

# High Performance Particle Tracking Velocimetry for Fluidized Beds

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**Abstract:** Fluidized beds are used in wide variety of industrial applications. These applications range from energy production to chemical industry. Particle tracking velocimetry (PTV) is an efficient way to study small scale behavior inside fluidized beds. An accurate PTV algorithm has to be able to perform also in relatively dense suspensions where particles may overlap and form clusters. PTV algorithms typically proceed from locating the particles to tracking their motion. Typically the particle locating has been based on either profile matching or image intensity thresholding. This study proposes a combined method that tries to take advantage of the both methods to overcome difficulties associated with dense suspensions. The method was tested in a synthetic case and in an experimental fluidized bed case. The synthetic tests showed a slight increase in error when the number of particles increased, but the error level remained acceptable. Results obtained from the fluidized bed were visually inspected. Visual inspection showed that most of the particles were tracked correctly, which suggests that the proposed method performs well also in practice.

## 1 INTRODUCTION

In a fluidized bed, the upward air flow from the grate (located at the bottom of the bed) causes sand and other solid particles to behave in a more fluid-like manner. Fluidized beds are used in many industrial applications, such as fuel boilers and other multiphase chemical reactors. Due to the complexity of the flow inside a fluidized bed, most of the research in this field is more or less dependent on experimental information.

Particle Tracking Velocimetry (PTV), as the name suggests, is a measurement method where individual particles are tracked. The particles may be already present in the flow, for instance in a fluidized bed, or they are inserted into the flow as tracer particles. The examined flow must be illuminated by a powerful light source (typically a laser) to obtain high enough frame rate to capture quickly moving particles within the short exposure time of a high-speed camera. Displacement vectors are obtained by tracking the movement of particles between sequential frames. Since the frame rate is known, the velocity can be obtained from the displacement.

PTV has been used in a wide variety of applications ranging from laser machining (Viitanen et al., 2012) to biomedical research (Smal et al., 2007). In the fluidized bed research, PTV is effective analysis

method for low volume fraction flows, where the solid suspension is not too dense. This type of flow condition typically exists in the upper parts of the fluidized bed. One of the main advantages of PTV is that it can be used to describe the very small scale behavior, such as the solid phase turbulence associated to so called granular temperature (Dijkhuizen et al., 2006).

PTV is typically based on either profile matching based on cross correlations, see for instance (Marxen et al., 2000), or thresholding image greyscale values, as done by (Feng et al., 2007). While the thresholding approach is effective when there are only few particles present, and their profile is simple, it fails in dense suspensions. On the other hand, the profile matching yields in poor accuracy for particle center points if no interpolation or distribution fitting is applied. Reliable sub-pixel accuracy is a crucial property in high frame rate applications, where the consecutive displacements are small.

In this study, the solid phase of the fluidized bed consisted of small spherical glass particles. The transparent glass particles tend to generate non-Gaussian profiles to image plane when light is supplied from behind the particles relative to camera. The particles can be observed from the images as dark rings which have bright centers. Although the profile is non-Gaussian in general, the profile of a dark ring can be approximated as Gaussian. The described shadow

profile of a particle is relatively easy to detect, and the bright center points increase the performance of PTV in dense suspensions.

In this work, a combined approach based on both thresholding and profile matching is proposed. The goal is to formulate a credible PTV algorithm that would have excellent sub-pixel accuracy, while being able to perform also in dense suspensions, where some particles are inevitably overlapped. The latter property is crucial in fluidized bed research, since the particles tend to form clusters, where they are very close to each other. The performance of the proposed method is improved by parallel computing on a Graphics Processing Unit (GPU).

## 2 EXPERIMENTAL SETUP

The experimental setup of this study consists of a nearly 2-dimensional fluidized bed, a diode laser and a high-speed camera. A schematic illustration of the experimental setup is shown in Figure 1. The fluidized bed is 6mm thick, and 100mm wide. It was operated with inlet pressure of 0.8bar. The high speed camera was placed in front of the bed. The bed was illuminated from behind by Cavitar HF diode laser with wavelength of 810nm. The laser beam was expanded with Cavitar micro optics. An optical bandpass filter was also used to reduce random noise. The bandpass filter was designed for the mean value of 810nm with the bandwidth of  $\pm 10$ nm. Pressurized air was supplied to an expansion zone beneath the bed and blown to the fluidized bed via four grate nozzles. Before entering the grate, air flow passes a humidifier, which was used to decrease static electricity. Static electricity can cause unwanted clustering and particle sticking to the walls of the fluidized bed.

