Generalized Pythagoras Trees for Visualizing Hierarchies

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Abstract: Pythagoras Trees are fractals that can be used to depict binary hierarchies. But this binary encoding is an obstacle for visualizing hierarchical data such as file systems or phylogenetic trees, which branch into *n* subhierarchies. Although any hierarchy can be modeled as a binary one by subsequently dividing *n*-ary branches into a sequence of $n-1$ binary branches, we follow a different strategy. In our approach extending Pythagoras Trees to arbitrarily branching trees, we only need a single visual element for an *n*-ary branch instead of spreading the binary branches along a strand. Each vertex in the hierarchy is visualized as a rectangle sized according to a metric. We analyze several visual parameters such as length, width, order, and color of the nodes against the use of different metrics. The usefulness of our technique is illustrated by two case studies visualizing directory structures and a large phylogenetic tree. We compare our approach with existing tree diagrams and discuss questions of geometry, perception, readability, and aesthetics.

1 INTRODUCTION

Hierarchical data (i.e., trees) occurs in many application domains, for instance, as results of a hierarchical clustering algorithm, as files organized in directory structures, or as species classified in a phylogenetic tree. Providing an overview of possibly large and deeply nested tree structures is one of the challenges in information visualization. An appropriate visualization technique should produce compact, readable, and comprehensive diagrams, which ideally also look aesthetically appealing and natural to the human eye.

A prominent visualization method are node-link diagrams, which are often simply denoted as *tree diagrams*; layout and aesthetic criteria have been discussed (Reingold and Tilford, 1981; Wetherell and Shannon, 1979). Although node-link diagrams are intuitive and easy to draw, visual scalability and labeling often is an issue. An alternative, in particular easing the labeling problem, are indented trees (Burch et al., 2010) depicting the hierarchical structure by indentation. Further, layered icicle plots (Kruskal and Landwehr, 1983) stack boxes on top of each other for encoding a hierarchy, but waste space by assigning large areas to inner nodes on higher levels of the hierarchy. The Treemap approach (Shneiderman, 1992), which is applying the concept of nested boxes, produces space-efficient diagrams but complicates interpreting the hierarchical structure.

In this paper, we introduce Generalized Pythagoras Trees as an alternative to the above hierarchy visualization techniques. It is based on Pythagoras Trees (Bosman, 1957), a fractal technique showing a binary hierarchy as branching squares (Figure 1, a); the fractal approach is named after Pythagoras because every branch creates a right triangle and the Pythagorean theorem is applicable to the areas of the squares. We extend this approach to *n*-arily branching structures and use it for depicting information hierarchies (Figure 1, b). Instead of triangles, each recursive rendering step produces a convex polygonal shape where the corners are placed on a semi circle. The size of the created rectangles can be modified for encoding numeric information such as the number of leaf nodes of the respective subhierarchy (Figure 1, c).

We implemented the approach as an interactive tool and demonstrate its usefulness by applying it to large and deeply structured abstract hierarchy data from two application domains: a file system organized in directories and the NCBI taxonomy, a phylogentic tree that structures the living organisms on earth in a tree consisting of more than 300,000 vertices. Furthermore, a comparison to existing hierarchy visualization approaches provides first insights

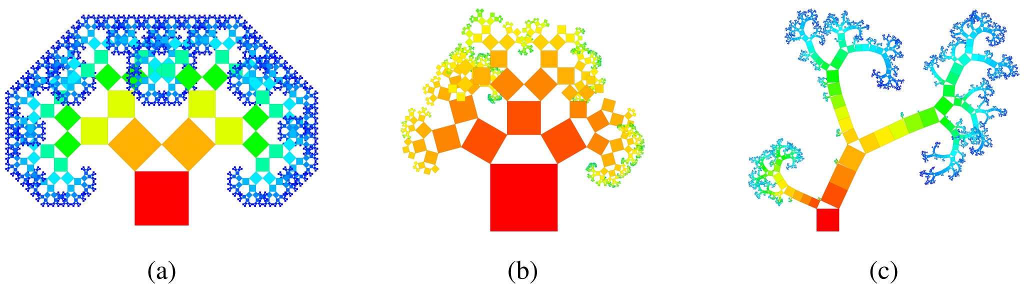


Figure 1: Extending Pythagoras Trees for encoding information hierarchies: (a) traditional fractal approach; (b) Generalized Pythagoras Tree applied to an n-ary information hierarchy; (c) additionally visualizing the number of leaves by the size of the inner nodes.

into the unique characteristic of Generalized Pythagoras Trees: a higher visual variety leads to more distinguishable visualizations, the fractal origin of the method supports identifying self-similar structures, and the specific layout seems to be particularly suitable for visualizing deep hierarchies. Finally, the created images are visually appealing as they show analogies to natural tree and branching structures.

2 RELATED WORK

The visualization of hierarchical data is a central information visualization problem that has been studied for many years. Typical respresentations include node-link, stacking, nesting, indentation, or fractal concepts as surveyed by Jürgensmann and Schulz (2010); Schulz (2011). Many variants of the general concepts exist, for instance, radial (Battista et al., 1999; Eades, 1992) and bubble layouts (Grivet et al., 2004; Lin and Yen, 2007) of node-link diagrams, circular approaches for stacking techniques (Andrews and Heidegger, 1998; Stasko and Zhang, 2000; Yang et al., 2003), or nested visualizations based on Voronoi diagrams (Balzer et al., 2005; Nocaj and Brandes, 2012).

Although many tree visualizations were proposed in the past, none provides a generally applicable solution and solves all related issues. For example, node-link diagrams clearly show the hierarchical structure by using explicit links in a crossing-free layout. However, by showing the node-link diagram in the traditional fashion with the root vertex on top and leaves at the bottom, much screen space stays unused at the top while leaves densely agglomerate at the bottom. Transforming the layout into a radial one distributes the nodes more evenly, but makes comparisons of subtrees more difficult. Node-link layouts of hierarchies have been studied in greater detail, for instance, Burch et al. (2011) investigated visual task solution strategies whereas McGuffin and Robert (2009) analyzed space-efficiency.

Indented representations of hierarchies are well-known from explorable lists of files in file browsers. Recently, Burch et al. (2010) investigated a variant as a technique for representing large hierarchies as an overview representation. Such a diagram scales to very large and deep hierarchies and still shows the hierarchical organization but not as clear as in node-link diagrams. Layered icicle plots (Kruskal and Landwehr, 1983), in contrast, use the concept of stacking: the root vertex is placed on top and, analogous to node-link diagrams, consumes much horizontal space that is as large as all child nodes together.

