Performance Analysis of Random Relaying of Partitioned MDS Codeword Block Applied to Persistent Relay CSMA over Random Error Channels

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Abstract: We propose incorporation of Random relaying of Partitioned Maximum Distance Separable codeword blocks (RP-MDS), which has been proposed for multi-hop cooperative relay networks, to Persistent Relay Carrier Sense Multiple Access (PRCSMA) over noisy channels. The proposed protocol elaborately employs the powerful error-correcting capability of MDS codes into cooperative communication systems and introduces the incremental redundancy concept to PRCSMA. A destination node can reinforce an error-correcting capability when it receives a new frame. The performance of the proposed protocol is analyzed with a Markov model in terms of the average duration of a cooperation phase and the energy efficiency. Numerical results indicate that the proposed protocol can significantly improve the performance, compared to the original PRCSMA.

1 INTRODUCTION

Cooperative communications with relay nodes have been recognized as one of effective and promising techniques in wireless/mobile communication systems. Relay standards are on the way to successful implementation in Long Term Evolution (LTE)-Advanced by the Third Generation Partnership Project (3GPP) and 802.16m by IEEE (Loa et al., 2010; Bhamri et al., 2011). Relay techniques have been enthusiastically investigated from the viewpoint of the physical (PHY) and data-link layers (Bhamri et al., 2011; Gómez-Cuba et al., 2012). In PHY layer perspective, Multiple-Input and Multiple-Output (MIMO) and diversity techniques are attractive. In the data-link layer perspective, a number of Cooperative Automatic Repeat reQuest (C-ARQ) protocols have been proposed and analyzed. Particularly, the design of Medium Access Control (MAC) protocols employed between relay nodes and the destination node influences the performance, when two or more relay nodes collaborate on an identical channel.

MAC protocols for C-ARQ systems have been proposed recently. Dianati et al. (Dianati et al., 2006) proposed a Node-Cooperation Stop-and-Wait (NCSW) ARQ protocol. The performance of NCSW with a single relay node was analyzed over two-state Markovian channels. Morillo and Garcia-Vidal (Morillo and Garcia-Vidal, 2011) proposed a C-ARQ scheme with an integrated frame combiner. They analyzed the performance with round-robin cooperation among relay nodes and with Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA). Alonso-Zarate et al. (Alonso-Zarate et al., 2009; Predojev et al., 2012) proposed Persistent Relay CSMA (PRCSMA), which elaborately incorporates well-known IEEE 802.11 Distributed Coordination Function (DCF) (IEEE Standard 802.11, 1999). In (Alonso-Zarate et al., 2009), the performance of PRCSMA was analyzed based on a steady-state two-dimensional Markovian model proposed by Bianchi (Bianchi, 2000). In the above literature (Dianati et al., 2006; Morillo and Garcia-Vidal, 2011; Alonso-Zarate et al., 2009; Predojev et al., 2012), it is basically assumed that a node can correctly receive a transmitted frame if no frame collisions occur. Thus, when we consider a scenario where a channel adds errors to a non-colliding frame, it is expected that the use of error-correcting codes can improve the performance.

In this paper, we propose incorporation of Random relaying of Partitioned Maximum Distance Separable codeword block (RP-MDS) (Sakakibara et al., 2011) to PRCSMA over noisy channels. The pro-

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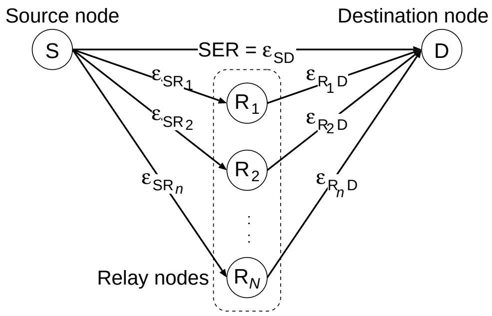


Figure 1: System model with N relay nodes.

posed protocol elaborately takes advantage of the powerful error-correcting capability of MDS codes. Incorporating RP-MDS into PRCSMA may introduce effective performance improvement in accordance with the concept of incremental redundancy (Pursley and Sandberg, 1989). A destination node can reinforce an error-correcting capability when it receives a new frame, even if it includes channel errors. The performance of the proposed protocol is analyzed with the aid of a Markov model. The accuracy of the model is verified by means of computer simulation.

The rest of the present paper is organized as follows: Section 2 presents a system model with relay nodes. PRCSMA is briefly reviewed in Section 3. In Section 4, after a short reminder of useful properties of MDS codes, the proposed protocol is described. Performance of the proposed protocol is analyzed in Section 5, based on the analysis in (Alonso-Zarate et al., 2009). Numerical results are presented in Section 6 in comparison with results obtained from computer simulation. Finally, Section 7 concludes the present paper.