Particles tracked from digital images are typically small tracer particles designed to follow closely the motions of the examined flow. These tracer particles range from  $< 1$  to  $30\mu\text{m}$  in diameter in gas flows according to (Melling, 1997). However, in the experimental case presented in this paper, the interest is aimed at tracking the motion of substantially larger spherical glass particles with the nominal diameter of  $200\mu\text{m}$ . Most of these particles are clearly visible, but some may be a bit out of focus, even though maximum possible depth of field (DOF) is achieved by minimizing the size of the camera aperture while ensuring sufficient exposure. Problems in detecting single particles arise when multiple particles are packed in clusters. Particles in a cluster can be only momentarily joined together while moving in different directions at different velocities. They can also be bonded

together by surface forces and move as a group, which is less usual based on the experimental data.

In general, an adequately high frame rate should be chosen when recording very fast particle motions. The frame rate of the camera was thus set to 1500fps, and the focal ratio to  $f/12$ . These settings kept the displacements of particles between consecutive frames relatively small, while allowing sufficient DOF as explained in the previous section. It should be noted that when a coherent laser, such as Nd-YAG laser, is used, setting the focal ratio too high can cause unwanted total reflections from the diffuser. However, the laser used in this study was not of the coherent type, and the mentioned problem does not occur.

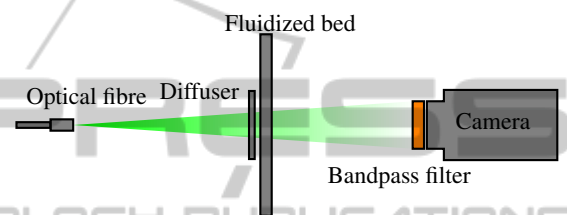


Figure 1: Schematic picture of the measurement setup.

## 3 PTV ALGORITHM

### 3.1 Profile Matching Algorithm

When particle suspension is illuminated from behind, there is usually a bright spot in the middle of the particle. The mid area between the boundary and the center of the particle has typically the smallest intensity. The detailed profile could be computed from the Mie scattering theory (Bohren and Huffman, 1983). However, camera causes Gaussian blur to the profile anticipated by the Mie theory. In addition, the sensor voltage of the camera cell and light intensity might not be linear. Therefore, in this study a more direct approach is taken.

A particle image is thought to consist of a torus of dark points with a diameter of  $d$ . Camera then sees a shadow of each of these points. If the lowest intensity of the particle is denoted by  $b$ , and the bright background intensity by  $w$  then the particle profile at point  $x$  can be given by superposition

$$J(x) = w(1 - \phi(x)) + b\phi(x), \quad (1)$$

where  $\phi(x)$  is a profile function.

The profile function for the above mentioned torus with Gaussian blur is given by

$$\phi(x) = \int_{\|y\|=d} N(x|y, \Sigma) dy, \quad (2)$$

where  $N(\cdot|y, \Sigma)$  denotes the density function of a 2D normal distribution with mean value  $y$ , and covariance matrix  $\Sigma$ . In this study, the covariance matrix had a diagonal structure.

The algorithm proceeds by looping over all the points in an image. From each point  $y$  a neighborhood of that point  $I_y(x)$ , also referred as the mask, is compared to the profile computed by Equation (1). The background intensity of the Equation (1) is chosen prior any computations. The lower intensity value  $b$  is estimated as the average color on a circle of radius  $d$  around the point  $y$ . The likelihood of point  $y$  being the center of a particle is estimated as

$$f(y) = \int \|J(x) - I_y(x)\|^2 N(x|0, \Sigma_I) dx, \quad (3)$$

where  $\Sigma_I$  is covariance matrix of the mask. The covariance matrix is used to increase the value of interior points over the boundary points, and is a common approach when computing image correlations.

The mask was chosen as a square, with size of  $12 \times 12$  pixels. A circular or ellipsoidal mask could also be interesting, but a square mask was used due to its simplicity. In Figure 2 an example particle and a suitable mask for profile matching has been plotted.

