SCATTERING OF ELECTROMAGNETIC WAVE BY OFFSET SPHERICAL PARTICLES

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Abstract: The Lorentz–Mie theory is applicable to calculating scattering characteristics of spherical shaped particles. It is often applied to slightly non-spherical particles where its range of validity is uncertain. This paper defines the range of validity of the T-matrix technique of Barber and Hill as applied to homogeneous spherical and non-spherical particles. Scattering calculations are made for a set of non-absorbing homogeneous spherical particles with the origin of the particle offset over a certain range. The numerical results show that even for small offset value with the same input parameters, the phase function, extinction and scattering cross sections differ quite significantly compared to the generalized Lorentz–Mie technique known to give accurate scattering characteristics for spherical particle.

1 INTRODUCTION

The scattering of electromagnetic waves by spherical object is a problem that has received increased attention in past and recent years. Knowledge of the scattered field is required in many areas of science and engineering applications. The idea was first developed by Gustav Mie in 1908 in order to understand the colours that resulted from light scattering of gold particles suspended in water. Applications of Mie solution has been extended from one end of the electromagnetic spectrum to the other, from Ultraviolent solar radiation backscattered by stratospheric aerosols to satellites, through visible and Infrared radiation scattered by clouds and aerosols, to microwaves and radar scattered from large hydrometeors. An excellent introduction to the theory is reported in (Kerker 1969; van de Hulst 1981; Mishchenko, Travis et al. 2002; Bohren and Huffman 2008). Although, the Mie-theory it is exact, but with the emergence of computing it has become practical to calculate various scattering characteristics (Wiscombe 1980). The Mie-theory has limitation of being restricted to spherical particles. However, it has served as a reference for validation of other techniques for evaluating scattering properties from scatterers, and implementation of this theory with slightly non-spherical particles has yielded similar results.

This paper deals with the range of validity of T-matrix method reported in (Barber and Hill 1990) as applied to a lossless dielectric spherical particle with the origin moved of centre over a certain range. Our aim is show the uncertainty with reference to particle shape when calculating scattering cross sections in which the origin is displaced from the centre of the spherical object as previously reported (Waterman 1965; Barber and Yeh 1975; Barber and Hill 1990) by adopting and implementing the code in (Barber and Hill 1990) and not the theoretical analysis as numerous papers have already addressed this aspect. Nevertheless, the results in the cited references differ with ours. Clearly, the final results given in this paper are not new. Rather, our contribution is based on offset range validity at various frequency bands with the goal of providing a consistent result regardless of the mathematics that led to their derivation.

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2 RELATED WORK

The approach adopted in this paper was originally introduced by P C Waterman (Waterman 1965) as a technique for computing electromagnetic scattering by a smooth, perfectly conducting, homogeneous, arbitrarily shaped particle illuminated by an incident plane electromagnetic wave. This technique is also known as null field method (Zheng 1988) or extended boundary condition method (EBCM) (Barber and Wang 1978), and is developed further by (Barber and Yeh 1975), (Mishchenko and Travis 1994; Mishchenko, Travis et al. 1996; Mishchenko and Travis 1998; Mishchenko, Hovenier et al. 1999). The technique has also gained wide acceptance in the field of electromagnetic waves scattering due to its capability to calculate the scattering properties of arbitrarily shaped scatterers. The approach of the T-matrix formulation utilizes vector spherical harmonic function expansions of the incident and scattered fields in conjunction with boundary conditions at the surface of the scattering particles to obtain a system of linear equations relating the unknown expansion coefficients of the scattered field to the known coefficients of the incident field. The most attractive feature of T-matrix technique starts as Lorenz-Mie theory when the scattering particle is homogeneous or layered sphere composed of isotropic materials.

Given a specific scattering object, first step is to select an internal origin on the scattering particle and surround the object with imaginary sphere of radius r large enough to circumscribe the scatterer (Barber and Hill 1990; Mishchenko, Travis et al. 2002), and numerically perform the surface integrations over the scatterer which are required to fill the coefficient matrix. The next step is to carry out matrix operation to obtain the scattered field coefficients f_v and g_v. The final step then is to substitute the scattered field coefficients into (1) to yeild the scttered field and other desire characteristics.

$$E^s(kr) = E_0 \sum_{\nu=1}^{\infty} D_\nu \times \left[f_\nu M_\nu^\lambda(kr) + g_\nu N_\nu^\lambda(kr) \right] \tag{1}$$

where M and N are the vector spherical harmonic functions and the superscript 3 on M and N indicates that these functions are of the type suitable for radiation or outgoing fields (Hankel function), v represents the spherical harmonic triple index σ (even or odd), m, n. The argument of the vector spherical wave functions kr, where $k = 2\pi/\lambda$ denotes wave number in the surrounding medium, λ is the incident wavelength, and r is the position vector which defines a point in three-dimensional space. The $\mathbf{E_0}$ is the amplitude of the incident electric field, and D_v denotes normalization constant.

