A Graph-based Algorithm for Three-way Merging of Ordered Collections in EMF Models

Felix Schwägerl, Sabrina Uhrig and Bernhard Westfechtel
Applied Computer Science 1, University of Bayreuth, Universitätsstr. 30, 95440 Bayreuth, Germany

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Abstract: Version control for models is not yet supported in an adequate way. In this paper, we address three-way merging of model versions. Based on a common base version b, two alternative versions a_1 and a_2 were developed by copying and modifying the base version. To reconcile these changes, a merged version m is to be created as a common successor of a_1 and a_2. We present a graph algorithm to solve an important subproblem which occurs in three-way model merging: merging of (linearly) ordered collections. To create the merged version, a generalized topological sort is performed. Conflicts occur if the order of elements cannot be deduced automatically; these conflicts are resolved either interactively or by default rules. We have implemented the merge algorithm in our tool BTMerge, which performs a consistency-preserving merge of versions of EMF models being instances of arbitrary Ecore models. By taking arbitrary move operations into account, the algorithm considerably goes beyond the functionality of contemporary merge tools which are based on common subsequences and thus cannot adequately handle move operations.

1 INTRODUCTION

Model-Driven Software Engineering (MDSE) (Stahl and Voelter, 2006) denotes a software engineering process which is driven by the development of high-level models being expressed in well-defined modeling languages. While MDSE is an active research area and more and more is making its way into industrial practice, several obstacles still impede its application. In particular, developing models in a team over a large period requires sophisticated *version control*: Each model evolves into multiple versions. Version control tools need to store these models, compute differences, and merge model versions which have been created concurrently on different branches by different software engineers.

Traditional version control tools such as Subversion (Collins-Sussman et al., 2004) or CVS (Vesperman, 2006) operate on *text files*. Even if models are stored as text files (e.g., XMI documents), applying traditional version control tools to models suffers from serious limitations, particularly concerning the *comparison* and *merging* of model versions (Förtsch and Westfechtel, 2007). Thus, tools for *comparing* and *merging models* which take the syntax and/or semantics of models into account are urgently needed. In contrast to text-based tools, *model-based tools* operate on the representation of models as sets of interconnected model elements. For comparing models, a variety of algorithms has been proposed and implemented (Kelter et al., 2005; Mehra et al., 2005; Xing and Stroulia, 2005; van den Brand et al., 2010), including e.g. the well known EMF Compare tool (Brun and Pierantonio, 2008). Likewise, quite a number of model-based merge tools have been developed (Alanen and Porres, 2003; Altmanninger et al., 2009; Altmanninger et al., 2010; Koegel et al., 2010; Taentzer et al., 2012; Schwägerl et al., 2013b).

In this paper, we focus on *three-way merging* of model versions (Figure 1). Based on a common *base version b*, two *alternative versions* a_1 and a_2 were developed by copying and modifying the base version. To reconcile the changes on both branches, a *merged version m* is to be created as a common successor of a_1 and a_2. Three-way merging is required e.g. for *optimistic version control* as supported by version control tools such as Subversion (Collins-Sussman et al., 2004) or CVS (Vesperman, 2006): Different software engineers may concurrently create different successors of the same base version without being delayed by locks on the base version. Later on, they reconcile their work by three-way merging such that non-conflicting changes are combined automatically and conflicts are detected and resolved.

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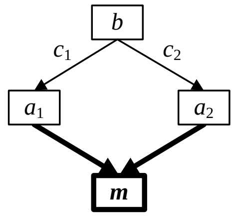


Figure 1: Three-way merging.

For three-way merging of models, we have developed the tool *BTMerge* (Schwägerl et al., 2013a; Schwägerl et al., 2013b) which is based on the theoretical foundations presented in (Westfechtel, 2010; Westfechtel, 2012) and is characterized by the following features: First, BTMerge is a *model-based* tool which internally relies on a *graph representation* of the input models to be merged. Second, the tool may be applied to *any EMF model*, regardless of the underlying Ecore model. Thus, BTMerge covers a large set of models, obviating the need for a specific merge tool for each model type. Third, BTMerge *preserves consistency*: Being supplied with input models which are consistent instances of a common Ecore model, BTMerge constructs a consistent merged model.

The current paper deals with an important subproblem which occurs in three-way model merging: *merging* of *(linearly) ordered collections*. Collections are contained in virtually any model; consider e.g. parameters of operations or structural and behavioral features of classes in Ecore models (which are model instances, too). In the case of ordered collections, a three-way merge algorithm has to consider not only insertions and deletions, but also *move operations*. If the elements of ordered collections are allowed to be moved in arbitrary ways, it is far from trivial to deduce their mutual order in the merged collection.

After clarifying the contribution of our paper, Section 3 provides relevant foundations. In Section 4, we introduce a motivating example problem and derive an expected solution. The description of our algorithm follows in Section 5, which also contains an analysis of its complexity. Furthermore, we explain how the algorithm handles the example. Section 6 describes the integration of the algorithm with BTMerge. Section 7 covers related work and includes a comparison based on a small set of examples; Section 8 concludes the paper.

2 CONTRIBUTION

In this paper, we present an *algorithm* for *merging ordered collections* in *EMF models* created in the Eclipse Modeling Framework (Steinberg et al., 2009).

For each ordered collection to be constructed in the merged version, a *graph* is built whose vertices and edges correspond to the elements and their ordering relationships, respectively. In general, this intermediate graph does not represent a linear order and may even contain cycles (in the case of conflicting move operations). To create the merged version, a *generalized topological sort* (GTS) is performed. *Conflicts* occur if the order of elements cannot be deduced automatically. In this case, either the end user has to perform a decision (interactive merge), or a default decision has to be applied (batch merge). Due to the generalized topological sort, we refer to our contributed algorithm as the *GTS algorithm* subsequently.

The problem of merging linear data structures has been studied for different kinds of artifacts such as models (the focus of this paper), text files, or XML documents (Lindholm, 2004; Khanna et al., 2007; Koegel et al., 2010; Brun and Pierantonio, 2008; Taentzer et al., 2012; Westfechtel, 2012). The GTS algorithm is distinguished from related approaches by the following properties:

P1 State-based Approach. Our algorithm is exclusively state-based. It neither relies on change logs, nor does it reconstruct change sequences. Therefore, the produced result depends neither on the actual editing history nor on the way in which changes are reconstructed.

P2 Separation of Matching and Merging. The merge algorithm relies on a *matching* which identifies corresponding elements of the input versions. Any algorithm for matching may be used without affecting the merge algorithm.