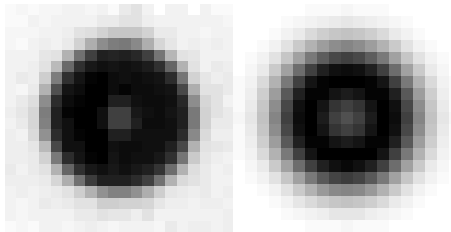


Figure 2: Example particle (left) and suitable profile matching mask (right).

Since the  $f$  values computed by Equation (3) are independent on other  $f$  values, they can be computed quite efficiently by a Graphics Processing Unit (GPU). In the study, it was found out that most of the time was spent in transferring the image to the GPU memory from the Random Access Memory (RAM).

The bright particle centers were recognized from the  $f$  values by searching local minimums. Since the background is not completely homogenous due to auto interference of the laser, there is typically a large number of local minimums in the background. However, the local minimums appearing in the background are typically larger by a magnitude than the actual particles, and therefore are relatively easy to distinguish from the actual particles.

The downside of the above described method is that it is very sensitive to diameter value  $d$ . This can cause multiple recognitions inside a single particle and some systematic errors, where the algorithm

places the center of particle systematically too near the boundary of the particle. To remedy this problem a clustering method can be used to connect too-near recognitions.

## 3.2 Thresholding Method

In order to identify particles from a greyscale image, it must be first decided whether a pixel of the image contains information about a particle or not. In this process, a binary image with ones representing the informative particle regions and zeros representing the non-informative background region is generated.

The most common binarization method is probably the single threshold binarization. In this method, a single threshold value is selected and the binary image is computed based on whether a pixel is above or below the selected threshold value. Although this method is simple and fast to execute, it requires clearly separable particle greyscale values from the background greyscale values. Even though the signal to noise ratio (SNR) in the images would be sufficient, this method lacks the ability to take account the non-uniform illumination conditions typically present in experimental setups.

In this paper, a correction of inhomogeneous illumination is combined with the single threshold binarization, leading to an efficient and reliable binarization method.

### 3.2.1 Correction of Inhomogeneous Illumination

In experimental setups, the inhomogeneous illumination conditions can be caused by several factors. For example, the amount of illumination decreases towards the light direction due to absorption and scattering from the measured particles and the fluid, when a laser light sheet is passed from one side to the camera's field of view. Also possible reflections from the solid structures cause problems when using the single threshold method. In the experimental setup presented in this paper, the uneven background illumination profile is caused by the diverging optics of the back laser. The unevenness can be observed from the decreasing greyscale values towards image edges.

The inhomogeneous illumination conditions can be easily corrected if it is possible to obtain images where the illumination conditions are otherwise similar to the experimental setup but no particles are present (Jähne, 2004). These images are usually called as reference or background images. In order to reduce the effect of random noise in the background images, it is recommended to compute pixelwise averages across a reasonable set of background images.

The inhomogeneous illumination is corrected by dividing every pixel of the image  $G$  by the corresponding pixel of the average background image  $R$

$$C_{mn} = c \frac{G_{mn}}{R_{mn}}, \quad (4)$$

where scalar  $c$  is used to scale the grey values of the corrected image. The scaling factor  $c$  was selected as 100 in this work. Values of the corrected image  $C$  can be thus considered as percentage of the image  $G$  greyscale value to the background image  $R$  greyscale value. Notice that the corrected image  $C$  is stored as double precision data type instead of the original image's 8-bit unsigned integers. An example of a random particle image corrected by the average background image is given in Figure 3.



Figure 3: Particle image, average background image and illumination corrected particle image.

### 3.2.2 Single threshold Binarization

After the correction of inhomogeneous illumination, it is possible to use the single threshold binarization method with outstanding results compared to the non-corrected images. The single threshold binarization for corrected images is defined as logical operation

$$B_{mn} = \begin{cases} 1, & \text{if } C_{mn} \leq C_{th} \\ 0, & \text{if } C_{mn} > C_{th}, \end{cases} \quad (5)$$

where  $C_{th}$  is the optimum threshold value.

While the correction of inhomogeneous illumination enables the usage of the single threshold binarization, it still leaves us the problem of selecting an optimal threshold value. Basically the thresholded pixels should contain as much information about the particle as possible, but false particles stepping out of the background should not occur. On the one hand, when decreasing the threshold value (in the case of dark particles), some boundary pixels are left outside the thresholded region. In this case some information of the particle has been lost and the shape of the thresholded boundary starts to have an effect on the particle center point computation, leading to increased pixel locking effect as shown by (Feng et al., 2007). On the other hand, when increasing the threshold value (in the case of dark particles) beyond the optimum value, false particles start to arise from the background. The size of the false particles range from 1 pixel to several

pixels, and the size increases as the threshold value increases.