Treemaps (Shneiderman, 1992), a space-filling approach, are a prominent representative of nesting techniques for encoding hierarchies. While properties of leaf nodes can be easily observed, a limitation becomes apparent when one tries to explore the hierarchical structure because it is difficult to retrieve the exact hierarchical information from deeply nested boxes: representatives of inner vertices are (nearly) completely covered by descendants. Treemaps have been extended to other layout techniques such as Voronoi diagrams (Balzer et al., 2005; Nocaj and Brandes, 2012) producing aesthetic diagrams that, however, suffer from high runtime complexity.

Also, 3D approaches have been investigated, for instance, in Cone Trees (Carrière and Kazman, 1995), each hierarchy vertex is visually encoded as a cone with the apex placed on the circle circumference of the parent. Occlusion problems occur that are solved by interactive features such as rotation. Botanical Trees (Kleiberg et al., 2001), a further 3D approach, imitate the aesthetics of natural trees but are restricted to binary hierarchies, that is, n-ary hierarchies are modeled as binary trees by the strand model of Holton (1994); it becomes harder to detect the parent of a node.

The term fractal was coined by Mandelbrot (1982) and the class of those approaches has also been used for hierarchy visualization due to their self-similarity property (Koike, 1995; Koike and Yoshihara, 1993). With OneZoom (Rosindell and Harmon, 2012), the authors propose a fractal-based technique for visualizing phylogenetic trees; however, *n*-ary branches need to be visually translated into binary splits. Devroye and Kruszewski (1995) visualize random binary hierarchies with a fractal approach as botanical trees; no additional metric value for the vertices is taken into account; instead, they investigate the Horton-Strahler number for computing the branch thicknesses.

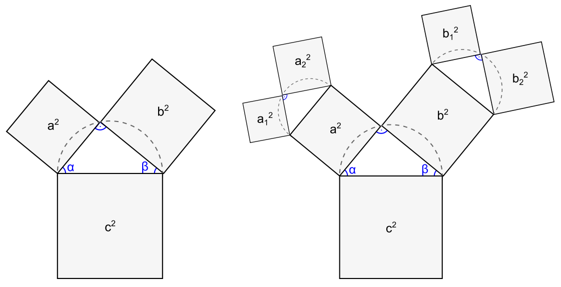
The goal of our work is to extend a fractal approach, which is closer to natural tree structures, towards information visualization. This goal promises embedding the idea of self-similarity and aesthetics of fractals into hierarchy visualization. Central prerequisite—and in this, our approach differs from existing fractal approaches—is that *n*-ary branches should be possible. With respect to information visualization, the approach targets at combining advantages of several existing techniques: a readable and scalable representation, an efficient use of screen space, and the flexibility for encoding additional information. A downside of the approach, however, is that overlap may occur similar as in 3D techniques (though it is a 2D representation)—only varying the parameters of the visualization or using interaction alleviates this issue.

3 VISUALIZATION TECHNIQUE

Our general hierarchy visualization approach extends the idea of Pythagoras Trees. Instead of basing the branching of subtrees on right triangles, we exploit convex polygons with edges on the circumference of a semi circle.

3.1 Data Model

We model a hierarchy as a directed graph $H = (V, E)$ where $V = \{v_1, \dots, v_k\}$ denotes the finite set of k vertices and $E \subset V \times V$ the finite set of edges, i.e., parent–child relationships. One vertex is the designated root vertex and is the only vertex without an incoming edge; all other vertices have an in-degree of one. We allow arbitrary hierarchies, that is, the out-degree of the vertices is not restricted. A maximum branching factor $n \in \mathbb{N}$ of H can be computed as the maximum out-degree of all $v \in V$. For an arbitrary vertex $v \in V$, H_v denotes the subhierarchy having v as root vertex; $\mid H_v \mid$ is the number of vertices included in the H_v (including v). The depth of a vertex v' in H_v is the number of vertices on the path through the hierarchy from v to v'. We allow positive weights to be attached to each vertex of the hierarchy $v \in V$ representing metric values such as sizes. We model them as a function $w : V \to \mathbb{R}$. The weight $w(v) \in \mathbb{R}^+$ of an inner vertex v does not necessarily need to be the sum of its children, but can be.



(a) (b)

Figure 2: Illustration of the traditional Pythagoras Tree approach: (a) a single binary branch; (b) recursively applied branching step.

3.2 Traditional Pythagoras Tree

The Pythagoras Tree is a fractal approach describing a recursive procedure of drawing squares. In that, it was initially not intended to encode information, but its tree structure easily allows representing binary hierarchies: each square represents a vertex of the hierarchy; the recursive generation follows the structure of the hierarchy and ends at the leaves.

Drawing a fractal Pythagoras Tree starts with drawing a square of side length c. Then, two smaller squares are attached at one side of the square—usually, at the top—according to the procedure illustrated in Figure 2 (a): Then, a right triangle with angles α and β where $\alpha + \beta = \frac{\pi}{2}$ is drawn using the side of the square as hypotenuse, which also becomes a diameter of the circumcircle of the triangle. The two legs of the triangle are completed to squares having side lengths a and b. In the right triangle, the Pythagorean theorem $a^2 + b^2 = c^2$ holds, i.e., the sum of the areas of the squares over the legs is equal to the area of the square over the hypotenuse. Applying this procedure recursively to the new squares as depicted for the next step in Figure 2 (b) creates a fractal Pythagoras Tree (the recursion is only stopped for practical reasons at some depth). The angles α and β can be set to a constant value or be varied according to some procedural pattern. Figure 1 (a) provides an example of a fractal Pythagoras Tree where $\alpha = \beta = \frac{\pi}{4}$.

Transforming the fractal approach into an information visualization technique, the squares are interpreted as representatives of vertices of the hierarchy, called *nodes*. As a consequence, the fractal encodes a complete binary hierarchy, the recursion depth being the depth of the hierarchy. If the generated image should represent a binary hierarchy that is not completely filled to a certain depth, the recursion has to stop earlier for the respective subtrees. If the hierarchy is weighted as specified in the data model, the weights can be visually encoded by adjusting the sizes of the squares, i.e., the corresponding angles α and β.

