NON-UNIFORM SAMPLING FOR SPECTRAL ANALYSIS OF MULTI-BAND SIGNALS

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Abstract: Recently, sub-Nyquist sampling of wideband signals has gained much attention in wireless communications. One of the main reasons for this popularity is the reduction in energy consumption in the ADCs. In this paper, we present an overview of the best known non-uniform sub-Nyquist sampling techniques and the associated spectral analysis. Spectral analysis using sub-Nyquist samples can be done in two ways. We can reconstruct the original Nyquist samples from the sub-Nyquist samples and then by using Fourier transform we can obtain the estimated power spectral density using classical methods like Welch method. We can also obtain the power spectral estimate directly from the sub-Nyquist samples without reconstructing the Nyquist samples. Both approaches can be used based on the application.

1 INTRODUCTION

In recent years, spectral estimation has attracted more and more interest due to its importance in the field of cognitive radio, where secondary users (unlicensed) can opportunistically use the frequency spectrum unused (holes) by primary users (licensed). For this purpose, the secondary user is forced to scan the radio environment of broadband in order to detect the holes. In [Zhang, 2010] authors use Filters Banks in order to share the wide band spectrum in many narrow bands and to comput in each narrowband in parallel the PSD. In [Tian and Giannakis, 2006] author apply wavelet transform in order to detect transitions between different subband of the wide band signal. But this type of wideband analysis involves a very high sampling rate and consequently a large energy consumption. This situation encouraged the researchers to examine the exploitable characteristics of the spectrum for a reduction of the requirements of the ADCs. A lot of papers propose to use the spectrum sparsity property to reduce the sampling rate [Bourdoux A. and der Perre L., 2011] [Miar et al., 2012]. Multi- Coset sampling studied in [Rashidi Avendi, 2010] [Venkataramani and Bresler, 2000] [Venkataramani and Bresler, 2001] is an effective way to reduce the frequency sampling for multi-band signals whose frequency support is a finite union of intervals. Similarly, [Mishali and Eldar, 2010] evaluates the subnyquist sampling for multi-band analog signals using a sparse ModulatedWideband Converter (MWC), consisting of several branches, each of which uses a different mixing function followed by a low-pass filtering and a low rate uniform sampling. Both the above approaches can be combined with compressive sampling. The reconstruction process can be carried out using methods such as sparse recovery algorithm LASSO (Least Absolute Shrinkage and Selection Operator) [Tibshirani, 1996] or even using traditional methods such as MUSIC (MUltiple Signal Classification) [Bresler, 2008]. The methods described in [Mishali and Eldar, 2010] [Rashidi Avendi, 2010] are known as blind spectral sampling methods, whose objective is to sample the signal at a minimum rate and reconstruct the unknown spectrum from these samples, while considering the spectrum is sparse. It was found in this work that for most signals, the minimum sampling frequency is given by the average lower limit of Landau (as studied in [Feng and Bresler,1996] [Landau, 1967]), which is equal to the Nyquist frequency divided by the spectrum occupancy. But for optimal reconstruction, all these meth

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ods assume that the number of bands and the maximum bandwidth, a band can have, is known.

The methods presented before, assume that the signal to be analyzed is sparce in the frequency domain. There are other non-uniform sampling methods which do not make this assumption like Additive Random Sampling (ARS), Jittered Random Sampling (JRS). These methods are alias-free therefore we can sample at sub-nyquist rates. But we will show later that their performance is not good in terms of spectral analysis. In [Traoré et al., 2013], we proposed a new sampler based on Multi-Coset (MC) sampling scheme, which adjusts its sampling rate according to the changes in the frequency spectrum of the input signal. That is, if the signal is sparse in the frequency domain, it samples at sub-nyquist rate, else it samples at nyquist rate.

In this paper, we present an overview of the best known techniques of non-uniform sampling and the associated spectral analysis. This article is organized as follows. In Section 2, we present two non-uniform sampling techniques (Jittered Random Sampling and Additive Ransom Sampling) and the associated spectral estimation. In Section 3, we present a well known non-uniform sampling technique (Multi-Coset Sampling) and its limitations. In Section 4, we present an adaptative Multi-Coset Sampling technique. Numerical results are presented in Section 5 followed by the conclusion in Section 6.