2 SYSTEM MODEL

Consider a wireless network consisting of a pair of source node S and destination node D with N relay nodes; $R_1, R_2, \ldots, R_N$, as shown in Fig. 1. All channels are half-duplex, so that a node can not transmit and receive simultaneously. All nodes are located within their transmission range. Hence, each node can overhear ongoing transmission originating from other nodes. Let ε_{SD}, ε_{SR_n}, and ε_{R_nD} be the symbol error probabilities on channels between source node S and destination node D, between source node S and relay node R_n, and between relay node R_n and destination node D, respectively, for $n = 1, 2, \ldots, N$.[1] If frame transmission from source node S resulted in erroneous reception at destination node D and if one or more relay nodes succeeded in error-free reception of the frame, then such relay nodes can collaboratively serve as supporters for frame retransmission. For effective use of cooperative communications, we generally assume that $\varepsilon_{SD} > \varepsilon_{R_nD}$. The duration in which relay nodes collaborate frame retransmissions is referred to as a *cooperation phase* (Alonso-Zarate et al., 2009). Note that every frame is assumed to include an appropriate header and an ideal Frame Check Sequence (FCS) for error/collision detection,[2] in addition to the payload.

3 PERSISTENT RELAY CSMA (PRCSMA)

PRCSMA (Alonso-Zarate et al., 2009; Predojev et al., 2012) is a MAC protocol which elaborately resolves frame collisions among transmission from relay nodes, based on IEEE 802.11 DCF (IEEE Standard 802.11, 1999). Similarly to IEEE 802.11 DCF, each relay node in PRCSMA inserts random backoff delay before every frame transmission in a distributed manner according to its own contention window (CW). Let $\boldsymbol{m}$ denote a message block of k-symbol length, which is generated at source node S. A DATA frame consists of a header, payload $\boldsymbol{m}$, and FCS. Note that the terms “message block $\boldsymbol{m}$” and “DATA frame” are used interchangeably hereafter, unless ambiguity arises.

The operation in PRCSMA is summarized as follows. The detailed description can be found in (Alonso-Zarate et al., 2009). After erroneous reception of a DATA frame, destination node D broadcasts a Call For Cooperation (CFC) frame. If one or more relay nodes receive both the DATA frame and the CFC frame, then the cooperation phase is invoked. Relay nodes which join in the cooperation phase is referred to as *active relay nodes*. Active relay nodes simultaneously start the DCF operation, after the reception of the CFC frame followed by DIFS (Distributed Inter-Frame Space). When destination node D correctly receives a frame, it broadcasts an ACK frame to announce not only correct reception of the DATA frame to source node S but also completion of the cooperation phase to all the nodes.

An illustrative operational example with two active relay nodes, R_1 and R_2, is shown in Fig. 2. Both active relay nodes independently set their back-

[1] Using the symbol error rate ε, we can evaluate the bit error rate as $1 - \sqrt[m]{1-\varepsilon}$ when a symbol consists of m bits.

[2] The term “ideal” implies that the probability of undetected errors can be neglected.

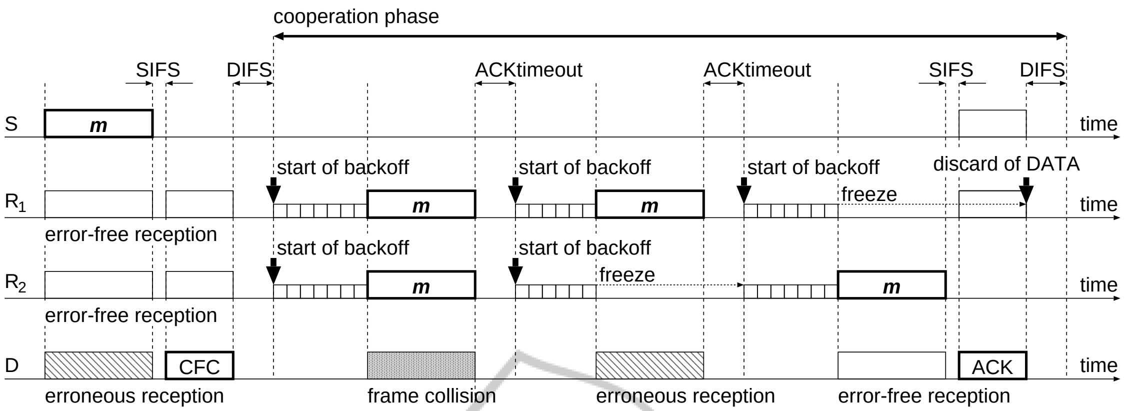


Figure 2: Illustrative example of PRCSMA.

off counter to seven and a cooperation phase is invoked. The first DATA frame transmission from these relay nodes results in collision. The second transmission from relay node R_1 suffers from channel errors. Finally, an ACK frame is returned by destination node D corresponding to error-free reception of the second transmission from R_2. It completes the cooperation phase. Notice that source node S does not participate in a cooperation phase (Alonso-Zarate et al., 2009).