Algorithm 1: Pythagoras Tree.

```
**PythagorasTree($H_v, S$):**
  // $H_v$: binary hierarchy
  // $S$: representative square $S = (c, \Delta s, \theta)$
    // $c = (x_c, y_c)$: center
    // $\Delta s$: length of a side
    // $\theta$: slope angle
  drawSquare($S$);    // draw square for current root vertex
  if $|H_v| > 1$ then
    // $v_1$ and $v_2$: children of $H_v$
    $\alpha := \frac{\pi}{2} \cdot \frac{w(v_2)}{w(v_1)+w(v_2)}$;
    $\beta := \frac{\pi}{2} \cdot \frac{w(v_1)}{w(v_1)+w(v_2)}$;
    $\Delta s_1 := \Delta s \cdot \sin\beta$;
    $\Delta s_2 := \Delta s \cdot \sin\alpha$;
    $c_1 := ComputeCenterLeft(c, \Delta s, \Delta s_1,)$;
    $c_2 := ComputeCenterRight(c, \Delta s, \Delta s_2)$;
    $S_1 := (c_1, \Delta s_1, \theta + \alpha)$;
    $S_2 := (c_2, \Delta s_2, \theta - \beta)$;

    PythagorasTree($H_{v_1}, S_1$);    // draw subhierarchy $H_{v_1}$
    PythagorasTree($H_{v_2}, S_2$);    // draw subhierarchy $H_{v_2}$
  end if
```

Algorithm 1 describes in greater detail how an arbitrary binary hierarchy (i.e., a hierarchy where each vertex either has an out-degree of 2 or 0) can be recursively transformed into a Pythagoras Tree visualization. It is initiated by calling **PythagorasTree(H_v, S):** where $H_v = (V, E)$ is a binary hierarchy and $S = (c, \Delta s, \theta)$ is the initial square with center c, length of the sides Δs, and slope angle θ. The recursive procedure first draws square S and proceeds if the current hierarchy still contains more than a single node. Then, encoding the node weights in the size of the squares, the angles α and β are computed according to the normalized weight of the node opposed to the angle. The angles form the basis for further computing the parameters of the two new rectangles S_1 and S_2. The drawing procedure is finally continued by recursively calling **PythagorasTree(H_{v_1}, S_1)** and **PythagorasTree(H_{v_2}, S_2)** for the two children v_1 and v_2 of the current root vertex.

When, for instance, using the number of leaf vertices as the weight of each vertex, the algorithm produces visualizations such as Figure 3 that encodes a

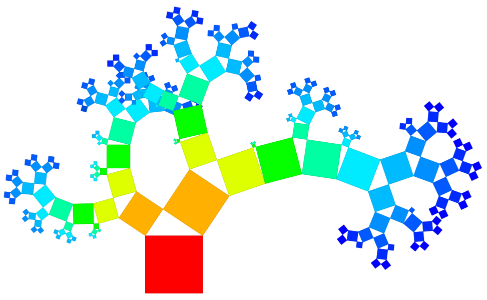


Figure 3: Random binary hierarchy visualized as a Pythagoras Tree that encodes the number of leaves in the size of the nodes.

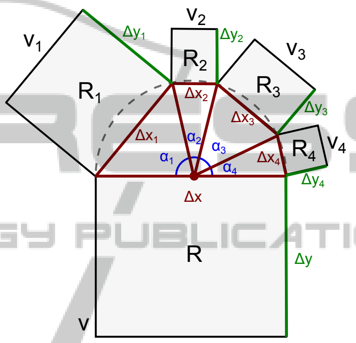


Figure 4: Polygonal split of Generalized Pythagoras Trees creating an n-ary branch.

random binary hierarchy. Like the fractal approach, the visualization algorithm still produces overlap of subtrees that, however, becomes rarer through sparser hierarchies.

3.3 Generalized Pythagoras Tree

The Generalized Pythagoras Tree, as introduced in the following, can be used for visualizing arbitrary hierarchies, that are hierarchies allowing n-ary branches. Right triangles are replaced by convex polygons sharing the same circumcircle; the former hypotenuse of the triangle becomes the longest side of the triangle. For increasing the visual flexibility of the approach, squares are exchanged for general rectangles.

Figure 4 illustrates an n-ary branch, showing the polygon and its circumcircle. The polygon is split into a fan of isosceles triangles using the center of the circumcircle as splitting point. While the number of rectangles is specified by the degree of the represented branch, the angles and lengths can be modified to encode further information. In particular, we have two degrees of freedom:

- **Width function $w_x : V \to \mathbb{R}^+$ of rectangles:** Similar to binary hierarchies, the width Δx_i of a rectangle R_i can be changed, here, by modifying the corresponding angle α_i accordingly. The angle α_i should reflect weight $w_x(v_i)$ of a vertex v_i in relation to the weight of its siblings:

$$\alpha_i := \pi \cdot \frac{w_x(v_i)}{\sum_{j=1}^{n} w_x(v_j)} \quad .$$

The width of the rectangle is $\Delta x_i := \Delta x \cdot \sin \frac{\alpha_i}{2}$ where Δx is the width of the parent node.

- **Length stretch function w_y of rectangles:** Analogously, the length Δy_i of the rectangle R_i can be varied. This length, in contrast to the width Δx_i, does not underly any restrictions such as the size of a cirumcircle. Nevertheless, we formulate the length dependent on the length of the parent Δy and the relative width $\sin \frac{\alpha_i}{2}$ in order to consider the visual context (otherwise, it would be difficult to define appropriate metrics not producing degenerated visualizations): the length of the rectangle is $\Delta y_i := w_y(v_i) \cdot \Delta y \cdot \sin \frac{\alpha_i}{2}$.

Algorithm 2: Generalized Pythagoras Tree.

```
GeneralizedPythagorasTree($H_v, R$):
  // $H_v$: hierarchy branching into $n \in \mathbb{N}_0$ subhierarchies $H_{v_1}, \ldots, H_{v_n}$
  // $R$: representative rectangle $R = (c, \Delta x, \Delta y, \theta)$
    // $c = (x_c, y_c)$: center
    // $\Delta x, \Delta y$: width and length
    // $\theta$: slope angle
  drawRectangle($R$);    // draw rectangle for parent vertex
  for all $H_{v_i}$ do
    $\alpha_i := \pi \cdot \frac{w_x(v_i)}{\sum_{j=1}^{n} w_x(v_j)}$;
    $\Delta x_i := \Delta x \cdot \sin \frac{\alpha_i}{2}$;
    $\Delta y_i := w_y(v_i) \cdot \Delta y \cdot \sin \frac{\alpha_i}{2}$;
    $c_i :=$ComputeCenter($c, \Delta x, \Delta y, (\alpha_1, \ldots, \alpha_{i-1}), \Delta x_i, \Delta y_i$);
    $\theta_i :=$ComputeSlope($\theta, (\alpha_1, \ldots, \alpha_i)$);
    $R_i := (c_i, \Delta x_i, \Delta y_i, \theta_i)$;
    GeneralizedPythagorasTree($H_{v_i}, R_i$);
  end for
```