2 NON-UNIFORM SAMPLING

Uniform Sampling (US) consists in representing a signal $x(t)$ with the values $x(k)$ at time instants which are integer multiples of T, called sampling period. In the case of non-uniform sampling, all $\{t_k\}_{k\in\mathbb{Z}}$, sampling instants, must be different from the set of instants $\{kT\}_{k\in\mathbb{Z}}$, for a sampling period T. The non-uniformly sampled digital signal obtained is given by:

$$x_s(t) = \sum_{k=-\infty}^{+\infty} x(t)\delta(t-t_k) \tag{1}$$

The estimate of the Fourier transform of the signal randomly sampled at the instants $\{t_k\}_{k\in\mathbb{Z}}$ consists only of the Fourier transform of continuous time $x(t)$ denoted $X(f)$ such that :

$$\hat{X}_s(f) = E\left[\sum_{k=-\infty}^{+\infty} x(t_k)e^{-j2\pi f t_k}\right] = \frac{1}{T}X(f) \tag{2}$$

Next we presente an overview of the well know non-uniform sampling techniques.

2.1 Jittered Random Sampling

In JRS Sampling, at the uniformly distributed sampling instants, a deliberate error obeying a statistical law of distribution defined on an interval $[-0.5T, 0.5T]$ is added. The n sampling instants by JRS mode are described by [Wojtiuk, 2000] [Ben Romdhane, 2009]:

$$t_k = kT + \tau_k, \quad 1 \le k \le n \tag{3}$$

with $E[t_k] = kT$, $Var[t_k] = \sigma^2$ and T is the average sampling period. For $n \ge 1$, $\{\tau_k\}_{k=1}^{n}$ is a set of random variables independent and identically distributed with probability distribution $p_1(\tau)$, with zero mean and variance σ^2. The probability density of the k^{th} sampling instant is given by:

$$p_k(t) = p_1(t-kT), \quad 2 \le k \le n \tag{4}$$

The digital signal obtained is given by :

$$x_s(t) = \sum_{k=-\infty}^{+\infty} x(t)\delta(t-kT-\tau_k) \tag{5}$$

There are several formulations of the Power Spectral Density (PSD) for random sampling using JRS mode [Wojtiuk, 2000] [Shapiro and Silverman, 1960]. However, the most exhaustive formulation which is based on justified hypotheses is that of Wojtiuk [Wojtiuk, 2000]. Wojtiuk demonstrated that in the case of JRS mode, the PSD of a randomly sampled signal is given by (6) :

$$P_{xJRS}(f) = P_x(f) \otimes \{\frac{1}{T^2}\sum_{n=-\infty}^{+\infty} |\Phi_1(\frac{n}{T})|^2\delta(f-\frac{n}{T}) + \frac{1}{T}\left[1-|\Phi_1(f)|^2\right]\} \tag{6}$$

where $P_x(f)$ is the PSD of the signal $x(t)$, $\otimes$ is the convolution operation, $\frac{1}{T}$ is the mean sampling rate and $\Phi_1(f)$ is the characteristic function of the distribution $p_1(\tau)$. The discrete term in (6) represents the aliasing operation and the continous term represents a wide band noise (Fig. 3). It was also verified that, for a uniform distribution, we improve the elimination of aliasing when we increase the average sampling rate and the standard deviation of the distribution (Fig. 4).

For the Gaussian distribution we have a very strong reduction for $\frac{\sigma}{T} = 0.2887$. The complete elimination of spectral aliasing achieved for uniform and poisson distribution (Fig. 5).

2.2 Additive Random Sampling

When the time interval between two successive sampling instants follow a statistical law of distribution defined on an interval $[0.5T, 1.5T]$, the sampling is said random cumulative (ARS, Additive Random Sampling). The n sampling instants by ARS mode are described by [Wojtiuk, 2000] [Ben Romdhane, 2009]:

$$t_k = t_{k-1} + \tau_k = t_0 + \sum_{i=1}^{+\infty} \tau_i, \quad 1 \leq k \leq n \tag{7}$$

where $E[t_k] = kT$ and $Var[t_k] = k\sigma^2$

For $n \geq 1$, $\{\tau_k\}_{k=1}^{n}$ is a set of random variables independent and identically distributed of probability distribution $p_1(\tau)$, with zero mean and variance σ^2. For $k = 0$, $p_0(t) = \delta(t)$. The probability density of the k^{th} sampling instant is given by:

$$p_k(t) = \otimes_{i=1}^{k} p_1(t) \tag{8}$$

The digital signal obtained is given by :

$$x_s(t) = \sum_{k=-\infty}^{+\infty} x(kT + \tau_k)\delta(t - kT - \tau_k) \tag{9}$$