P3 Uniqueness of Elements. When merging unique ordered collections (ordered sets), the algorithm guarantees that the merge result will preserve uniqueness; each element will occur only once.

P4 Consistent Propagation of Insertions and Deletions. By abstracting from the order of the merged collection, the same collection will be obtained which would be produced by merging the respective unordered collections. Thus, if an element was inserted/deleted on one branch, it will be inserted into/removed from the merged collection.

P5 Support of Move Operations. There is no constraint concerning the relative ordering of matched elements. Thus, when the alternative versions are created from the common base version, elements may be moved in arbitrary ways.

P6 Global Reasoning. In contrast to performing merge decisions locally, the GTS algorithm constructs a global data structure. In this way, conflicts may be recognized which may go unnoticed when local merge rules are applied without any coordination.

P7 Transitive Relationships. Our merge algorithm considers not only immediate neighbors of elements, but in addition takes transitive ordering relationships into account. In this way, more intuitive merge results may be obtained.

P8 Unified Two- and Three-way Merging. In addition to three-way merging, the merge algorithm covers two-way merging, as well. Two-way merging may be applied if the base version is not available. In this case, each difference requires a user decision[1].

3 FOUNDATIONS

In the *Eclipse Modeling Framework (EMF)* (Steinberg et al., 2009), *Ecore* can be used to define meta-models. In the current section, we make a minimal set of assumptions necessary to handle EMF models, particularly concerning multi-valued structural features.

3.1 EMF Models

An *EMF model* is a set of objects (instances of `EObject`), each of which consists of values for *structural features* defined in the meta-model. Structural features are divided into *attributes*, whose values are atomic, and *references*, which are used to interconnect objects by links. Depending on their multiplicities, structural features are either single-valued (upper bound of 1) or multi-valued (upper bound greater than 1, realized as *collections*).

3.2 Collections in EMF

Four types of collections are distinguished by the properties *ordered* and *unique* of their corresponding structural feature. A *set* is a unique collection which contains each of its elements exactly once; a *bag* allows for multiple occurrences of its elements. Unique and non-unique ordered collections are called *ordered sets* and *sequences*, respectively.

In EMF, only attribute values can be stored in bags; the values of references must be unique. For the realization of multi-valued features, EMF uses *ordered collections* (instances of `EList`), regardless of whether the corresponding structural feature is *ordered* or not.

[1]Due to the lack of space, however, we will confine our representation to the use case of three-way merging.

3.3 Matchings

Before alternative versions of a collection can be merged with respect to a common base version, it is necessary to identify their commonalities and differences. In state-based approaches, this is achieved by calculating *matchings*. A matching is a binary relation between the elements of two collections such that (**C1**) each element is matched at most once and (**C2**) only equal elements are matched. Two cases have to be distinguished:

- In the case of ordered sets, the calculation of the matching is trivial since it may simply be induced from their elements. In addition to **C1** and **C2**, the *induced matching* is maximal, i.e., (**C3**) all equal elements are matched.
- In the case of sequences, a match algorithm is required to calculate a matching. Match algorithms on sequences do not necessarily satisfy condition **C3**, i.e., it may happen that equal elements remain unmatched.

An *order-preserving* matching preserves the order of elements (**C4**), i.e., if two elements are matched, they appear in the same (transitive) order in both collections. For example, an algorithm computing the *longest common subsequence* (Hunt and Szymanski, 1977) returns an order-preserving matching. In general, the induced matching on two ordered sets may not be order-preserving. Furthermore, an order-preserving matching on sequences may leave equal elements unmatched, i.e., in general, the matching does not satisfy condition **C3**.

3.4 Merging

As explained above, collections may be classified into four categories (sets, bags, ordered sets, and sequences). However, all of these cases may be covered by a single algorithm:

- In EMF, all collections are stored as lists even if the underlying Ecore model designates the respective structural feature as unordered. Likewise, our GTS algorithm always considers the order of the collections to be merged. Even when the order is irrelevant, it makes sense to preserve it whenever possible as it may affect the external representation of the model and thus the model's interpretation by the user.
- If an element has multiple occurrences in a collection (which can only be the case for non-unique multi-valued attributes), the mapping refers to the

occurrences of elements rather than to the elements themselves. Therefore, we can handle bags analogously to sets.

In our work, matching is clearly separated from merging. The merge algorithm assumes a matching calculated by some match algorithm. In this way, the match algorithm may be replaced without affecting the merge algorithm. However, as far as ordered sets are concerned, the merge algorithm assumes the induced matching (which is trivial to compute). Otherwise, it cannot be guaranteed that merging of ordered sets delivers an ordered set. For example, if an element is inserted on both branches and remains unmatched, it would be inserted twice.

From the input collections and their pair-wise matchings, the set of elements of the merged collection may be calculated as follows:

1. If an element occurs in both a_1 and a_2, it is inserted into m.
2. If an element occurs in either a_1 or a_2 and is contained in the base b, it is not included into m.
3. If an element occurs in either a_1 or a_2 and not in b, it is inserted into m.

However, determining the order of elements is much more difficult (see subsequent sections).

4 MOTIVATING EXAMPLE

4.1 Example

The following small example serves as an introduction to the underlying ideas of the merge algorithm and will be revisited after the algorithm has been explained in detail. For reasons of space, we chose an abstract example, where same letters represent corresponding elements. We have a base version b of an ordered set, that has evolved into two different versions a_1 and a_2 as stated here:

$$\begin{aligned} b &= TKQNFBP, \\ a_1 &= KQTNJPFS, \\ a_2 &= TKMNPJFX. \end{aligned}$$

The author of the first modified version a_1 has reordered the first three elements, inserted J and S, deleted B and rearranged P and F. The second version a_2 has been modified as follows: the elements Q and B have been deleted, and M has been inserted. F and P have also been rearranged. The second author also inserted J, but at a different position. Finally, he/she inserted an element X at the last position.

4.2 The expected Merge Result

In order to consider the changes of both authors in a common version, the merge algorithm comes into play, but what is the expected result?

We expect the merge algorithm to make simple decisions on its own and let the user resolve only contradictory changes. The merge algorithm should prevent the user from generating a solution that may not be derived by the merge rules for sets with respect to contained elements (Section 3.4). Furthermore, the order of the elements has to be considered. This does not only hold for direct relations, but also for indirect/transitive relations. Transitive relations in the base version should be preserved unless they were destroyed in either of the alternative versions. In this way, a more intuitive merge result is produced.