The important formulas for Mie scattering are well defined (Kerker 1969). The quantities required at this level are summarized. The amplitude matrix S for spherical object is a diagonal matrix; due to symmetry it takes the form:

$$E_s = \begin{pmatrix} E_\perp^s \\ E_\parallel^s \end{pmatrix} = \frac{e^{ikr}}{r} \begin{pmatrix} S_{hh} & 0 \\ 0 & S_{vv} \end{pmatrix} \begin{pmatrix} E_\perp^i \\ E_\parallel^i \end{pmatrix} \tag{2}$$

where

$$S_{vv}(\Theta) = = \sum_{n=1}^{\infty} \frac{2_n+1}{n(n+1)} \left[a_n \pi_n(\cos\theta) + b_n \tau_n(\cos\theta) \right] \tag{3}$$

$$S_{hh}(\Theta) = = \sum_{n=1}^{\infty} \frac{2_n+1}{n(n+1)} \left[b_n \pi_n(\cos\theta) + a_n \tau_n(\cos\theta) \right] \tag{4}$$

which are respectively, scattered electric field and complex scattering amplitude for the two orthogonal directions of incident polarization. $|S_{vv}|^2$ and $|S_{hh}|^2$ are the scattered intensities, π and τ are the angular function, a_n and b_n are Mie expansion coefficients in terms of the vector spherical harmonic, depend on the size parameter and on the complex refractive index. They are also expressed in terms of spherical Bessel functions.

3 SIMULATION RESULTS

An incident plane polarized wave propagating in the $+z$ direction is assumed, with the origin of the scatterer coincides with the spherical coordinate system for the calculation of differential scattering cross section of spherical, and slightly non-spherical particles. Some of the input parameters such as size parameter

$$\left(x = 1 = 2\pi r/_{\lambda}\right)$$

and refractive index

$$(m = 2.0 + i0.0)$$

are chosen to compare results (van de Hulst 1981). For a sphere, r is the radius of the scattering object but for non-spherical particles (oblate or prolate spheroid), the choice of r gives users the option of defining r as the radius of a sphere of either equal volume, or equal surface area to that of the scattering object while for offset spherical particle, it is the distance from the origin of the spherical coordinate system to the surface of offset spherical scatterer calculated by applying Pythagoras or Cosine rule.

Numerical illustrations confirm that the results for spherical bodies are identical at different frequencies compared to those obtained by the Mie theory. The check was extended to slightly non-spherical particles (oblate and prolate spheroids); a similar agreement is generally observed for both particles. The vertical and horizontal polarizations are denoted **VV** and **HH** respectively.

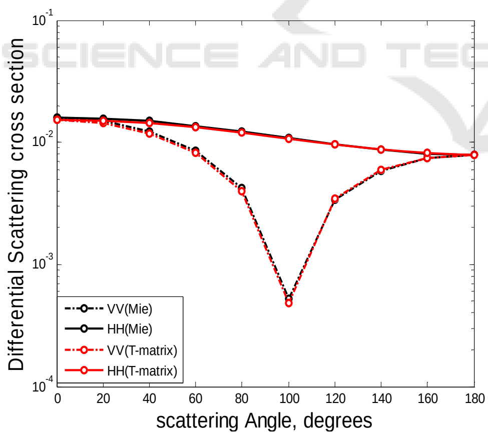

Figure 1: Comparison of Mie-theory and T-Matrix method for differential scattering cross sections of non-absorbing spherical and offset spherical particles at 220 GHz.

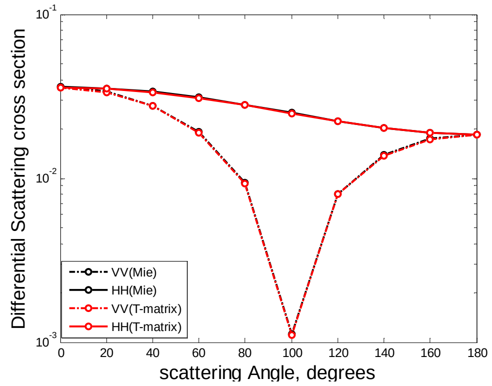

Figure 2: Comparison of Mie-theory and T-Matrix method for differential scattering cross sections of non-absorbing oblate spheroid at 94 GHz.

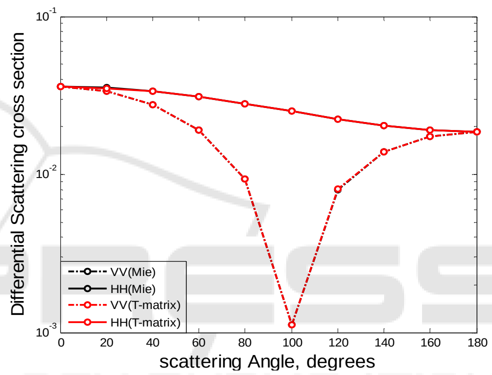

Figure 3: Comparison of Mie-theory and T-Matrix method for differential scattering cross sections of non-absorbing prolate spheroid at 94 GHz.