It is possible to search the optimum threshold value by comparing the total amount of found particles at different threshold values from an extensive sample of corrected particle images. If the minimum size of the measured particles is clearly larger than the size of the small false particles, containing only some pixels, it is practical to remove all particles below a selected minimum size in pixels (see Section 3.3.3). It should be noted that also some incomplete particles located at the image edges are thus considered as small particles. In the case of dark particles, the maximum threshold value, with which the total amount of found particles remains at fairly constant level, is selected as an optimum threshold. As pointed out by (Feng et al., 2007), it should be also checked from a sub pixel map that the pixel locking effect does not occur at the selected threshold value.

The optimum threshold is determined experimentally using every twentieth frame of the recorded data. As can be seen from Figure 4, the amount of all found particles begins to rise notably after the threshold value 72, which is thus selected as the optimum value. Interestingly, the amount of selected particles remains very stable even when the total amount of found particles starts a rapid rise. This is because the small particles below a selected minimum size are removed from the total amount of found particles leading to the number of selected particles. The observed stability in the amount of selected particles is a highly desirable feature of the algorithm.

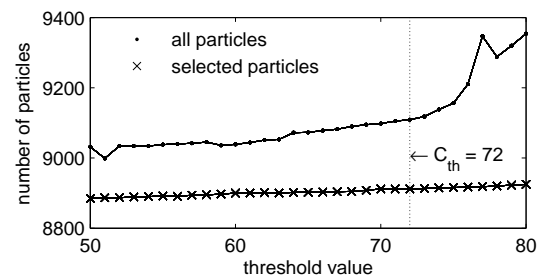


Figure 4: Amount of all particles and selected particles as function of threshold value.

A sub pixel map of particle locations in the optimum threshold selection is presented in Figure 5. The sample size of the corrected particle images was the same as in the optimum threshold determination. No visible pixel locking pattern (Feng et al., 2007) can be observed, so the selected optimum threshold value is adequate. In general, when the particle average size is quite large (as in the experimental case), pixel locking occurs in a smaller degree than in the case of particles containing only some pixels.

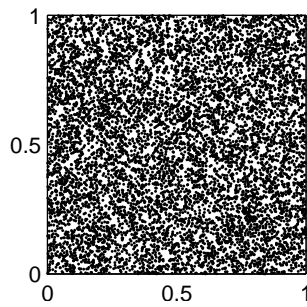


Figure 5: Subpixel image for particle center point locations at optimum threshold value.

### 3.3 Particle Indexing

In this work, the thresholding method is combined with the profile matching method. The particle indexing is thus performed in two steps. In the first step, thresholded particle regions in the binary image are indexed by tracking their outer boundaries. In the second step, particle clusters are separated based on the center points recognized by the profile matching algorithm. Each pixel of a particle cluster is re-indexed using the criteria of closest distance to a center point. After these steps, every pixel of every thresholded particle region is given an index that determines the particle in which it belongs to.

A major advantage in combining the profile matching method with the thresholding method is the sub pixel accuracy obtained by the calculation of the particle center points from thresholded and indexed pixels as explained in Section 3.4. Additionally, the separation of particles from particle clusters increases significantly the reliability of the identified particles in comparison to plain thresholding method.

#### 3.3.1 Thresholded Particle Region Indexing

The thresholded particle regions are indexed by tracking their outer boundaries using the Freeman chain code of eight directions, explained by (Jähne, 2004). In this algorithm, the binary image is scanned line by line and when a binary value one (that is not yet indexed) is found, the outer boundary binary ones are tracked in clockwise direction using 8 possible neighbor directions. In this work, all pixels inside the obtained closed boundary are indexed by a number that defines the particle. Even though there may be some binary zero pixels inside the closed boundary (typically the bright spot in the middle of the particle), the corresponding pixels of the corrected image  $C$  probably contain in some extent useful information and can be thus considered as a part of the particle region.

#### 3.3.2 Particle Cluster Separation

It is possible that an indexed particle region contains multiple particles. The recognized particle center points calculated with the profile matching algorithm are used here to separate single particles from particle clusters. If the indexed particle region contains more than one center point, the pixels inside are separated into re-indexed regions based on their closest distance to a center point. When particles are not too overlapped and the center points are correctly recognized, this method usually leads to well-separated particles.