So let's have a look at our example. The reordering of T and K realized by the first author should be preserved, i.e., we expect m to start with K. Q has to be deleted because it is deleted in a_2. M has been inserted by the second user after K and should be included into the merged version. Likewise, T should be included because it is contained in all input versions. Since M has been inserted after K in a_2 and T has been moved after K in a_2 (recall that Q is deleted), there is a conflict concerning the ordering of T and M, i.e., both elements may be arranged in arbitrary order. Furthermore, it is clear that T, K and M have to be placed before N. Next, we would like to have B deleted in the merged version as it has been deleted in both versions. As J has been inserted in both versions but at different positions, the decision whether J is positioned before or after P has to be left to the user. Both elements have to be placed before F. The insertion of S and X at the last position is contradictory and can only be resolved by the user.

In sum, we get a total of 8 different plausible solutions. These solutions can be expressed by the following regular-like expression including 3 user decisions:

$$m = K\{TM\}N\{JP\}F\{XS\}.$$

The operator $\{.\}$ stands for a *cluster* of elements; its operands may be arranged in an arbitrary order[2].

[2]The fact that our example allows for 8 plausible solutions reflects that in many cases, the existence of a single optimal solution is arguable as the expected result may depend on the specific application. In (Uhrig and Schwägerl, 2013), we have discussed a similar problem in the context of model matching.

5 THE GTS ALGORITHM

In this section, we present the generalized topological sort algorithm for merging ordered collections. It considers different versions as linear graphs. The elements of the collections are represented by vertices and the vertices of direct successors are connected by directed edges such that every vertex has an edge to its successor. The graph representations of these three ordered collections are combined into an initial merge graph which is traversed in order to identify a linear order of the elements. Due to conflicting changes, the graph usually contains cycles and/or alternative paths. With the help of a generalized topological sort, the traversal is executed automatically if possible. In case there are multiple candidates for the next step, the user is asked to decide.

5.1 Definitions

Let S denote a set of n elements. An *ordered set* is a bijective function $\vec{S} : S \to \{1..n\}$. $\vec{S}(i)$ denotes the element of $\vec{S}$ at position i. Furthermore, let $S' \subseteq S$. Then, $\vec{S}|_{S'}$ denotes the *restriction* of $\vec{S}$ onto the elements of S', i.e., the ordered set which is obtained by removing all elements not contained in S'. A *partition* π of a set S is a set of sets $S_1 \ldots S_m$ such that $S_1 \cup \ldots \cup S_m = S$ and $S_i \cap S_j = \emptyset (i \neq j)$. Finally, a *directed graph* $g = (V, E)$ is a pair consisting of a vertex set $V = \{v_1, \ldots, v_n\}$ and an edge set $E \subseteq V \times V$.

Our algorithm is supplied with three ordered sets $\vec{V}_b$, $\vec{V}_1$, and $\vec{V}_2$. Equal elements are identified by the matchings having been calculated before the merge algorithm is executed. The output of the GTS algorithm will be an ordered set $\vec{V}_m$. The algorithm uses four graphs $g_j = (V_j, E_j), j \in \{b, 1, 2, m\}$ as auxiliary data structures. The merge graph g_m is transformed step by step until it represents an ordered set which is finally converted into the output $\vec{V}_m$.

The GTS algorithm contains potentially *non-deterministic* operations, which are underlined in the description below. We assume that either the user is involved to make a decision, or that a batch-like procedure resolves non-determinism.

5.2 Algorithm in Pseudo-Code

1. Calculate the unordered set of merged vertices: $V_m := (V_1 \cup V_2) \setminus ((V_b \setminus V_1) \cup (V_b \setminus V_2))$.
2. Modify the inputs in order to exclude deleted elements: $\vec{V}_j := \vec{V}_j|_{V_m}, \quad j \in \{1, 2, b\}$. Furthermore, initialize the output $\vec{V}_m := \emptyset$.
3. Construct edge sets for the input versions. Pairs of vertices that follow each other in the modified ordered sets are connected by an edge: $E_j := \{(\vec{V}_j(1), \vec{V}_j(2)), \ldots, (\vec{V}_j(n_j - 1), \vec{V}_j(n_j))\}, \quad j \in \{1, 2, b\}$.
4. Calculate the unordered set E_m of *semi-transitively* merged edges, which is defined as the union of E_1 and E_2, excluding edges that "represent an immediate order that has been deleted transitively on the other branch"[3]: $E_m := (E_1 \cup E_2) \setminus ((E_b^+ \setminus E_1^+) \cup (E_b^+ \setminus E_2^+))$.
5. For each vertex v_i inside V_m that has no incoming edge, find the *closest common predecessor* if possible, i.e. a vertex v_{ccp} that has a minimal distance to v_i in both V_1 and V_2. If v_{ccp} exists, add the edge (v_{ccp}, v_i) to V_m.
6. For each vertex v_i inside V_m that has no outgoing edge, find the *closest common successor* if possible, i.e. a vertex v_{ccs} that has a minimal distance from v_i in both V_1 and V_2. If v_{ccs} exists, add the edge (v_i, v_{ccs}) to V_m.
7. Identify the *strongly connected components* of g_m[4]. This part of the algorithm is described in detail in the Appendix. As a result, we obtain a partition $\pi_m = \{C_{m1}, \ldots, C_{mk}\}$ of the set of vertices V_m.
8. Perform a user-aided *topological sort* (Sedgewick and Schidlowsky, 2003, Chapter 19.6) based on π_m. While $V_m \neq \emptyset$, perform the following steps (for more details, see Appendix):
   (a) If the component selected in the previous iteration is not empty, continue with it. Else select a new non-empty strongly connected component C_{mj} from π_m that contains no vertex with an incoming edge from outside.
   (b) Select v_i as one of the vertices inside C_{mj}. If possible, restrict the candidate set to vertices that are ordered first in one of the input sequences $\vec{V}_1$ or $\vec{V}_2$, and/or to successors of the vertex selected in the previous iteration.
   (c) Append v_i to the end of $\vec{V}_m$.
   (d) Remove v_i and all of its incoming and outgoing edges from g_m.
9. The resulting ordered set is $\vec{V}_m$.

[3] E_j^+ denotes the *transitive closure* over the edge set E_j.

[4] A strongly connected component of a graph is a set of vertices where each pair of vertices is connected by a directed path. The algorithm used for the identification of strongly connected components was proposed by Kosaraju (Sedgewick and Schidlowsky, 2003, Chapter 19.8).