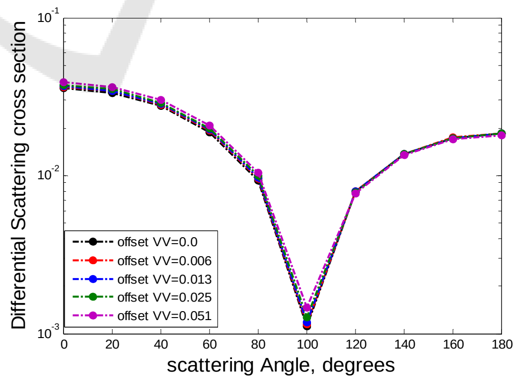

Figure 4: Comparison of Mie-theory and T-Matrix method for non-absorbing spherical and offset spherical particles at 94 GHz.

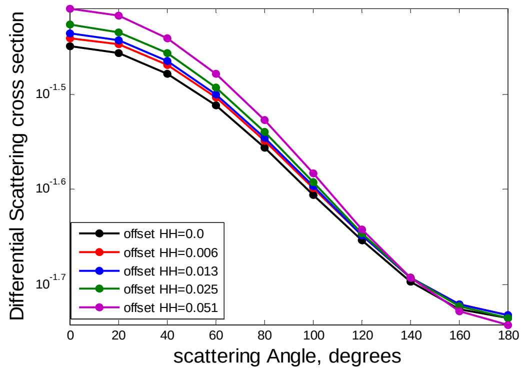

Figure 5: Comparison of Mie-theory and T-Matrix method non-absorbing spherical and offset spherical particles at 94 GHz.

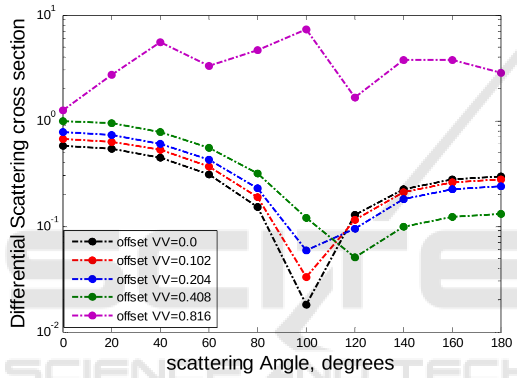

Figure 6: Comparison of Mie-theory and T-Matrix method for non-absorbing spherical and offset spherical particles at 5.8 GHz.

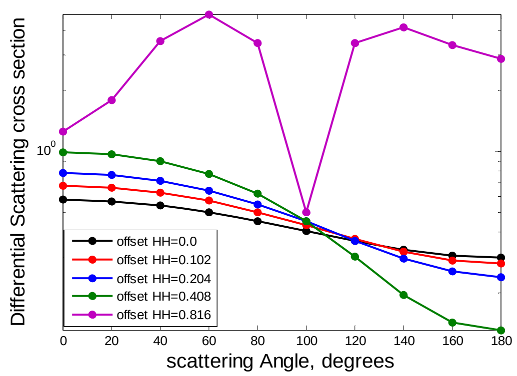

Figure 6: Comparison of Mie-theory and T-Matrix method for non-absorbing spherical and offset spherical particles at 5.8 GHz.

It is evident from Figure 1 that our results for both approach show good agreement with (Barber and Yeh 1975; Barber and Hill 1990) and (Waterman 1965) regardsless of the scatterer (i.e. spherical or offset spherical particles) at 220 GHz. This is also observed in Figure 2 and 3 comparing Mie-theory and T-Matrix method for differential scattering cross sections of non-absorbing spheroid at 94 GHz.

Evaluation of results from Mie-theory and T-Matrix method for non-absorbing spherical and with the spherical particle origin moved over a range at 94 GHz for vertical polarization still show reasonable agreement, but with horizontal polarization in Figure 5, it is obvious that the differential scattering cross-sections increases as the offset range increases, this effect is least noticed with increase in scattering angle. This shows that at higher operating frequencies the effect is insignificant; however, Figure 6 and 7 results show that even for small offset value with the same input parameters, the phase function, extinction and scattering cross sections differ quite significantly at 5.8 GHz compared to the generalized Lorentz–Mie technique.

4 CONCLUSIONS

We have demonstrated in our results that the effect of offset values relative to the frequency bands and how the scattering calculation in terms of geometric properties of the particles; (shapes and size parameter) for spherical and slightly non-spherical particles adopting Mie theory and T-matrix techniques are similar with previous works at higher frequency bands. Furthermore, the same trends of results are observed in terms of vertical polarization for non-absorbing offset spherical scatterer at 94 GHz. On the other hand, scattering characteristics for horizontal polarization at 94 GHz, and at lower frequency bands (i.e. 5.8 GHz) differs quite significantly with the same input parameters. Hence, scattering calculation from non-absorbing homogeneous spherical particles with the origin of the particle moved over a certain range should be used with caution depending on the wave frequency. This is particularly important due to previous concept that the same scattered cross section is obtained with the origin of the spherical scatterer at the centre. Obviously, the difference in our results with the former at lower frequency (i.e. 5.8 GHz) would lead to erroneous values being generated as the offset value increases and tends toward the radius of scattering object, and inaccurate prediction of hydrometeor shapes are likely if the previous concept is applied in radar and remote sensing applications.

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