The estimate of the Fourier transform of the randomly sampled signal is given by [Wojtiuk, 2000] (10):

$$P_{xARS}(f) = P_x(f) \otimes \{\frac{1}{T^2}\delta(f) + \frac{1}{T}\Re[\frac{1+\Phi_1(f)}{1-\Phi_1(f)}]\} \tag{10}$$

From (10), the PSD of a randomly sampled signal according to the ARS mode is different from that of the JRS mode, because there is no sum of discrete components. Furthermore, the elimination of aliasing is better when we increase the ratio $\frac{\sigma}{T}$ (Fig. 6). For example, for gaussian distribution with $\frac{\sigma}{T} = 0.2887$, aliasing is completely eliminated (Fig. 7).

3 MULTI-COSET SAMPLING

In this section, we give an overview of the sampler based on Multi-Coset (MC) sampling [Mishali and Eldar, 2010, Rashidi Avendi, 2010] and discuss its limitations. MC sampling is a periodic non-uniform sampling technique which samples the signal $x(t)$ at a rate lower than the Nyquist rate, thereby capturing only the amount of information required for an accurate reconstruction of the signal. MC sampling starts by first choosing an appropriate sampling period T, less than or equal to the Nyquist period associated to $x(t)$. Two parameters L and p are selected such that $L \geq p > 0$. Then the input signal $x(t)$ is non-uniformly sampled at $t_i(n) = (nL + c_i)T$, where $1 \leq i \leq p$ and $n \in \mathbb{Z}$ [Mishali and Eldar, 2010] . The set $\mathcal{C} = \{c_i\}$ contains p distinct integers from $\mathbb{L} = \{0, 1, , L-1\}$ and is termed as the *sampling pattern*. In short, the process of MC sampling can be viewed as first sampling the input signal at a uniform rate with period T and then selecting only p non-uniform samples from L equidistant uniform samples. The process is repeated for consecutive segments of L uniform samples such that the p selected samples have a sampling period L. The set $\mathcal{C}$ specifies the p samples that are retained in each segment of length L such that $0 \leq c_1 < c_2 < ... < c_p \leq L-1$.

MC sampler is usually implemented by placing p ADCs in parallel as shown in Fig.1. Each ADC operates uniformly at a period $T_s = LT$. The $\Delta_i = c_i T$ represent the time shifts in sampling instants introduced in each branch. Thus we have

$$t_i(n) = (nL + c_i)T = (n + \frac{c_i}{L})T_s$$

where $1 \leq i \leq p$. It should be noted that MC sampling is highly sensitive to even small errors [Mishali and Eldar, 2010]. Because of this sensitivity, an apparently possible perfect reconstruction is not achievable and the signal easily becomes corrupted and unrecognizable in certain practical conditions. A good choice of the sampling pattern $\mathcal{C}$ reduces the margin of error due to spectral aliasing and sensitivity to noise in the reconstruction process [Rashidi Avendi, 2010]. It is quite evident from Fig.1 that once the sampling parameters (such as p) are selected, architecture of the MC sampler will remain unchanged irrespective of the input signal characteristics modifications. In other words, once designed, the sampler in Fig.1 cannot be changed because of hardware limitations. If the input signal changes, the MC sampler does not adapt, which results in sub-optimal sampling of the signal.

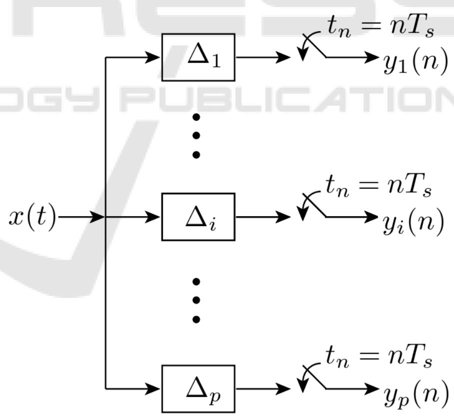

Figure 1: Multi-Coset (MC) sampler implemented as a multi-channel system.