The presented particle separation process combines the advantages of the thresholding method and the profile matching method. The thresholded particle regions are considered to contain reliable information of the particles being measured. By checking that a center point recognized by the profile matching algorithm is inside a thresholded region, it is ensured that no false particle center point matches arising from the background, are considered as actual particles. If the profile matching algorithm has failed to recognize a center point inside an indexed particle region, it is assumed that the thresholded region is correct and contains a single particle.

When particles are partially overlapped, some pixels evidently belong to both particles, but are straightforwardly divided to different particles using the presented separation method. This clearly leads to some error in the calculation of the particle center point in Section 3.4. In cases where particles are so overlapped that only one or no center point at all is recognized by the profile matching method, some error also occurs in the calculation of the center point of the overlapped particle cluster. The magnitude of these errors can be examined by testing the method using synthetic particle data.

#### 3.3.3 Removing Small Particles

Even with the optimal threshold value some small false particles can still occasionally arise from the background. To ensure that no false particles are considered as real particles, all particles below a selected minimum size in pixels are removed. Evidently, the particle minimum size should be notably smaller than the minimum size of the measured particles. The particle minimum size should be selected with care, because also some incomplete particles are inevitably removed from the image edges. In this paper, approximately 20% of the average size of the measured particles is used as the particle minimum size.

### 3.4 Particle Center Point Computation

The center point position of a particle is determined by the greyscale value weighted position of pixels

$$\mathbf{X} = \frac{\sum_i \mathbf{X}_i (C_i - C_{base})}{\sum_i (C_i - C_{base})}, \quad (6)$$

where  $i$  represents the pixels belonging to an indexed particle,  $\mathbf{X}_i$  is the pixel center point location vector,  $C_i$  is the corrected greyscale value of a pixel of the particle  $i$  and  $C_{base}$  is the base line value.

It has been shown by (Feng et al., 2007) that selecting  $C_{base} = C_{th}$  minimizes the total error in the sub pixel accuracy caused by the pixel locking effect associated with the pedestal part of the intensity that remains below the threshold value. Also in this work, the base line value equal to the optimum threshold value is selected.

### 3.5 Particle Tracking

Several particle tracking algorithms have been developed for tracking the motion of single particles from digital images. Although promising results have been obtained using many of these methods, the authors decided to demonstrate the performance of a relaxation based tracking method. In this work, the original relaxation method (ORX), summarized by (Ohmi and Li, 2000) and originally developed by (Barnard and Thompson, 1980), is selected for tracking the particles. The implementation of the original relaxation method used in this work follows closely to the one presented by (Ohmi and Li, 2000). The new relaxation method introduced by (Ohmi and Li, 2000) was recently further improved by (Jia et al., 2013). For the sake of simplicity, the additional relaxation parameters and other improvements to the original relaxation method, discussed by (Ohmi and Li, 2000) and (Jia et al., 2013) among other authors, are not studied in this work. The original relaxation algorithm is easily expandable to include the latest improvements on demand.

The relaxation methods calculate particle matching probabilities iteratively until the probabilities have converged to nearly constant levels. During these iterations, the probability of a correct matching particle is increased close to one. A major advantage of the relaxation methods is the no-match probability formulated in the relaxation algorithm, which reduces clearly the amount of false matching particles compared to other particle tracking methods. A useful feature is also the ability to define the search radiuses  $R_s$ ,  $R_n$  and  $R_c$  (Ohmi and Li, 2000) to suitable values depending on the examined case in order to achieve

high quality tracking results. In general, the relaxation methods work well in complicated flows and dense particle regions in comparison to other methods (Jia et al., 2013), which suggest that these methods are applicable to the turbulent two-phase flow of the fluidized bed examined in this article.

