in radially flowing media, *IEEE Transactions on Microwave Theory and Techniques*, Vol. MTT-17, pp. 374-378.

Censor, D., 1969b. Scattering of a plane wave at a plane interface separating two moving media, *Radio Science*, Vol. 4, pp. 1079-1088.

Censor, D., 1972. Interaction of electromagnetic waves with irrotational fluids, *Journal of the Franklin Institute*, Vol. 293, pp. 117-129.

Censor, D., 2005. Non-relativistic scattering: pulsating interfaces, In *PIER—Progress In Electromagnetic Research*, Vol. *54*, pp. 263-281.

Censor, D., 2010. The need for a first-order Quasi Lorentz Transformation. In *AIP Conference Proceedings*, Vol. 1301, pp. 3-13.

Collier, J. R., Tai C.T., 1965. Guided waves in moving media. *IEEE Transactions on Microwave Theory and Techniques*. Vol. MTT-13, pp. 441-445.

Einstein, A., 1905. Zur Elektrodynamik bewegter Körper. *Ann. Phys. (Lpz.), 17, pp. 891-921*. English translation: On the Electrodynamics of moving bodies. In *The Principle of Relativity*, 1952, Dover.

Minkowski, H., 1908. Die Grundgleichungen für die elektromagnetischen Vorgänge in bewegten Körpern. In *Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen*, pp. 53-116.

Nathan, A., Censor, D., 1968. Extended ∇ relations with reference to EM waves in moving simple media, *IEEE Transactions on Microwave Theory and Techniques*, Vol. MTT-16, pp. 883-884.

Pauli, W., 1958. *Theory of Relativity*, Pergamon.

Sommerfeld, A., 1964. *Electrodynamics*, p. 280ff. Academic Press.

Stratton, J.A., 1941. *Electromagnetic Theory*, McGraw-Hill, 1941.

Tai, C.T., 1964. A study of electrodynamics of moving media, *Proceedings of the IEEE*, Vol. 52, pp. 685-689.

Van Bladel, J., 1984. *Relativity and Engineering*, Springer.