This motivated the author of [Traoré et al., 2013] to look for an optimal system which conforms with the spectrum of the input signal. In the next section, we explain the functioning of this new sampling scheme and show that it is more flexible compared to the MC sampler in Fig.1.

4 DYNAMIC SINGLE BRANCH MULTI-COSET SAMPLER

With the exponential growth in the means of communications, modifying radio devices easily and cost-effectively has become business critical. Software radio (SW) technology brings the flexibility, cost efficiency and power to drive communications forward, with wide-reaching benefits realized by service providers and product developers through to end users [Palicot and al, 2010]. One of the main objectives of SW is to propose new technologies to design radio terminals and wireless infrastructure able to support hardware-independent, multi-service operations and are remotely reconfigurable. Furthermore,

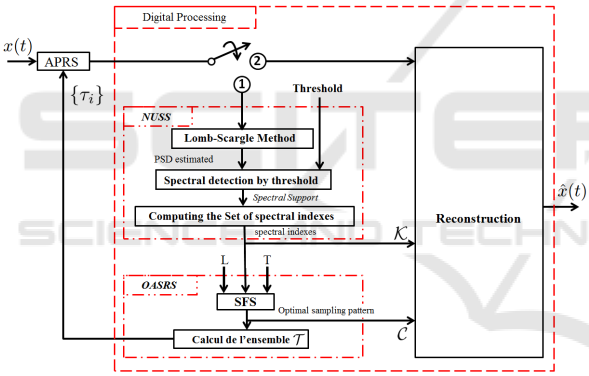

Figure 2: Dynamic Single Branch Non-Uniform Sampler

Cognitive radio continues to gain popularity as it adapts intelligently to the radio environment, there by dynamically managing the spectrum [Palicot and al, 2010]. As a result, the spectrum changes continuously and rarely remains constant. Keeping the aspects of Software radio and Cognitive radio in mind, authors in [Traoré et al., 2013] present a sampler, that not only adapts to the changes in the input signal but is also remotely reconfigurable and is, therefore, not constrained by the inflexibility of hardwired circuitry. [Traoré et al., 2013] proposed non-uniform sampler is shown in Fig. 2. They call it the *Dynamic Single Branch Non-Uniform Sampler* (DSB-NUS) or simply the DSB sampler. It operates in blind mode, without any knowledge of the input signal's spectral support and the number of bands. The DSB-NUS works in two phases:

1. Adaptation phase: When the switch is in position 1, the system has no information about $x(t)$ (number of bands N, bands location, set $\mathcal{K}$ of spetral indices, maximum bandwidth B_{max}. It performs a spectral analysis using block NUSS (Non Uniform Spectrum Sensing) (estimate the Spectral support of the signal and $\mathcal{K}$, using Lomb-Scargle Method Fig. 8). In the Optimal Average Sampling Rate Search block (OASRS), according to (L, T, $\mathcal{K}$), used the Sequential Forward Selection (SFS) algorithm to find the optimal sampling pattern $\mathcal{C}$ and then the optimal set of sampling instants $\{\tau_i\}$.
2. Reconstruction Phase: When the switch is in position 2, the DSB-NUS sampler performs multi-coset reconstruction.

5 NUMERICAL RESULTS

For simulations, we consider a multi-band signal with 6 bands, each with a maximum bandwidth of $80MHz$. $16-QAM$ modulation symbols are used that are corrupted by the additive white Gaussian noise with $\frac{E_b}{N_0} = 50dB$. The wideband of interest is in the range of $\mathcal{B} = [-1.0, 1.0]GHz$ i.e. $f_{nyq} = 2GHz$.

Figures 3-4-5 show the PSD of the non-uniform samples obtained using JRS method, for different values of the average sampling rate i.e. at Nyquist, $\frac{4}{5}$ of Nyquist and $\frac{3}{5}$ of Nyquist. Spectral analysis was performed using Welch method with 50% overlap. Missing samples were all replaced with zeros.