## 4 SYNTHETIC IMAGES

In this work, an artificial particle grayscale distribution is used to generate synthetic particles that resemble the particle profile observed in the actual images. The artificial greyscale distribution is defined as a twin normal distribution expanded into two dimensions by the equation

$$z(x, y) \propto \mathbf{N}(r(x, y) | \mu, \sigma) + \mathbf{N}(r(x, y) | -\mu, \sigma), \quad (7)$$

where

$$r(x, y) = \sqrt{(x - x_0)^2 + (y - y_0)^2} \quad (8)$$

is the radius from the center point of the distribution  $(x_0, y_0)$  to any specific point  $(x, y)$ . The normal distribution  $\mathbf{N}$  parameters were selected as  $\mu = 2.6$  and  $\sigma = 1.7$  and the distribution values  $z(x, y)$  were multiplied by an appropriate scaling constant  $c_s$  to make the greyscale values computed from the given distribution similar to measured greyscale intensity profile. Accurate values in every pixel of the synthetic image were obtained by numerically evaluating the double integral

$$G_{p, mn} = c_s \int_{m-0.5}^{m+0.5} \int_{n-0.5}^{n+0.5} z(x, y) dx dy \quad (9)$$

using a 9-point Gauss quadrature rule. The resulting center line profile of a synthetic particle  $G_p$  with center point placed at origin  $(X_0, Y_0) = (0, 0)$  is presented in Figure 6. Notice the slight difference between the values integrated over every center line pixel presented with bars and the values of the distribution function multiplied with  $c_s$  and marked with plus signs.

The synthetic particle images are generated by randomly spreading a selected amount of the distribution center points  $(x_0, y_0)$  into an image area of  $1024 \times 1024$  pixels. Particles are then integrated one by one from the distribution values using Equation (9). When combining overlapped particles to a single synthetic image, the maximum greyscale value in the corresponding pixels is selected. In reality, the scattering light does not behave this way, but this is a reasonable simplification since the actual scattering profile is not modeled.

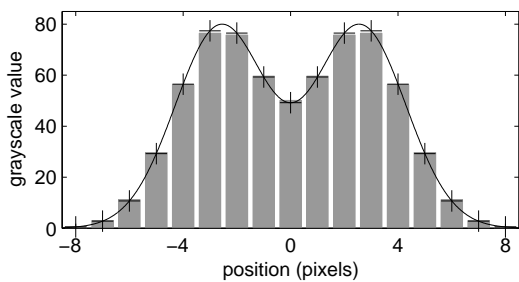


Figure 6: Cross-sectional grayscale profile of synthetic particle.

The standard deviation of the background grayscale values is estimated based on an image, where no particles are present, corrected by Equation (1). The mean of the normal distribution in the synthetic background images is selected as  $\mu = 100$  and the standard deviation is approximated as  $\sigma = 1.5$ . As a result, a synthetic particle image including the background random noise is generated by subtracting the synthetic particle image from the background image.

## 5 RESULTS

### 5.1 Generic Test

The errors encountered in particle center point calculation can be divided into two different groups. This is achieved by computing the nearest computed center point for every synthetic center point. If the amount of synthetic center points connected to a single computed center point is more than one, the synthetic center point is considered to belong to multi-match group. Otherwise it belongs to single-match group. Obviously the sum of synthetic single-match particles and synthetic multi-match particles equals the total number of synthetic particles in a single frame.

The overall error in the synthetic test is computed for each frame as the average of all distances between a synthetic center point location and the closest computed center point location. The average overall error increases at nearly linear rate from 0.13 pixels (250 particles) to 0.42 pixels (1000 particles) as Figure 7 indicates. In Figures 7 and 8 there are 250, 500, 750 and 1000 synthetic particles in the frames 1-100, 101-200, 201-300 and 301-400, respectively. The average value for every set of 100 frames is plotted as a straight line to these figures.

In Figure 8 the effect of multi-match particles to the overall error is compared. The increase of the overall error is mostly caused by the synthetic center points that are connected to multiple computed cen-

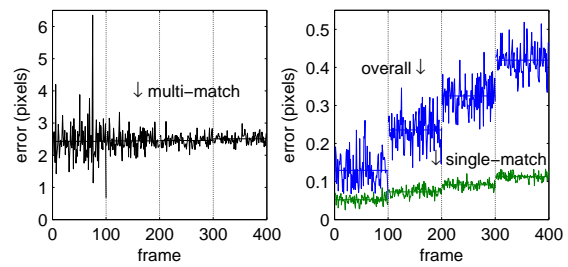


Figure 7: Multi-match, single-match and overall error in synthetic test.

ter points. The average effect of multi-match particles to the overall error increases as the total amount of particles increases. Although the average error of multi-match particles remains at almost constant level around 2.45 pixels as Figure 7 indicates, the effect to total error increases due to the increasing ratio of multiple matching particles presented in Figure 8.