4 PRCSMA WITH RANDOM RELAYING OF PARTITIONED MDS CODEWORD BLOCK

In a cooperation phase in PRCSMA over noisy channels, destination node D may successively receive erroneous frames one by one in between backoff intervals. It suggests possibility to effectively utilize the concept of incremental redundancy (Pursley and Sandberg, 1989), where the error-correcting capability at a receiving node is reinforced upon frame reception. In this context, we propose incorporating RP-MDS into PRCSMA. RP-MDS has been proposed for multi-hop cooperative relay networks over noisy channels (Sakakibara et al., 2011). The proposed protocol, designated as PRCSMA+RP-MDS, is described after some properties of MDS codes are reviewed.

4.1 MDS Codes

Denote a linear block code of length n and dimension k over a certain finite field by an $[n,k]$ code. An $[n,k]$ code is MDS if its minimum distance is $n-k+1$. A class of MDS codes, including Reed-Solomon codes, is known to be fruitful in advantageous properties (Wicker, 1995). Among them, the following two theorems; Theorems 8-4 and 8-6 in (Wicker, 1995), respectively, are used afterward:

Theorem 1. *For an $[n,k]$ MDS code, a receiver can recover the encoded message of length k, if it receives at least k code symbols with no errors.* □

Theorem 2. *Punctured MDS codes are also MDS, that is, the minimum distance of an $[n-p,k]$ punctured MDS code is $n-p-k+1$, if $n-p \geq k$.* □

Suppose a systematic $[Lk,k]$ MDS code $\mathcal{C}$.[3] Let $\boldsymbol{G}$ be a generator matrix of $\mathcal{C}$. It is clear that $\boldsymbol{G}$ is a $k \times Lk$ matrix. Let

$$\boldsymbol{G} = [\, \underbrace{\boldsymbol{I}}_{k} \mid \underbrace{\boldsymbol{G}_1}_{k} \mid \underbrace{\boldsymbol{G}_2}_{k} \mid \cdots \mid \underbrace{\boldsymbol{G}_{L-1}}_{k} \,] \tag{1}$$

be a partition of $\boldsymbol{G}$ into L blocks of identical size, where $\boldsymbol{I}$ and $\boldsymbol{G}_\ell$ are an identity matrix and a square matrix of order k for $\ell = 1,2,\ldots,L-1$, respectively. Then, for a message block $\boldsymbol{m}$ of length k to be encoded, a codeword of $\mathcal{C}$ can be also partitioned into L codeword blocks $\boldsymbol{c}_\ell$ of length k;

$$\boldsymbol{c} = \boldsymbol{m}\boldsymbol{G} = [\, \underbrace{\boldsymbol{c}_0}_{k} \mid \underbrace{\boldsymbol{c}_1}_{k} \mid \underbrace{\boldsymbol{c}_2}_{k} \mid \cdots \mid \underbrace{\boldsymbol{c}_{L-1}}_{k} \,], \tag{2}$$

where $\boldsymbol{c}_0 = \boldsymbol{m}$ and $\boldsymbol{c}_\ell = \boldsymbol{m}\boldsymbol{G}_\ell$ for $\ell = 1,2,\ldots,L-1$. From Theorem 1 and Theorem 2, the following corollary holds at a receiver when one or more codeword blocks $\boldsymbol{c}_\ell$ are received:

Corollary 1. *Assume that u distinct codeword blocks, $\boldsymbol{c}_{\ell_1}$, $\boldsymbol{c}_{\ell_2}, \ldots,$ $\boldsymbol{c}_{\ell_u}$, are received and that a receiver*

[3]Using a systematic code, an encoded message appears explicitly in the corresponding codeword vector. It implies that its generator matrix includes an identity matrix, as its submatrix. In the case that a given generator matrix is nonsystematic, we can convert it into a systematic form with the aid of appropriate elementary row operations (Peterson and Weldon, 1972).