5.3 Example Revisited

Before analyzing the particular steps of the GTS algorithm in general, let us return to our running example introduced in Section 4 and apply the algorithm to it. The (ordered) input sets of vertices were given as:

$$\begin{aligned}\vec{V}_b &= TKQNFBP,\\ \vec{V}_1 &= KQTNJPFS,\\ \vec{V}_2 &= TKMNPJFX.\end{aligned}$$

The application of the vertex set rule in **step 1** gives us the following unordered result set of vertices:

$$V_m = \{K,T,N,J,P,F,S,M,X\}.$$

The restriction of the inputs to V_m in **step 2** results in the following modified inputs:

$$\begin{aligned}\vec{V}_b &= TKNFP,\\ \vec{V}_1 &= KTNJPFS,\\ \vec{V}_2 &= TKMNPJFX.\end{aligned}$$

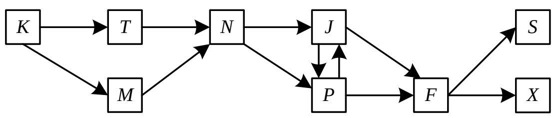
Next, the creation of edges for succeeding elements in **step 3** results in the following edge sets:

$$\begin{aligned}E_b &= \{(T,K),(K,N),(N,F),(F,P)\},\\ E_1 &= \{(K,T),(T,N),(N,J),(J,P),(P,F),\\ &\quad (F,S)\},\\ E_2 &= \{(T,K),(K,M),(M,N),(N,P),(P,J),\\ &\quad (J,F),(F,X)\}.\end{aligned}$$

In **step 4**, the edge set E_m is created using the *semi-transitive* merge rule. For each edge contained in E_1 or E_2, we have to check if it has been "deleted transitively" in the opposite version. In this example, this is only the case for the edge (T,K): It exists in E_b and E_2, but not (transitively) in E_1. All the other edges from E_1 and E_2 are added to E_m. The resulting edge set is:

$$\begin{aligned}E_m = \{&(K,T),(T,N),(N,J),(J,P),(P,F),(F,S),\\ &(K,M),(M,N),(N,P),(P,J),(J,F),(F,X)\}.\end{aligned}$$

After step 4, g_m can be visualized as follows:



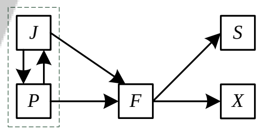
During **steps 5 and 6**, additional transitive edges (closest common predecessors and successors) are added in general. In this example, these steps do not have any effect because K and S/X have no common predecessor/successor in E_1 and E_2.

Step 7 identifies the strongly connected components of g_m. In this example, there is only one component that contains two or more vertices: the cycle $\{J,P\}$.

$$\pi_m = \{\{K\},\{T\},\{N\},\{J,P\},\{F\},\{S\},\{M\},\{X\}\}.$$

In **step 8**, a user-aided topological sort is performed on S_m as described by the following increments:

1. $\{K\}$ is the only strong component without incoming edge from outside. The vertex K is selected automatically and removed from g_m. The current result is $\vec{V}_m = K$.
2. Either $\{T\}$ or $\{M\}$ can be chosen as the next component. Depending on the user's choice, either T or M is added to the result.
3. The component that has not been chosen in the previous increment is selected next deterministically. Using the notation introduced at the end of Section 4.2, we may represent the set of possible results processed so far: $\vec{V}_m = K\{TM\}$[5].
4. N is selected deterministically. Now, $\vec{V}_m = K\{TM\}N$.
   The current graph g_m can be visualized as follows:



5. The component $\{J,P\}$ is chosen next. Both candidates, P and J appear first in one alternative (J in V_1, P in V_2) and are direct successors of the previously processed N. Consequently, the decision is up to the user.
6. The selected component still contains a vertex, which is added deterministically: $\vec{V}_m = K\{TM\}N\{JP\}$.
7. F is selected deterministically: $\vec{V}_m = K\{TM\}N\{JP\}F$.
8. Either $\{S\}$ or $\{X\}$ can be chosen as next component. Depending on the user's choice, either S or X is added to $\vec{V}_m$.
9. The last vertex (X or S) is added to $\vec{V}_m$ deterministically. After that, g_m is an empty graph. Coinciding with the expected result derived in Section 4.2, the overall result is:

$$\vec{V}_m = K\{TM\}N\{JP\}F\{XS\}.$$

[5] Note that a single run of the algorithm produces just one result, ordering either T before M or vice versa.

5.4 Properties

Particular properties of the GTS algorithm have already been claimed in the introduction (**P1** until **P8**). Now we revisit the algorithm in order to show which properties can be asserted to respective (intermediate) results. The algorithm is divided into three major parts: *vertex set calculation*, which takes place in step 1, *edge set calculation*, which is performed by steps 2 until 6, and *graph traversal*, beginning with step 7.

Vertex Set Calculation: In step 1, the set of elements to be contained by the resulting sequence is fixed. It consists of a union of the vertex sets V_1 and V_2, excluding vertices that have been deleted in the first ($V_b \setminus V_1$) or second ($V_b \setminus V_2$) alternative version. By the application of the given set operations, it is impossible to produce a result that contains several instances of a matched element, so the *uniqueness of elements* property is fulfilled (**P3**). Furthermore, our algorithm produces a result that is consistent with the reasoning for three-way merging sets of elements described at the end of Section 3.4. The set of vertices cannot be modified by later steps; consequently, the *propagation of insertions and deletions* is *consistent* (**P4**).

Edge Set Calculation: Steps 2 and 3 are necessary to construct (linear) graphs for the input sequences. The properties of *global reasoning* (**P6**) and *transitive relationships* (**P7**) require that the merged edge set does not only include edges that emerge from direct, but from transitive relationships encoded in *paths* inside E_1 and E_2. The GTS algorithm considers this fact by a twofold strategy. Step 4 ensures that non-contradictory transitive information that is already stored in direct edges of the alternative versions is preserved. Nevertheless, it is not possible to maintain each possible transitive path this way. In some cases, a “transitive deletion” of an edge destroys a path that expresses another valid relationship between two elements. Therefore, in steps 5 and 6, additional relationships are added to vertices that have an undetermined transitive predecessor/successor set.