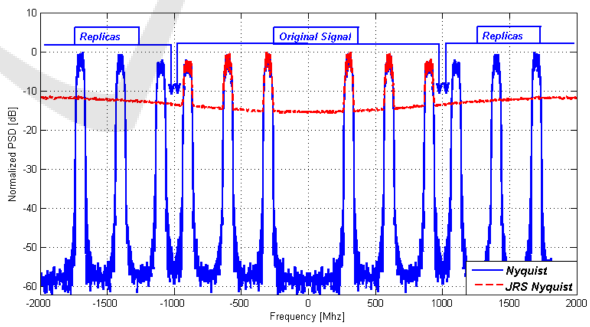

Figure 3: JRS presented free-aliasing property

We clearly see in Figure 3 that the curves obtained from JRS non-uniform samples are alias free. The curve obtained from uniform Nyquist samples present spectral replicas at multiples of the Nyquist rate.

In Figure 4, we show that the quality of the estimated PSD increases with increase in the average sampling rate of JRS technique. It can be seen that the wideband noise level increases and decreases in direct relation with the average sampling rate of the JRS

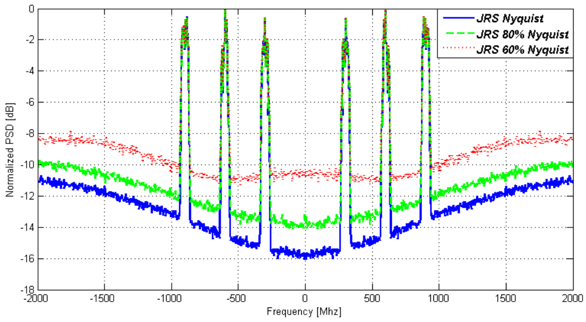

Figure 4: JRS with Uniform distribution and different average sampling rates

mode. For average sampling rate of 60% of Nyquist rate, the noise level is very high compared to the average sampling rate of 80% of Nyquist rate. However, this improvement in quality of the estimated PSD comes at the expense of more samples. Moreover, it should be noted that the results presented in Figure 4 are averaged over 100 iterations.

Next, in Figure 5, we present the effect of distributions of sampling instances, used for JRS sampling, on the estimated PSD. We compare three distributions, namely, Gaussian, Poisson and Uniform. We can see that when Gaussian distribution is used, spectral replicas appear, however, there amplitude is small. When Poisson distribution is used, no spectral replicas are observed, however the noise level has increased compared to the Gaussian distribution. Finally we present the estimated PSD obtained using uniform distribution and it can be seen that there are no spectral replicas. Furthermore, with unifrom distribution the noise level remains approximately the same as obtained with Gaussian distribution.

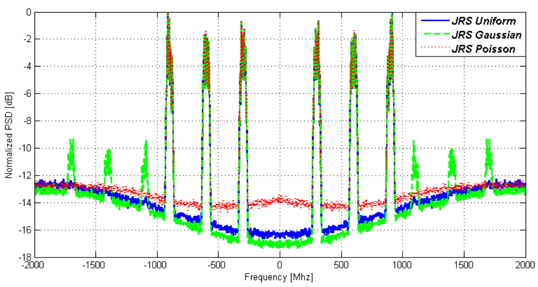

Figure 5: JRS with Uniform, Gaussian($\frac{\sigma}{T} = 0.2887$) and poisson distribution

The results presented in Figures6-7, are for ARS sampling. In Figure 6, the effect of the average sampling frequency of ARS is shown on the estimated PSD. It can be seen clearly, just as was observed in the case of JRS sampling that with increase in sampling rate the PSD estimates improve. Another important point to note is that the noise in the estimated PSD for ARS sampling is less than that of JRS sampling.