Algorithm 2 extends Algorithm1 and describes the generation of Generalized Pythagoras Tree visualizations. Again, it is a recursive procedure and is initialized by calling **GeneralizedPythagorasTree(H_v, R)** where $H_v = (V, E)$ is an arbitrary hierarchy and $R = (c, \Delta x, \Delta y, \theta)$ represents the initial rectangle that, in contrast to the previous case, has a width Δx and a length Δy. For an n-ary branching hierarchy H_v with root vertex v, the algorithm first draws the respective rectangle before all children $v_1, \ldots, v_n$ are handled: for each child v_i, the computation of angle α_i forms the basis for deriving the width Δx_i and length Δy_i of the respective rectangle R_i as described above. Furthermore, the center and slope

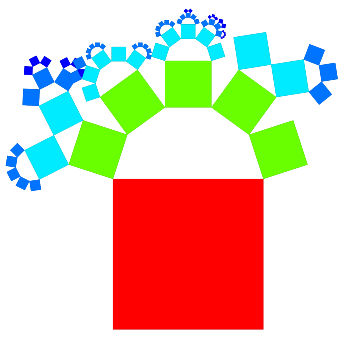


Figure 5: Generalized Pythagoras Trees showing n-ary hierarchy using a constant width and length stretch function.

of the new rectangle need to be retrieved. Finally, **GeneralizedPythagorasTree(H_{v_i}, R_i)** can be recursively applied to subhierarchy H_{v_i} having rectangle R_i as root node.

Figure 5 shows a sample visualization created with the algorithm. For this initial image width function w_x is set to a constant value and the length stretch function w_y is defined as 1. As a consequence, the nodes are squares again, equally sized for each branch but n-arily branching. An example with a similar configuration can be found in Figure 1 (a); the same dataset is shown in Figure 1 (b) applying the number of leaf nodes as the width function w_x. Further configurations are discussed more systematically below. The discussion also includes the usage of color, which, in all figures referenced so far, visualizes the depth of the nodes. Furthermore, the order of rectangles can be modified and has an impact on the layout; in the generalized approach, we have a higher degree of freedom ($n!$ possibilities) than in the standard Pythagoras Trees where only a flipping between two angles can be applied.

3.4 Excursus: Fractal Dimension

The fractal dimension is typically used as a complexity measure for fractals. Looking back to the origin of the Generalized Pythagoras Tree visualization and interpreting it as a fractal approach, the extended fractal approach can be characterized by this dimension. To this end, however, not an information hierarchy can be encoded, but the approach needs to be applied for infinite n-arily branching structures; for simplification we do not consider scaling of rectangles. The following analysis shows that the fractal dimension, which is 2 for traditional Pythagoras Tree fractals, asymptotically decreases to 1 for a branching factor approaching infinity.

Any fractal can be characterized by its fractal dimension $D \in \mathbb{R}$ that is defined as a relation between

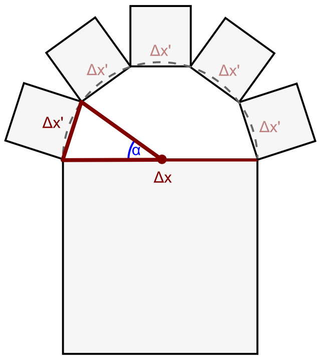


Figure 6: Illustrating the fractal dimension of an n-ary branching hierarchy by showing the splitting into equally sized angles.

the branching factor n and the scaling factor r given by $D = -\log_r n$. In our scenario, we have to first compute the scaling factor r depending on the branching factor n. Figure 6 illustrates the following formulas and shows an n-ary branch.

First of all, the n-ary branch creates a convex polygon, which is split into isosceles triangles as described before. Since all rectangles have the same width, the angle at the tip of the triangle is $\alpha = \frac{\pi}{n}$. The width of the rectangle then is

$$\Delta x' = \Delta x \cdot \sin\frac{\alpha}{2} = \Delta x \cdot \sin\frac{\pi}{2n} \quad .$$

Relating the size of the square to the original square, the scaling factor can be derived as follows:

$$r = \frac{\Delta x'}{\Delta x} = \sin\frac{\pi}{2n} \quad .$$

The fractal dimension finally is

$$D_n = -\frac{\log n}{\log \sin\frac{\pi}{2n}} \quad .$$

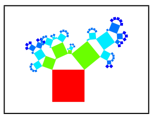
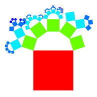
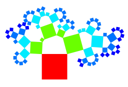
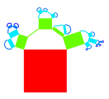
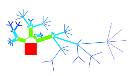
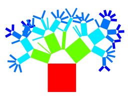
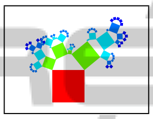
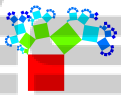
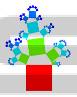
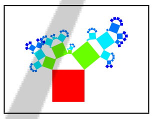
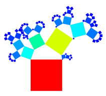
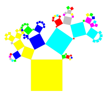
This result confirms $D_2 = 2$ (traditional binary branches) and shows that the fractal dimension is approaching 1 for increasing n, i.e.,

$$\lim_{n\to\infty} D_n = 1 \quad .$$

3.5 Visual Parameters

The visualization approach has been described precisely but still has some degrees of freedom that shall be explored in the following. For example, the size of the rectangles can be varied, the order of the subhierarchies in a branch is not restricted, or the coloring of the nodes is open for variation. These parameters help optimizing the layout and support the visualization by extra information in form of weights assigned to each node. For illustrating the effect, Table 1 shows the

Table 1: Exploring different parameter settings such as size, order, and color of rectangles for a sample dataset; framed images represent the default setting and are equivalent; the number of leaf nodes is applied as weight.

size (squares)	 (S1) sides: weight	 (S2) sides: equal size	 (S3) sides: weight; enlarged circle
size (rectangles)	 (S4) width: equal size; length: weight	 (S5) width: equal size; area: weight	 (S6) width: weight; area: weight
order	 (O1) external	 (O2) ascending weight	 (O3) maximum weight in the center
color	 (C1) depth	 (C2) weight	 (C3) category

same random hierarchy (75 nodes; maximum depth of 5) in different parameter settings. As a weight, the number of leaf nodes is applied; but the metric is interchangeable, for instance, by the number of subnodes, the depth of the subtree, or a domain-specific weight. One setting (Table 1, S1 = O1 = C1), which seemed to work most universally in our experience, is selected as default and applied in all following figures of the paper if not indicated otherwise.

3.5.1 Size

Already for the traditional Pythagoras Tree approach, rectangles can be split in uniform size or non-uniform size. For the generalized approach, we define a width function as well as a length function (Section 3.3). When employing the same metric for both, all nodes are represented as squares. Table 1 (S1) uses the number of leaf nodes as the common metric, which seems to be a good default selection because more space is assigned to larger subtrees. In contrast, when all subnodes are assigned the same size (i.e. a constant function is employed), small subtrees become overrepresented as depicted in Table 1 (S2). A variant of the approach, which is shown in Table 1 (S3), extends the approach from using semi circles to larger sectors of a circle.