In step 4, the merged edge set E_m is calculated by a *semi-transitive* merging rule. As already described in the pseudo-code, the intuition behind this rule is to delete an “immediate order that has been deleted transitively on the other branch”. While immediate orders are expressed by edges in E_1 and E_2, transitive deletions can be expressed by ($E_b^+ \setminus E_2^+$) or ($E_b^+ \setminus E_1^+$), respectively. For the merged edge set, the condition $(E_1 \cap E_2) \subseteq E_m \subseteq (E_1 \cup E_2)$) holds.[6]

After step 4, there might be additional non-contradictory transitive relationships expressed in the input versions which are not yet considered in E_m, and neither contained in $E_1 \cup E_2$. Due to cross-over moves, g_m might contain vertices that have no predecessor or successor, even if there exist one or more common predecessors or successors in g_1 and g_2. The basic idea behind steps 5 and 6 is to reconstruct these (so far ignored) relationships by finding the *closest common predecessor* or *successor*. By including an additional edge from/to that identified vertex, a subset of $E_1^+ \cap E_2^+$ is included that expresses exactly the missing information.[7]

Graph Traversal: After step 6, the transitive closure over the edge set E_m contains all non-contradictory transitive relations from E_1 and E_2. Based on this graph, a linear order has to be derived in the subsequent steps. As described above, this is performed by means of a topological sort. Due to the support of arbitrary *move operations* (**P5**), g_m may contain *cycles*, particularly in case of cross-over moves. Unfortunately, cycles inhibit the application of a topological sort. To overcome this problem, the topological sort in step 8 is not performed on the original graph, but on its *strongly connected components* which are calculated in advance by step 7: Cycles are virtually replaced by atomic vertices. When the topological sort enters a cycle, the order of its contained elements needs to be derived in a reasonable way. Sub-step 8b of the GTS algorithm makes sure that the order in which the vertices of a cycle are traversed is consistent with the order of one of the alternative input sequences $\vec{V_1}$ and $\vec{V_2}$. At the same time, step 8a makes sure that a cycle is only left after all of its vertices have been processed.[8]

[6]In the running example, the deletion of the edge (T,K) is necessary because otherwise, the user could produce an order that starts with T, which has obviously been removed from the first position of the collection.

[7]The running example does not require this step. As a substitute, the interested reader may apply the GTS algorithm to Example 2 from the Related Work section (Section 7). Here, an additional transitive edge (A,D) is inserted due to the interacting moves of B in $\vec{V_1}$ and D in $\vec{V_2}$.

[8]Again, the running example does not require this particular traversal strategy. In Example 1 from the Related Work section, however, the absence of step 8b would allow an order that does not contain the edge (A,C), which occurs (transitively) in each of the three versions and should definitely be preserved in the merged version.

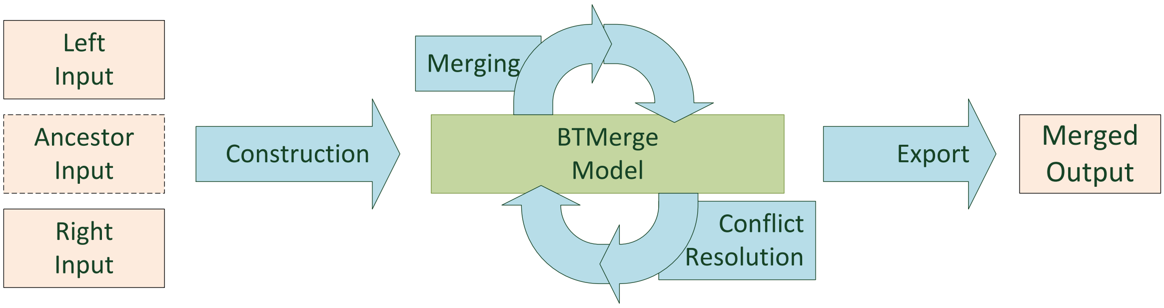


Figure 2: Our three-way model merge tool *BTMerge*, which includes an implementation of the GTS algorithm, decomposes the merging problem into three phases.

5.5 Complexity

We outline an analysis of runtime complexity and memory consumption for the GTS algorithm, proceeding step by step. The number n is considered equal to $|V_m|$, the number of vertices contained in the resulting collection, which is equal or less than $|V_1|+|V_2|$.

1. Provided that the presence of a vertex in a set can be determined in constant time (e.g. by hashing), we can compute V_m in $O(n)$. The same upper bound obviously applies to memory consumption.
2. With the same assumption, we can compute and store three restrictions of a maximum of $3n$ elements in $O(n)$.
3. A maximum of $3(n-1)$ edges are inserted. Assuming that edges are stored in an *adjacency list*, we have $O(n)$ for both runtime complexity and memory consumption.
4. The number of edges to be inserted into E_m will not exceed the upper bound $3(n-1)$ and thus require $O(n)$ of memory. The transitive closures E_j^+, however, would temporarily consume $O(n^2)$ of memory. We can avoid this by rewriting the merge rule as follows:
$E_m := (E_1 \setminus (E_b^+ \setminus E_2^+)) \cup (E_2 \setminus (E_b^+ \setminus E_1^+))$.
The sets part of the union can be easily computed by index comparisons, which are performed in constant time. Consequently, we get $O(n)$ for both runtime and memory.
5. During step 5, vertices without predecessors are traversed. In the worst case, finding a closest common predecessor requires the traversal of all n vertices of the input sequences V_1 and V_2. The number of vertices without predecessor has an upper bound of n, too, resulting in a complexity of $O(n^2)$. However, the worst case will only be reached if the number of cross-over move operations is close to n. A maximum of n new edges are inserted, so the memory consumption is $O(n)$.
6. Step 6 equals step 5 in runtime complexity and memory consumption.
7. Using an adjacency list, the algorithm of Kosaraju has a runtime of $O(|V_m|+|E_m|)$ (Sedgewick and Schidlowsky, 2003, Chapter 19.8). Here, an upper bound for $|E_m|$ is $3n$, and so we get linear runtime of $O(n)$. Both the resulting strongly connected components and the auxiliary search sequence will consume additional memory of $O(n)$.
8. A topological sort can be performed in $O(|V_m| + |E_m|)$ (Sedgewick and Schidlowsky, 2003, Chapter 19.6), which is $O(n)$ in our special case (see above). However, the selection of a vertex out of the component has an additional worst-case complexity of $O(n)$ because the conditions (first vertex in V_1 or V_2, successor of previously inserted element) require traversing the vertex set of the current component in each of the n iterations. In sum, we have a computational complexity of $O(n^2)$. Nevertheless, if no cycles occur in g_m, selection can be done in constant time, and runtime complexity can be reduced to $O(n)$. In any case, the resulting sequence will consume $O(n)$ of memory.

In sum, the GTS algorithm has a memory consumption of $O(n)$, which allows for three-way merging of large ordered collections. The computational worst-case complexity is bounded to $O(n^2)$.