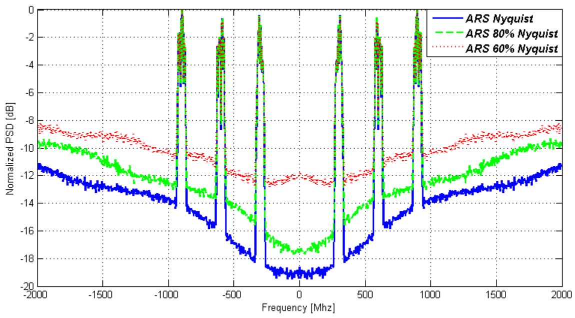

Figure 6: ARS with Uniform distribution and different average sampling rates

In Figure 7, the estimated PSD obtained with ARS sampling is plotted for three different distributions of the sampling instances. Again results are presented for Gaussian, Poisson and Uniform distributions. For Poisson distribution we have a high noise level. An interesting result to note is that, contrary to the JRS case, no spectral replicas are observed for the Gaussian distribution and furthermore, its performance is better compared to the uniform distribution.

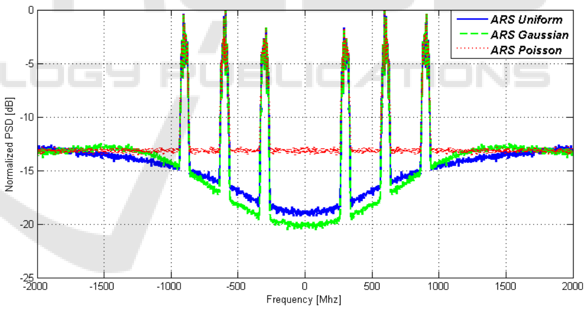

Figure 7: ARS with Uniform, Gaussian($\frac{\sigma}{T} = 0.2887$) and poisson distribution

Next in Figure 8, the estimated PSD obtained from DSB samples, is presented. The results are compared with the Welch method which uses uniform Nyquist samples. It must be noted that for the DSB sampler, no prior knowledge of the spectral support of the signal is available. The spectral support is computed in the adaptation phase using Lomb-Scargle method []. Note that while operating in adaptation phase $L = 128, p = 90$. The interesting aspect of the DSB sampler is that it will adjust p for the reconstruction phase, based on the information obtained in the adaptation phase. Therefore, the DSB sampler adjusts the sampling rate based on the spectral support of the

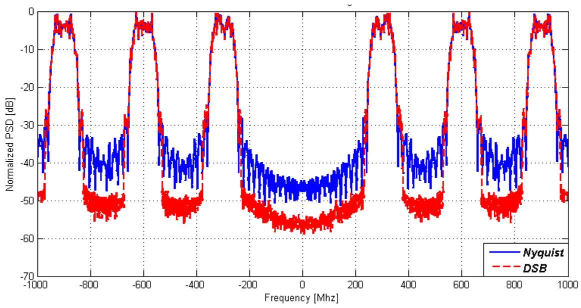

Figure 8: PSD with Lomb-Scargle method

input signal. From Figure 8, it can be seen that the results obtained using DSB sampler are quite good.

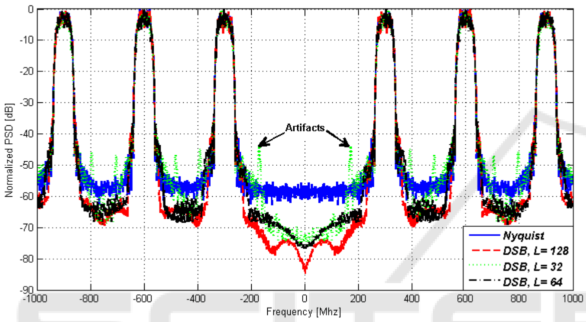

Figure 9: Influence of the parameter L in the variation of the PSD after DSB

Figure 9 shows the effect of the parameter L on the estimated PSD from DSB samples. For $L = 32$, spectral artifacts are observed in the estimated PSD, as shown in Figure 9. But as L is increased these artifacts are eliminated. Therefore, it is clear that the reconstructed spectrum is better when L increases.

6 CONCLUSIONS

In this paper we have presented some best known non-uniform sampling techniques. JRS and ARS mode gives approximately the same PSD without spectral replicas but with a much smaller dynamic than Uniform samples PSD. These two techniques can be used for spectrum sensing, condition not having a large difference in power between the bands. We dissassed about a new sampler based on Multi-Coset sampling scheme, which adjusts its sampling rate according to the changes in the frequency spectrum of the input signal. That is, if the signal is sparse in the frequency domain, it samples at sub-nyquist rate, else it samples at nyquist rate. We also saw the effect of the parameter L on the estimated PSD from DSB samples.

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