Inserting different functions for width and length further increases the flexibility—nodes are no longer squares, but differently shaped rectangles. For instance, Table 1 (S4) encodes the number of leaf nodes in the height and applies a constant value to the width. When defining the length function relative to the (constant) width so that the area of the rectangle is proportional to the number of leaves, those leaf nodes are emphasized as depicted in Table 1 (S5). A similar variant shown in Table 1 (S6) has a constant length and chooses the width accordingly for encoding the number of leaf nodes in the area.

3.5.2 Order

The subnodes of an inner node of a hierarchy are visualized as an ordered list. While, for some applications, there exist a specific, externally defined order, many other scenarios do not dictate a specific order. In case of the latter, the subnodes can be sorted according to a metric, which again is the number of leaf nodes in this example. The sorting criterion mainly influences the direction in which the diagram is growing but also influences overlapping effects. Often the external order, at least in case it is random or independent of size, creates quite balanced views as depicted in Table 1 (O1). When, for instance, applying an ascending order, the image like the one shown in Table 1 (O2) grows to the right. More symmetric visualizations such as in Table 1 (O3) are generated when placing the vertices with the larger size in the center.

3.5.3 Color

The areas of the rectangular nodes can be filled with color for encoding some extra information. Selecting the color on a color scale according to the depth of the node in the hierarchy helps comparing the depth of subtrees: for instance, in Table 1 (C1) this encoding reveals that the leftmost main subtree, though being shorter, is as deep as the rightmost one. Alternatively, the weight of a node can be encoded in color like shown in Table 1 (C2), which, however, is more suitable if the size of the node not already encodes the weight. If categories of vertices are available, also these categories can be color-coded by discrete colors as depicted in Table 1 (C3).

3.6 Analogy to Node-Link Diagrams

Though being derived from a fractal approach, Generalized Pythagoras Trees can be adapted—without changing the position of nodes—to become variants

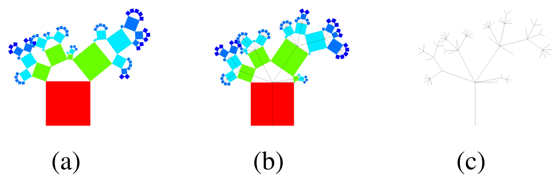


Figure 7: Relationship between Generalized Pythagoras Trees and node-link diagrams: (a) Generalized Pythagoras Tree; (b) Generalized Pythagoras Tree and analogous node-link diagram; (c) analogous node-link diagram.

of node-link diagrams. An analogous diagram can be created as illustrated in Figure 7 by connecting the circle centers of the semi circles of branches by lines. The circle centers become the nodes, the lines become the links of the resulting node-link diagram. Like the subtrees of a Generalized Pythagoras Tree might overlap, the analogous node-link drawing is not guaranteed to be free of edge crossings. We prefer the Pythagoras variant over the analogous node-link variant because it uses the available screen space more efficiently (which is important, for instance, for color coding) and shows the width of a node explicitly.

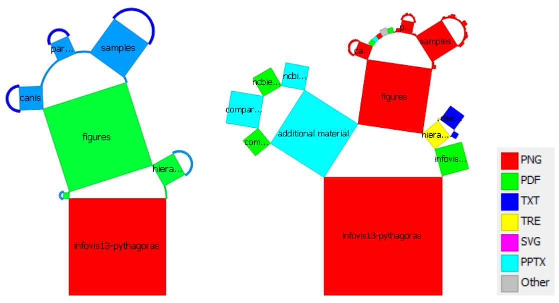
4 CASE STUDIES

To illustrate the usefulness of our Generalized Pythagoras Tree visualization, we applied it to two datasets from different application domains—file systems with file sizes as well as the NCBI taxonomy that classifies species. In these case studies we demonstrate different parameter settings and also show how interactive features can be applied for exploration.

4.1 File System Hierarchy

While the approach can be applied to any directory structure, we decided to demonstrate this use case by reading in the file structure of an early version of this particular paper. Since we use LaTeX for writing, the paper directory contains multiple text files including temporary files as well as a list of images. Also included are supplementary documents and a script used for creating exemplary random information hierarchies. All in all, the directory structure contains 139 vertices (7 directories and 132 files) having a maximum depth of 4 and a maximum branching factor of 38 (*figures* directory). Figure 8 shows two visualizations of this directory structure employing different parameter settings.

In Figure 8 (a), we applied the default settings sizing the vertices in relation to the number of leaf nodes and using color for encoding depth. The image shows that, among the main directories, the *figures* directory



(a) (b)

Figure 8: Directory hierarchy of this paper on the file system: (a) size based on the number of leaf nodes with color-coded depth information; (b) size encoding the file and directory sizes with color-coded file types.

contains by far the most leaf nodes (94) and itself is split into three further directories, which include the images needed for the three more complex figures and tables of this paper: *canis* (Figure 9), *parameters* (Table 1), and *samples* (Table 2). Additionally, *figures* also directly includes a number of images, which are needed for the other figures. The only other directory containing a reasonable number of files is the *hierarchy_generator* folder; besides the generator script it contains a number of generated sample datasets.

Customizing the parameters of the visualization for the use case of investigating file systems, we assigned the file size to the size of the vertices (directory sizes are the sum of the contained file sizes). Moreover, the file type is encoded in the color of the vertex (category coding) a legend providing the color–type assignments; directories are encoded in the color of the dominating file type of the contained files. The resulting visualization as depicted in Figure 8 (b) shows that the *figures* directory is also one of the largest main directories, but there exist other files and directories that also consume reasonable space such as the *additional material* directory. Comparing the size of the main PDF document to the *images* directory, it can be observed that not all image files contained in the directory are integrated into the paper because the paper is smaller than the *images* directory. The color-coded file types reveal that the most frequently occurring type are PNG files, not only in the *images* directory but also in general. The *hierarchy_generator* directory mostly includes TRE files (Newick format), but is dominated with respect to size by two TXT files (an alternative hierarchy format not as space-efficient).

4.2 Phylogenetic Tree

Moreover, our approach is tested on a hierarchical dataset commonly used by the biology and bioinformatics communities. The taxonomy here used has been developed by NCBI and contains the names of all organisms that are represented in its genetic database (Benson et al., 2010). The specific dataset encoding the taxonomy contains 324,276 vertices (60,585 classes and 263,691 species) and has a maximum depth of 42. The Generalized Pythagoras Tree visualization applied to this dataset (Figure 9 I) creates a readable overview visualization of the very complex and large hierarchical structure. The vertices of the tree have different sizes according to the number of leaves of their subtrees. Each inner vertex represents a class of species and it is easy to point out the class that contains more species. The root node is an artificial class of the taxonomy that contains every species for which a DNA sequence or a protein is stored in the NCBI digital archive.