6 IMPLEMENTATION

We implemented the GTS algorithm for merging ordered collections as a generic standalone component based on (but not restricted to) the Eclipse Modeling Framework. The following explanations refer to the integration of the component with our tool *BTMerge* (Schwägerl et al., 2013b; Schwägerl et al., 2013a), which performs a consistency-preserving three-way merge of EMF models. As shown in Figure 2, model merging is realized as a three-phase process. First, the merge model is created (*construction*) as a superimposition of the three input versions. For the identification of corresponding objects, we rely on a match algorithm, e.g. *EMF Compare* (Brun and Pierantonio, 2008). The identification of corresponding values inside *unique* collections is induced by equality for attribute values, and by the object matching for reference values, respectively. For *non-unique* collections, a heuristic sequence comparison is performed that produces a *maximal*, but not necessarily an *order-preserving* matching (see Section 3.3). The second phase, *merging*, follows an incremental design. The preliminary merge model is modified alternately by the merge algorithm and the user; the merge algorithm applies *merge rules*, which can be applied automatically, or stops in case of *conflicts*. These can in turn be resolved by the user, who chooses one of the proposed resolution methods. Only after all conflicts have been resolved, the merge model is *exported* as an EMF instance.

The user interface of the interactive merge tool, called the *resolution tool* (cf. Fig. 3), allows the user to communicate resolution decisions for specific conflicts during the second phase. A dedicated *conflicts* view (not shown in the screenshot) outlines pending merge decisions. The user can resolve a conflict by means of a wizard that describes the consequences of the proposed resolution methods. After resolution by the user, the next merge increment is performed automatically.

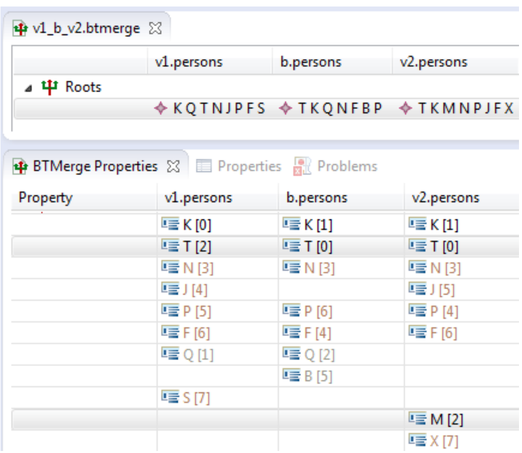


Figure 3: The resolution tool of BTMerge. The screenshot depicts the situation after K has been inserted and the user has to choose between T and M.

Steps 1 until 7 of the GTS algorithm are executed during the *construction* phase, where an initial graph g_m is created for each multi-valued structural feature. During *merging*, the respective graph is transformed step by step in order to calculate the resulting ordered

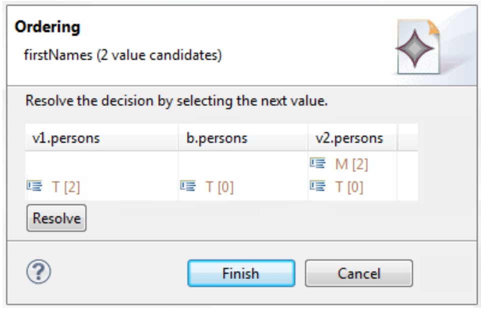


Figure 4: A screenshot of a conflict wizard for the resolution of an ordering conflict. The user is asked to select either T or M as the preferred next value.

set $\vec{V_m}$ (step 8). For steps 8a/b, our tool provides a conflict resolution wizard page that lets the user choose the value he/she wants to appear previously to all other selectable values (cf. screenshot Fig. 4). The algorithm will continue with the selected v_i until the next conflict is detected. During *export*, the final ordering described by $\vec{V_m}$ is considered when converting an ordered structural feature back into its EMF representation.

The fact that EMF even stores the values of *unordered* features in ordered lists led us to an alternative implementation that possibly skips the user interaction in steps 8a/b and selects an element by applying a default rule (*left* or *right* version) instead.

7 RELATED WORK

7.1 Related Approaches

Text-based merging has been used for long in version control systems such as RCS (Tichy, 1985), CVS (Vesperman, 2006), or Subversion (Collins-Sussman et al., 2004). To this end, the tool *diff3* (Khanna et al., 2007) is used frequently. diff3 compares both alternative versions to the base, relying on an algorithm which computes a longest common subsequence (Hunt and Szymanski, 1977) of elements (text lines). A longest common subsequence (LCS) is a maximal matching among the elements of the subsequences without cross-overs, i.e., two elements of the longest common subsequence appear in the same order in both sequences. Next, diff3 computes a sequence of stable and variable chunks (consecutive sequences of elements). A stable chunk is contained in all three versions; a variable chunk has at least two different versions. The merged version is constructed from the sequence of chunks. A conflict occurs if a chunk has three different versions. In the case of a non-interactive merge, as performed in version control systems, conflicting versions of chunks are written to the merged version, and the user has to resolve these conflicts post mortem.

3dm (Lindholm, 2004) is a tool for three-way merging of XML documents. 3dm considers documents as ordered trees. A document is represented by a set of facts for the contents of nodes and the tree structure, the latter of which is expressed by *pcs triples* (parent, child, successor). After a matching phase, which identifies elements of different documents, the sets of facts are united. A conflict occurs if the contents, the parent, predecessor, or successor of a node are not unique. All facts from the base version which are involved in conflicts are removed. If the resulting set of facts is free of conflicts, a document is created from the top to the bottom and (within lists) from left to right. In (Lindholm, 2004), resolution of conflicts among "new" facts is not considered.

Alanen and *Porres* (Alanen and Porres, 2003) describe algorithms for comparing and merging MOF models. It is assumed that elements carry universally unique identifiers. Elements from different model versions are matched if their identifiers are equal. For three-way merging, two directed deltas from the base version to each of the alternative versions are calculated. In the case of ordered multi-valued features, deltas are composed of insert and delete operations. The merge algorithm merges the input deltas into a single delta from the base to the merged version. During the merge, indices are updated if an operation from the opposite delta is selected. A conflict occurs if two elements are inserted at the same position. A conflict is resolved by ordering the left before the right element or vice versa. In the case of conflicts, all possible deltas are computed, and a delta with minimal length is returned.

As part of the version control system *EMF Store*, a tool for three-way merging of EMF models is provided (Koegel et al., 2010). In EMF Store, all model elements are identified by universally unique identifiers. Furthermore, the model editor integrated with EMF Store records change logs, which are stored in the repository of the version control system. The merge tool receives a base version and two operation sequences, which are combined into a single sequence from the base to the merged version. Changes to ordered collections are described in terms of insertions, deletions, and move operations. Except for deletions of the same element, change operations to the same collection on different branches always conflict. To resolve a conflict on an ordered collection, the user may select either the left or the right version of the whole collection.