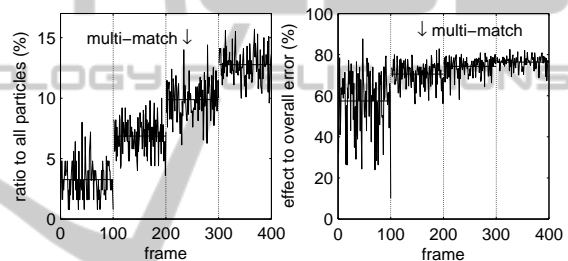


Figure 8: Ratio of multi-match particles to all synthetic particles. Effect of multi-match particles to overall error in synthetic test.

The average error of single-match particles increases as the amount of particles increases as can be seen from Figure 7. Although the ratio of single-match particles to all synthetic particles decreases as Figure 8 conversely indicates, the average error of single-match particles increases from 0.05 to 0.11 pixels. This is probably caused by the increased probability of neighboring particles, which inevitably reduces the sub-pixel accuracy as the thresholded pixels of particle clusters are divided to separate particles.

### 5.2 Experimental Test

Figure 9 shows the selected amount of particles after thresholding, particle cluster separation and removal of small particles. The selected amount of particles shows a high fluctuation because also the actual amount of particles varies with a large amplitude in the measurement area of the highly turbulent upper part of the fluidized bed. However, the amount of selected particles in sequential frames shows only moderate fluctuations, which means that the amount of

computed particles in sequential frames is quite continuous, as it should. No unexpected outliers are observed, which proves the robustness of the algorithm. The amount of separated particles from the thresholded particle regions is also presented in Figure 9. As expected, the amount of separated particles increases notably when the total amount of particles increases and more particles become close to each other.

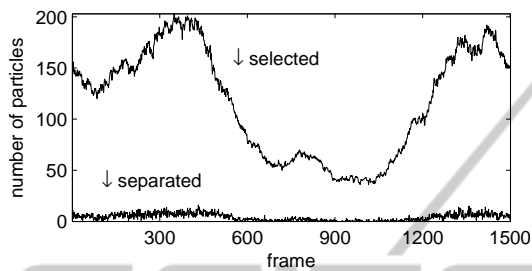


Figure 9: Selected amount of particles and separated amount of particles from particle clusters.

The sub-pixel image for all computed center points in every frame of the experimental test shows no signs of pixel locking. This implies that the computed center points matching a single particle are within excellent sub-pixel accuracy (Feng et al., 2007). As the synthetic test demonstrates, the accuracy drops to pixel scale in cases where multiple actual particles computed as one particle. However, the degraded accuracy in the case of particle clusters is at some degree diminished at the particle tracking stage when suitable matching particles are not found between sequential frames.

The particle size distribution is plotted as a histogram in Figure 10. The distribution contains a single peak at the particle size of 141 pixels and has a mean value of approximately 150 pixels. The number of particles is thus slightly more concentrated on the right-hand side from the peak value. The reason for this is probably that the algorithm is not able to separate all particle clusters to single particles. If the particles are somewhat normally distributed around the nominal value, the measured particle size distribution should also resemble this distribution. The non-separated particle clusters evidently shift the balance towards larger size particles.

In Figure 11 a vector field computed from frames 319-320 is presented. The particle tracking is computed by the original relaxation method (Ohmi and Li, 2000). Parameters used in the experimental test are  $R_s = 28$  for sequential radius (sequential frames),  $R_n = 90$  for neighbor radius (first frame) and  $R_c = 20$  for parallel motion radius (sequential frames). The selected value for parallel motion radius allows some clearly erroneous displacement vectors. On the other

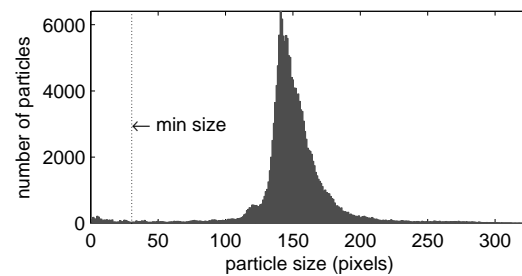


Figure 10: Particle size histogram in experimental test. Particles below minimum size are removed.

hand, decreasing this value will cancel out many good matching particles in sequential frames. In general, when applied to highly turbulent flow where the motion of single particles is chaotic, this parameter should have clearly higher value than in flows where the particle random motion is considerably smaller.