At the first level of the tree (see Figure 9 I), a big node represents *cellular organisms* and further nodes the *Viruses*, *Viroids*, unclassified species, and others (this information can be retrieved by using the geometric zoom). Selecting nodes and retrieving additional information facilitate the exploration of the tree. For instance, the biggest node at level 2 is the *Eukaryota* class, which includes all organisms whose cells contain a membrane-separated nucleus in which DNA is aggregated in chromosomes; it still contains 177,258 of the 263,691 species.

Besides gaining an overview of the main branches of the taxonomy, the visualization tool allows for analyzing subsets of the hierarchy down to the level of individual species by applying semantic zooming. As a concrete example, we demonstrate the exploration process in the right part of Figure 9; in each step we selected the subtree of the highlighted node (red circle): Figure 9 II shows the *Amniota* class, which belongs to the *tetrapoda vertebrata* taxis (four-limbed animals with backbones or spinal columns). In the next steps (Figure 9 III-V), we followed interesting branches until we reach the *Carnivora* class in Figure 9 V, which denotes meat-eating organisms; the subtree contains 301 species. From here, it is simple to proceed the exploration towards a well-known animal, such as the common dog, defined as *Canis Familiaris*, by zooming in the subtrees of *Caniformia*, literally “dog shaped” (Figure 9 VI), then through *Canidae*, the family of dogs (Figure 9 VII) with 45 species, and finally *Canis Familiaris*.

5 DISCUSSION

The introduced technique for representing hierarchical structures is discussed by taking existing other hierarchy visualization approaches into account.

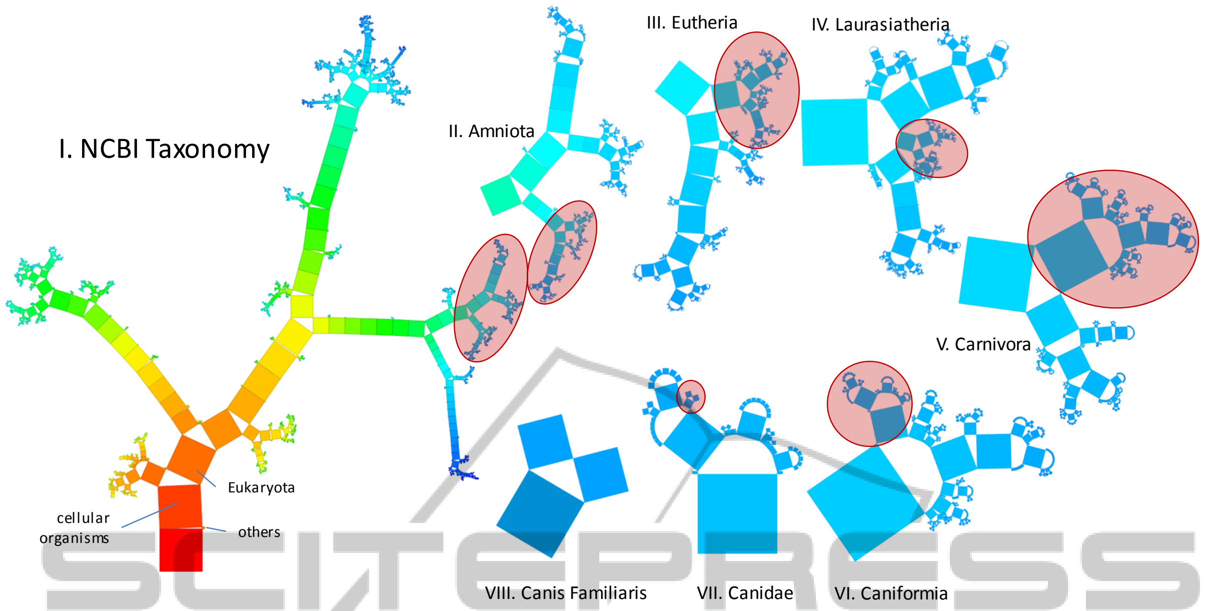


Figure 9: NCBI taxonomy hierarchically classifying species; rectangles sizes indicate the number of species in a subtree, color encodes the depth; an example for exploring the taxonomy by semantic zooming is provided.

We applied different standard hierarchy visualization techniques to a number of randomly generated artificial datasets. The results are listed in Table 2. Each column represents a different data set with some characteristic feature: a binary hierarchy with a branching factor of 2, a deep hierarchy with many levels, a flat hierarchy with a high maximum branching factor, a degenerated hierarchy that grows linearly in depth with the number of nodes, a symmetric hierarchy having two identical subtrees, and a self-similar hierarchy following the same pattern at each level. The rows show standard visualization techniques in comparison to Generalized Pythagoras Trees. Though the graphics can only act as previews in a printed version of the paper, they are included in high resolution and are explorable in a digital version. The following analysis considers multiple levels of abstraction from geometry and perception to readability and aesthetics.

5.1 Geometry and Perception

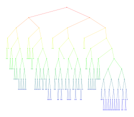
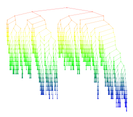
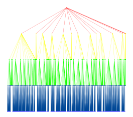
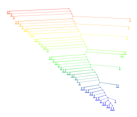
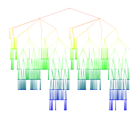
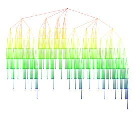
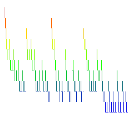
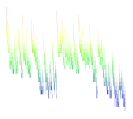
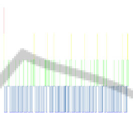
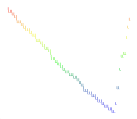
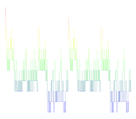
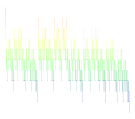
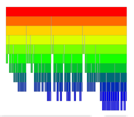
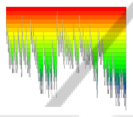
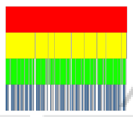
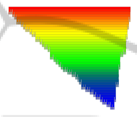
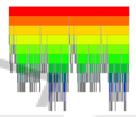
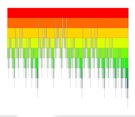
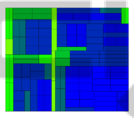
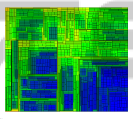
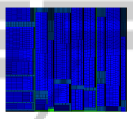
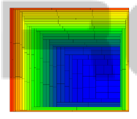
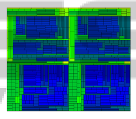
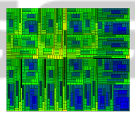
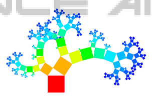
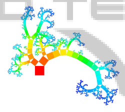
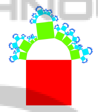
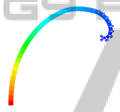
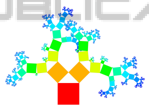
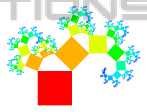
Hierarchy visualizations aim at showing containment relationships between nodes and their descendants. Considering Gestalt theory (Wertheimer, 1923), different approaches exist for visually encoding relationships: for instance, node-link diagrams use *connectedness* to express containment, while Treemaps are based on *common region* for showing that several nodes belong to the same parent. In contrast, Generalized Pythagoras Trees do neither directly draw a line between the nodes nor nest one node into the other, but they draw rectangles of decreasing size onto an imaginary curve. The human reader automatically connects the rectangles on the curve, which is denoted as the *law of continuation*. In all hierarchy visualization approaches shown in Table 2, *proximity* also plays a certain role (i.e., related nodes are placed next to each other) but should not be overinterpreted (i.e., nodes placed next to each other are not necessarily related).