EMF Compare (Brun and Pierantonio, 2008) is a tool for comparing versions of EMF models. When supplied with a base version and two alternative versions, EMF Compare may also be used for three-way merging. To this end, both alternative versions are compared to the base. From the resulting match model, a difference model is derived which is composed of deltas from the base version to the alternative versions. As in two-way merging, the user has to decide for each change whether it is applied to the opposite branch. In addition, EMF Compare detects conflicting changes in the case of three-way merging. EMF Compare may merge insertions and deletions on ordered collections. The tool recognizes when the order of elements has been changed. However, this results in an unspecific message concerning the whole collection ("the order has changed"), and the user may propagate the order to the opposite branch only as a whole.

AMOR (Taentzer et al., 2012) provides a merge tool for EMF models which follows EMF Compare's basic process of merging (see above). Moves are recognized, but reduced to insertions and deletions, which provide the foundation for merging. When merging ordered collections, different types of conflicts are detected: (1) insertions of different elements at the same index, (2) insertion of an element vs. deletion of a predecessor/successor, or (3) deletions on opposite branches with subsequent indices.

The algorithm presented in this paper constitutes an improvement over the original algorithm for merging ordered collections that we proposed in (Westfechtel, 2012) as a part of the EMF model merge algorithm underlying a previous version of the BTMerge tool. Like the GTS algorithm, the described algorithm constructs three linear graphs representing the input versions and combines them into a union graph. Next, it removes vertices and edges having been deleted on at least one branch, and constructs a linear order by a series of graph transformations. These transformations aggregate mutually unrelated vertices as well as vertices on cycles into clusters, whose elements may then be serialized in any order. Therefore, we refer to this algorithm as *cluster algorithm* below.

Compared to our GTS algorithm, the cluster algorithm suffers from several limitations. In particular, since clusters may be serialized in any order, an order of the elements may be constructed which was not present in any input version. Furthermore, the cluster algorithm may form clusters which are too large. Instead of transforming the merge graph into a sequence of clustered nodes, the GTS algorithm performs a generalized topological sort; furthermore, it takes transitive in addition to direct neighbor relation-

Table 1: Expected and actual results for three example problems.

	Example 1	Example 2	Example 3 (Running Example)
b	ABC	$ABCDEF$	$TKQNFBP$
a_1	BAC	$ACDEFB$	$KQTNJPFS$
a_2	ACB	$ABDECF$	$TKMNPJFX$
m (expected)	$[BAC \vert ACB]$	$ADECFB$	$K\{TM\}N\{JP\}F\{XS\}$
diff3	$BA[C \vert CB]$	$A[C \vert B]DECFB$	$K[QT \vert M]N[JP \vert PJ]F[S \vert X]$
Alanen and Porres	$BACB$	$ADECFB$	$K\{TM\}NJPJF\{XS\}$
EMF Store 1.0.0	$[BAC \vert ACB]$	$[ACDEFB \vert ABDECF]$	$[KQTNJPFS \vert TKMNPJFX]$
EMF Compare	$[BAC \vert ACB]$	$[ACDEFB \vert ABDECF]$	$[TKM \vert TKQ \vert KQT \vert KMT]N\{JP\}F\{SX\}$
Cluster algorithm	$\{ABC\}$	$A\{BCDEF\}$	$K\{TM\}N\{JPF\}\{SX\}$
GTS algorithm	$[BAC \vert ACB]$	$ADECFB$	$K\{TM\}N\{JP\}F\{XS\}$

ships into account. Instead of transforming the merge graph into a sequence, it performs a generalized topological sort. These differences result in considerably improved behavior, as we will demonstrate below.

7.2 Comparison

Table 1 shows the results produced by applying most of the tools/algorithms described above to a set of three examples. The last row lists the outcomes of the GTS algorithm presented in this paper. The results for 3dm were omitted; manual execution of all examples results in conflicts, whose resolution is not described in (Lindholm, 2004). AMOR was not included because the information given in (Taentzer et al., 2012) does not suffice to perform a manual conflict resolution.

In the case of conflicts, multiple results may be produced. These result sets are represented by regular expressions, where $\{.\}$ stands for an arbitrary permutation of its operands and $[.|.]$ indicates an exclusive alternative.

In Example 1, B has been moved to the head and the tail on different branches. Since only one of these operations may be applied, we assume that a conflict is reported and only one of these operations may be applied. Depending of the conflict resolution, one of BAC and ACB should be returned. In Example 2, B has been moved to the tail in a_1, and C has been moved behind E in a_2. These cross-over moves may be merged without conflict, resulting in the output $ADECFB$. Example 3 is our running example.

The *GTS algorithm* for ordered collections is the only one which precisely produces the expected results in all examples.

diff3 does not support move operations, does not guarantee the uniqueness of elements in the case of ordered sets, and does not consistently propagate insertions and deletions (properties **P3, P4 and P5**, respectively; see Section 1). Conflicts are reported even in Example 2 (non-conflicting moves), where any conflict resolution results in a duplicate element (B or C). In Example 3, Q may be part of the merged version although it was deleted in a_2.

The algorithm of *Alanen* and *Porres* does not support moves, either; however, it behaves differently from diff3. In Example 1, B is deleted once and inserted twice, without reporting a conflict. In Example 2, the correct result is delivered. In Example 3, J is inserted twice (duplicate insertion of J, resulting in the subsequence JPJ).

EMF Store does consider moves, but defines conflicts on a coarse-grained level. Since in all examples both alternatives were modified, a conflict is raised in each case (even in Example 2, where the moves could be combined without conflict). The user may select one of the inputs, but may not combine the change operations.

With respect to re-orderings, *EMF Compare* follows a coarse-grained approach, as well. Therefore, in Examples 1 and 2 EMF Compare merely reports order changes, and the user may only establish the order in one of the alternative versions. In particular, the moves are not merged in Example 2. In Example 3, no conflicts are reported. By applying all changes from the base in different ways, a set of results may be produced which are as expected from N through to the end. At the beginning, EMF Compare derives a substitution of Q by M rather a deletion of Q and insertion of M. Thus, Q may be included into the merge result. Furthermore, T may be ordered before K.

All three examples demonstrate different shortcomings of the *cluster algorithm*. In Example 1, the cluster algorithm constructs the cycle $A \rightarrow C \rightarrow B \rightarrow A$ and collapses this cycle into a cluster, whose elements may arranged in any order. Clusters result in loss of information and may allow unexpected permutations such CAB (in both alternatives, A precedes C). In Example 2, the algorithm constructs a cluster resulting from the cycle $B \rightarrow D \rightarrow E \rightarrow C \rightarrow F \rightarrow B$, consisting of new immediate neighborhood edges. This example demonstrates the shortcomings of considering direct neighborhood relations only. Finally, in Example 3 the algorithm forms the cluster JPF, allowing to order F before J or P. In this case, the cluster is too large.