Figure 11: Sample vector field. Green equals initial and red final computed center point locations.

The sequential radius of the particle tracking algorithm is selected based on the observed particle motion between two sequential frames. It should be pointed out, that displacements exceeding this value are not considered as possible matching particles. The selection of the neighbor radius  $R_n$  proves out to be quite tricky, because the particle density varies highly in different locations of a single frame and in different frames of the experimental test. This problem is rarer in tracer particle experiments, where the density of the seeding particles remains typically at constant level. As a solution to this problem, the authors propose increasing the radius  $R_n$ , until an adequate number of neighbor particles are inside the perimeter. However, in the experimental test of this work, the neighbor radius is simply set to a high enough value to include some neighbor particles also in dilute regions.



## 6 CONCLUSIONS

The proposed method was tested in a synthetic test case with computer generated random data, and with a cold model fluidized bed. The synthetic test case allowed one to inspect the behavior of the PTV algorithm with varying number of particles. From the synthetic test cases one can conclude that there are two major sources of error, namely error caused by finite sub pixel accuracy, and error caused by inability to detect the particle. Both errors increased when the number of particles was increased. However, the latter error type became quickly dominant.

In statistical studies, usually the number of detections is not of main concern, but rather the credibility of the detections. As shown in the synthetic test case results, sub pixel accuracy showed only slight increase with the increase of particles. On the other hand, most PTV based studies in fluidized beds are of the statistical type. Thus the proposed method is well suited for the fluidized bed research, or other similar problems, where particles exist in relatively dense suspensions.

The proposed method was also tested in a cold-model of a fluidized bed. Since the real displacements are not known the data was only visually inspected. Visual inspections showed that the method most of the time able to detect particles, but failed occasionally to detect all of the particles from clusters. On the other hand, number of false positives (particle detections on the background) was very small, which is especially important in statistical studies.

Structure of the algorithm is easy to parallelize, as particles interact with each other in very limited way on the detection phase. The particle tracking algorithm uses information of the neighboring particles, which is difficult to parallelize, but is computationally much less expensive than the detection phase. Profile matching algorithm used in this study does assume that the particle profiles are independent of the neighboring particles. While this assumption is the key for computational efficiency, it limits the maximum particle density. Future developments in this field are likely to include interacting particles models, for further improvements in accessible particle density.

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## REFERENCES

- Barnard, S. and Thompson, W. (1980). Disparity analysis of images. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 2:333–340.
- Bohren, C. and Huffman, D. (1983). *Absorption and Scattering of Light by Small Particles*. John Wiley and Sons, New York, 1st edition.
- Dijkhuizen, W., Bokkers, G. A., Deen, N. G., van Sint Annaland, M., and Kuipers, J. A. M. (2006). Extension of piv for measuring granular temperature field in dense fluidized beds. *American Institute of Chemical Engineering*, 53:108–118.
- Feng, Y., Goree, J., and Liu, B. (2007). Accurate particle position measurement from images. *Review of Scientific Instruments*, 78.
- Jähne, B. (2004). *Practical Handbook on Image Processing for Scientific and Technical Applications*. CRC Press, Boca Raton, 2nd edition.
- Jia, P., Wang, Y., and Zhang, Y. (2013). Improvement in the independence of relaxation method-based particle tracking velocimetry. *Measurement Science and Technology*, 24:055301 (13pp).
- Marxen, M., Sullivan, P. E., Loewen, M. R., and Jahne, B. (2000). Comparison of gaussian particle center estimators and the achievable measurement density for particle tracking velocimetry. *Experiments in Fluids*, 29:145–153.
- Melling, A. (1997). Tracer particles and seeding for particle image velocimetry. *Measurement Science and Technology*, 8:1406–1416.
- Ohmi, K. and Li, H. (2000). Particle-tracking velocimetry with new algorithms. *Measurement Science Technology*, 11:603–616.
- Smal, I., Niessen, W., and Meijering, E. (2007). Advanced particle filtering for multiple object tracking in dynamic fluorescence microscopy images. *Biomedical Imaging: From Nano to Macro*, 4:1048–1051.
- Viitanen, T., Kolehmainen, J., Okamoto, Y., and Piche, R. (2012). Spatter tracking in laser machining. *Proceedings of 8th International Symposium on Visual Computing*, pages 626–635.