In node-link diagrams, indented tree diagrams, or icicle plots, each level in the hierarchy creates another layer in the visualization. As a consequence, the amount of (vertical) space available for a layer is reduced when adding further levels. In Generalized Pythagoras Trees, however, there are no global layers for levels of nodes: adding a level only produces a kind of local layer that is arranged on a semi circle. With respect to this characteristic, Generalized Pythagoras Trees are similar to Treemaps, which neither have global layers but split the area of a node for introducing the next level.

Like in icicle plots and Treemaps, larger areas are used to encode the nodes in Generalized Pythagoras Trees. This makes it easier to use color for encoding some metrics (such as the hierarchy level) in the nodes because colors are easier to perceive for larger areas (Ware, 2004) (*Color for Labeling*). In contrast to Treemaps (and complete icicle plots), Generalized Pythogoras Trees do not create space-filling images. Areas, however, might overlap, which is discussed in detail below.

Comparing the images shown in Table 2 with respect to uniqueness, Generalized Pythagoras Trees

Table 2: Comparison of hierarchy visualization approaches for representatives of a selected set of hierarchy classes.

	binary hierarchy	**deep hierarchy**	**flat hierarchy**	**degenerated hierarchy**	**symmetric hierarchy**	**self-similar hierarchy**
	node degree of 2	high number of hierarchy levels (25)	high maximum node degree (20)	linearly growing depth	two equivalent subtrees	self similar tree structure
node-link						
indented tree						
icicle plot						
Treemap						
Generalized Pythagoras Tree						

show a high visual variety: not only the subtrees vary in size, they are also rotated. Only the splitting approach in Treemaps creates similarly varying images, however, just with respect to texture but not shape. A positive effect of a high visual variety is that the different datasets can be distinguished more easily—the visualization acts as a fingerprint. Together with the fractal roots of the approach, the uniqueness helps detect self-similar structures: Table 2 (last column) shows a tree having a self-similar structure, which is generated according to the same recursive, deterministic procedure for every node; the self-similar property of the hierarchy is best detectable in the Generalized Pythagoras Trees because every part of the tree is just a rotated version of the complete tree.

5.2 Readability and Scalability

A hierarchy visualization is readable if the users are able to efficiently retrieve the original hierarchical data from it and easily observe higher-level characteristics. However, readability is also related to visual scalability, which means preserving readability for larger datasets. While, for smaller datasets, the exact information is usually recognizable in any hierarchy visualization, the depicted information often becomes too detailed when increasing the scale of the dataset. The visualization approach, hence, needs to use the available screen space efficiently and has to focus on the most important information.

Generalized Pythagoras Trees clearly emphasize the higher-level nodes of the tree (i.e., the root node and its immediate descendants): most of the area that is filled by the visualization is consumed by these higher-level nodes, which can be easily perceived because surrounded by whitespace. Lower-level nodes and leaf nodes, however, become very small and are not visible. But the visualization allows for sizing the nodes according to their importance by using the number of leaf nodes as a metric as done in Table 2. Node-link diagrams, indented trees, and icicle plots are similar in their focus on the higher-level nodes; as well, lower-level nodes become difficult to discern because of lack of horizontal space. Since the vertical space assigned to each level does not become smaller in these visualizations, it is easier to retrieve the maximum depth of a subtree. Treemaps focus on leaf nodes and show largely different characteristics.

The ability of a visualization technique to display also large datasets in a readable way considerably widens its area of application. As shown in the case study, Generalized Pythagoras Trees can be used for browsing large hierarchies such as the NCBI taxonomy. While it is possible to interactively explore large hierarchies in a similar way with the other paradigms listed in Table 2, Generalized Pythagoras Trees show some characteristic scalability advantages: for specifically deep hierarchies such as the one in the second column of Table 2, it adaptively expands into the direction of the deepest subtree, here in spiral shape. Comparing it to the other approaches, deep subtrees are still readable in surprising detail. In contrast for flat hierarchies, which have a specifically high branching factor, Generalized Pythagoras Trees do not seem to be as suitable: the size of the nodes decreases too fast which constrains readability.

For a degenerated hierarchy (Table 2, fourth column), which grows linearly in depth with the number of nodes, Generalized Pythagoras Trees create an idiosyncratic but readable visualization, similar as it is the case for the other visualization approaches. Also a symmetry in a hierarchy such as two identical subtrees (Table 2, fifth column) can be detected: the identical tree creates the same image, which is rotated in contrast to the other approaches, where it is moved but not rotated.

A problem limiting the readability of Generalized Pythagoras Trees is that, depending on the visualized hierarchy, subtrees might overlap. The other visualization approaches do not share this problem; only Treemaps also employ a form of overplotting: inner nodes are overplotted by its direct descendants. While Treemaps use overplotting systematically, overlap only occurs occasionally in Generalized Pythagoras Trees and is unwanted. A simple way to circumvent the problem using the interactive tool is selecting the subset of the tree that is overdrawn by another. Also, reordering the nodes or adapting the parameters of the algorithm could alleviate the problem.