As mentioned above, the *GTS algorithm* delivers the expected results in all example problems. In Example 1, the GTS algorithm constructs the cycle $A \rightarrow C \rightarrow B \rightarrow A$. However, the cycle is not collapsed into a cluster. Rather, one element is selected as the start vertex; then, all remaining elements follow in the order defined by the edges of the cycle. Only A and B are offered as start vertices. Thus, the algorithm returns the expected outputs BAC or ACB. In Example 2, the GTS algorithm constructs an acyclic graph: New edges are inserted only into the graph if their transitive order has not been flipped in the opposite version. For example, the graph does not contain the edge (B,D). The resulting graph may be serialized in a unique way, delivering the expected result $ADECFB$. Example 3 – the running example – has been explained in detail before.

8 CONCLUSIONS

In this paper, we have investigated an important subproblem of model merging: the creation of a linear order for collections which occur as the values of multi-valued structural features in EMF models. Given a common base version b, the expected behavior for a merge algorithm for ordered collection is the propagation of insertions, deletions and moves from the alternative versions a_1 and a_2 to a merged version m of the same collection.

The presented GTS algorithm is able to handle arbitrary move operations. Conflicts are only reported to the user in case decisions cannot be made automatically. It is purely *state-based*, i.e. it does neither require change logs nor unique object identifiers. In contrast, it relies on a *matching*, which can be calculated by any sequence comparison algorithm that delivers an one-to-one mapping of equal elements. Our algorithm is graph-based; from given inputs, a graph g_m for the merged version m is calculated by means of set formulas. For the calculation of its edge set, not only immediate, but also transitive successor relationships are taken into account. Information stored in more than one edge, i.e. in *paths*, is propagated into the merged version, too. From the merge graph g_m, a linear order is derived by means of a *generalized topological sort* that involves the user in case a decision cannot be made automatically due to conflicting insertions or moves. The fact that a topological sort requires an acyclic graph motivates the calculation of the *strongly connected components* of g_m beforehand. This way, *cycles* can be handled which result from contradicting moves. The GTS algorithm has a worst-case runtime complexity of $O(n^2)$. However, for large sequences with few cyclic moves, we expect a runtime close to $O(n)$ in practice.

Concerning the merging of ordered collections, the GTS algorithm goes considerably beyond related approaches to three-way merging of ordered collections in models, including the *cluster algorithm* contained in the three-way model merge algorithm we presented in (Westfechtel, 2012). But even when compared to tools dedicated to a related but more popular problem, merging of sequences of lines in text-based version control systems, our approach outperforms its competitors with respect to the accuracy of the calculated results. We demonstrated this fact by means of a running example and two additional examples, for which our algorithm is able to produce the expected result.

We integrated an implementation of the GTS algorithm in the tool *BTMerge*, which is dedicated to consistent three-way merging of EMF models. A graph is constructed for each multi-valued feature that is contained by the model to be merged, and the algorithm is executed in an incremental way in order to allow the user to commit his/her merge decisions step by step. The implementation has been tested by means of a test set of adequate size (29 model versions, 64 three-way merges).

Although we implemented the GTS algorithm in the context of EMF models, our plans for future include to make use of its generic nature. In particular, we plan to support three-way merging text files. This way, a detailed evaluation against main-stream algorithms used in common version control systems will be possible.

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APPENDIX

Calculation of Strongly Connected Components (Algorithm of Kosaraju[9])

1. Initialize an auxiliary search sequence $\vec{D} := \emptyset$. Perform a post-ordering depth-first search in g_m, beginning with an arbitrary vertex that is not contained in $\vec{D}$. Each time the expansion of a vertex is finished, append it to the search result sequence $\vec{D}$. While $\vec{D}$ does not contain all vertices in V_m, repeat this step.

[9](Sedgewick and Schidlowsky, 2003, Chapter 19.8)

2. Initialize the partition π_m of the set of vertices V_m. At the beginning, it contains only one set, a copy C_{m0} of V_m. $\pi_m := \{C_{m0}\}$.
3. While $\vec{D} \neq \emptyset$, perform the following steps:
   (a) Obtain the last vertex v_i in $\vec{D}$.
   (b) Identify the subset C_{mi} of $\vec{D}$ which contains vertices that that have a path to v_i but are not yet contained by any set of the partition π_m. The set C_{mi} includes v_i itself.
   (c) Remove from $\vec{D}$ all vertices in C_{mi}.
   (d) Add the set C_{mi} to the partition π_m.
   (e) Remove from C_{m0} all vertices in C_{mi}. Taking additionally into account the previous step, π_m remains a valid partition of V_m.
4. Remove the empty set C_{m0} from the partition π_m.

User-aided Topological Sort on Strongly Connected Components

1. Initialize an auxiliary current component set $C_{mj} := \emptyset$,
   and an auxiliary successor set $Q := \emptyset$.
2. For each vertex $v_i \in V_m$, provide a boolean flag $m(v_i) \in \{\top, \bot\}$ that indicates if v_i is *marked*. The default value is $\bot$.
3. Within each strongly connected component (each set of the partition π_m), *mark* each vertex v_o that appears first either in V_1 or V_2 by setting $m(v_o) := \top$.
4. While $V_m \neq \emptyset$, perform a user-aided topological sort as described by the following steps:
   (a) If $C_{mj} = \emptyset$, <u>select</u> a strongly connected component in π_m that contains no vertex with an incoming edge from outside that component. The new value of C_{mj} is the identified set. Set $Q := \emptyset$.
   (b) <u>Select</u> as v_i one of the *marked* vertices $m(v_i) = \top$ inside C_{mj}. For the selection, prefer vertices that are contained in Q.
   (c) *Mark* all successors v_s of v_i in C_{mj} by setting $m(v_s) := \top$.
   (d) In case of three-way merging, append v_i to the end of $\vec{V}_m$.
   (e) In case of two-way merging:
      i. If $v_i \in (V_1 \cap V_2)$, append v_i to the end of $\vec{V}_m$.
      ii. Else <u>decide</u> whether to append v_i to the end of $\vec{V}_m$.
   (f) Modify Q to contain all direct successors of v_i that are still contained in C_{mj}.
   (g) Remove v_i from C_{mj}. If $C_{mj} = \emptyset$, remove it from the partition π_m.
   (h) Remove v_i and all of its incoming and outgoing edges from g_m. Taking additionally into account the previous step, π_m remains a valid partition of V_m.