5.3 Aesthetics

Fractals often show similarities to natural structures such as trees, leaves, ferns, clouds, coastlines, or mountains (Peitgen and Saupe, 1988). Among the images shown in Table 2, the Generalized Pythagoras Trees clearly show the highest similarity to natural tree and branching structures. Since, according to the biophilia hypothesis, humans are drawn towards every form of life (Wilson, 1984), this similarity suggests that Generalized Pythagoras Trees might be considered as being specifically aesthetic. Also the property of self-similarity that is partly preserved when generalizing Pythagoras Trees supports aesthetics: *"fractal images are usually complex, however, the propriety of self-similarity makes these images easier to process, which gives an explanation to why we usually find fractal images beautiful."* (Machado and Cardoso, 1998)

6 CONCLUSIONS AND FUTURE WORK

In this paper, we introduced an extension of Pythagoras Tree fractals with the goal of using these for visualizing information hierarchies. Instead of depicting only binary trees, we generalize the approach to arbitrarily branching hierarchy structures. An algorithm for generating these Generalized Pythagoras Trees was introduced and the fractal characteristics of the new approach were reported. A set of parameters allows for customizing the approach and creating a variety of visualizations. In particular, metrics can be visualized for the nodes. The approach was implemented in an interactive tool. A case study demonstrates the utility of the approach for analyzing large hierarchy datasets. The theoretical comparison of Generalized Pythagoras Trees to other hierarchy visualization paradigms, on the one hand, suggested that the novel approach is capable of visualizing various features of hierarchies in a readable way comparably to previous approaches and, on the other hand, might reveal unique characteristics of the approach such as an increased distinguishability of the generated images and detectabiltiy of self-similar structures. Further, the approach may have advantages for visualizing deep hierarchies and provides natural aesthetics.

An open research questions is how the overplotting problem of the approach can be solved efficiently and how the assumed advantages can be leveraged in practical application. Moreover, formal user studies have to be conducted to further explore the characteristics of the approach.

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REFERENCES

Andrews, K. and Heidegger, H. (1998). Information slices: Visualising and exploring large hierarchies using cascading, semicircular disks. In *Proceedings of IEEE Symposium on Information Visualization*, pages 9–11.

Balzer, M., Deussen, O., and Lewerentz, C. (2005). Voronoi treemaps for the visualization of software metrics.

In *Proceedings of Software Visualization*, pages 165–172.

Battista, G. D., Eades, P., Tamassia, R., and Tollis, I. G. (1999). *Graph Drawing: Algorithms for the Visualization of Graphs*. Prentice-Hall.

Benson, D. A., Karsch-Mizrachi, I., Lipman, D. J., Ostell, J., and Sayers, E. W. (2010). Genbank. *Nucleic Acids Research*, 38(suppl 1):D46–D51.

Bosman, A. E. (1957). *Het wondere onderzoekingsveld der vlakke meetkunde*. Breda, N.V. Uitgeversmaatschappij Parcival.

Burch, M., Konevtsova, N., Heinrich, J., Höferlin, M., and Weiskopf, D. (2011). Evaluation of traditional, orthogonal, and radial tree diagrams by an eye tracking study. *IEEE Transactions on Visualization and Computer Graphics*, 17(12):2440–2448.

Burch, M., Raschke, M., and Weiskopf, D. (2010). Indented Pixel Tree Plots. In *Proceedings of International Symposium on Visual Computing*, pages 338–349.

Carrière, S. J. and Kazman, R. (1995). Research report: Interacting with huge hierarchies: beyond cone trees. In *Proceedings of Information Visualization*, pages 74–81.

Devroye, L. and Kruszewski, P. (1995). The botanical beauty of random binary trees. In *Proceedings of Graph Drawing*, pages 166–177.

Eades, P. (1992). Drawing free trees. *Bulletin of the Institute for Combinatorics and its Applications*, 5:10–36.

Grivet, S., Auber, D., Domenger, J., and Melançon, G. (2004). Bubble tree drawing algorithm. In *Proceedings of International Conference on Computer Vision and Graphics*, pages 633–641.

Holton, M. (1994). Strands, gravity, and botanical tree imaginery. *Computer Graphics Forum*, 13(1):57–67.

Jürgensmann, S. and Schulz, H.-J. (2010). A visual survey of tree visualization. *IEEE Visweek 2010 Posters*.

Kleiberg, E., van de Wetering, H., and van Wijk, J. J. (2001). Botanical visualization of huge hierarchies. In *Proceedings of Information Visualization*, pages 87–94.

Koike, H. (1995). Generalized fractal views: A fractal-based method for controlling information display. *ACM Transactions on Information Systems*, 13(3):305–324.

Koike, H. and Yoshihara, H. (1993). Fractal approaches for visualizing huge hierarchies. In *Proceedings of Visual Languages*, pages 55–60.

Kruskal, J. and Landwehr, J. (1983). Icicle plots: Better displays for hierarchical clustering. *The American Statistician*, 37(2):162–168.

Lin, C. C. and Yen, H. C. (2007). On balloon drawings of rooted trees. *Graph Algorithms and Applications*, 11(2):431–452.

Machado, P. and Cardoso, A. (1998). Computing aesthetics. In *Advances in Artificial Intelligence*, volume 1515 of *Lecture Notes in Computer Science*, pages 219–228. Springer Berlin Heidelberg.

Mandelbrot, B. (1982). *The Fractal Geometry of Nature*. W.H. Freeman and Company. New York.

McGuffin, M. and Robert, J. (2009). Quantifying the space-efficiency of 2D graphical representations of trees. *Information Visualization*, 9(2):115–140.

Nocaj, A. and Brandes, U. (2012). Computing Voronoi Treemaps: Faster, simpler, and resolution-independent. *Computer Graphics Forum*, 31(3):855–864.

Peitgen, H.-O. and Saupe, D., editors (1988). *Science of Fractal Images*. Springer-Verlag.

Reingold, E. and Tilford, J. (1981). Tidier drawings of trees. *IEEE Transactions on Software Engineering*, 7:223–228.

Rosindell, J. and Harmon, L. (2012). OneZoom: A fractal explorer for the tree of life. *PLOS Biology*, 10(10).

Schulz, H.-J. (2011). Treevis.net: A tree visualization reference. *IEEE Computer Graphics and Applications*, 31(6):11–15.

Shneiderman, B. (1992). Tree visualization with tree-maps: 2-D space-filling approach. *ACM Transactions on Graphics*, 11(1):92–99.

Stasko, J. T. and Zhang, E. (2000). Focus+context display and navigation techniques for enhancing radial, space-filling hierarchy visualizations. In *Proceedings of the IEEE Symposium on Information Visualization*, pages 57–65.

Ware, C. (2004). *Information Visualization, Second Edition: Perception for Design (Interactive Technologies)*. Morgan Kaufmann, 2nd edition.

Wertheimer, M. (1923). Untersuchungen zur Lehre von der Gestalt. II. *Psychological Research*, 4(1):301–350.

Wetherell, C. and Shannon, A. (1979). Tidy drawings of trees. *IEEE Transactions on Software Engineering*, 5(5):514–520.

Wilson, E. O. (1984). *Biophilia*. Harvard University Press.

Yang, J., Ward, M. O., Rundensteiner, E. A., and Patro, A. (2003). InterRing: A visual interface for navigating and manipulating hierarchies. *Information Visualization*, 2(1):16–30.