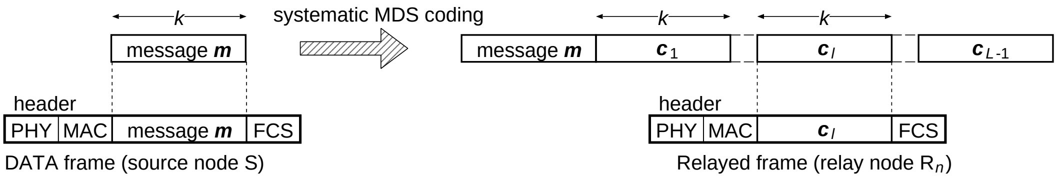


Figure 3: Frame format.

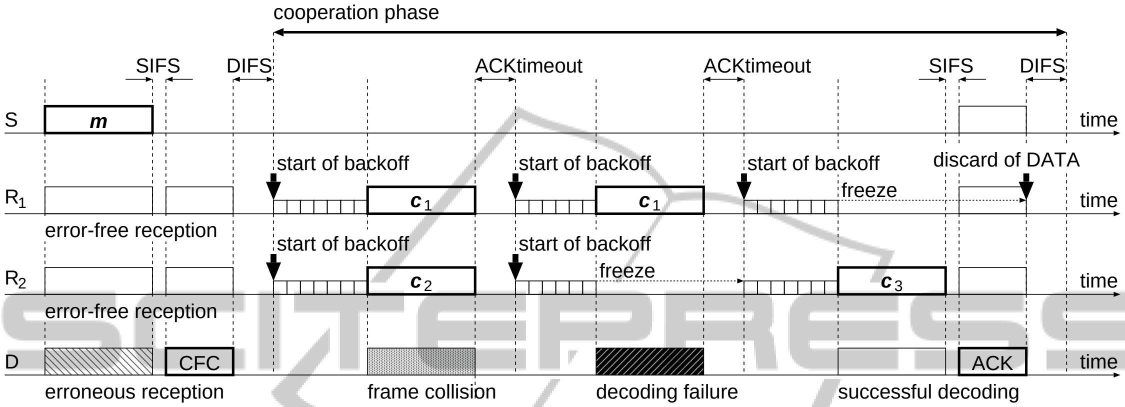


Figure 4: Illustrative example of proposed protocol (PRCSMA+RP-MDS).

can identify the received codeword block number, $\ell_1, \ell_2, \ldots, \ell_u$*, for* $u \leq L$ *and* $0 \leq \ell_1 < \ell_2 < \cdots < \ell_u \leq L-1$*. Then, a k-symbol message* $\boldsymbol{m}$ *can be recovered, if either of the following conditions is satisfied: (i) at least one codeword block* $\boldsymbol{c}_\ell$ *is error-free; and (ii) the total number of errors occurred in the u codeword blocks is less than or equal to*

$$t_u = \left\lfloor \frac{(u-1)k}{2} \right\rfloor, \tag{3}$$

where $\lfloor x \rfloor$ *is the maximum integer not greater than x.* □

Proof. Since every codeword block $\boldsymbol{c}_\ell$ consists of k symbols, it is straightforward from Theorem 1 that a receiver can recover the message $\boldsymbol{m}$ from one or more error-free codeword blocks. This leads to the first condition.

Next, aggregation of the u distinct received codeword blocks results in a codeword of a $[uk, k]$ punctured MDS code. Thus, t_u or less errors can be corrected according to Theorem 2, which provides the second condition. □

4.2 Proposed Protocol (PRCSMA+RP-MDS)

In PRCSMA, as described in Section 3, what a relay node transmits is a replica of the message block $\boldsymbol{m}$. Therefore, it is required for destination node D to receive a frame with no errors in order to complete the cooperation phase. By contrast, in the proposed protocol, an active relay node randomly transmits one out of $L-1$ redundant MDS codeword blocks; $\boldsymbol{c}_1, \boldsymbol{c}_2, \ldots, \boldsymbol{c}_{L-1}$, after encoding the received message block $\boldsymbol{m}$ by $\mathcal{C}$, as in (2). Furthermore, destination node D stores erroneously received frames in the buffer rather than discard.

A frame format used in the proposed protocol is depicted in Fig. 3. The codeword block number ℓ should be appropriately embedded in a header part, which can be digitized by $\lceil \log_2 L \rceil$ bits, where $\lceil x \rceil$ is the minimum integer not less than x. For small L, it can be negligible.

We describe the proposed protocol with the aid of an illustrative operational example with the same scenario as shown in Fig. 4. Destination node D stores an erroneous message block $\boldsymbol{m}$ into its buffer. Two active relay nodes R_1 and R_2 independently encode $\boldsymbol{m}$ and randomly select one codeword block. In Fig. 4, R_1 selects $\boldsymbol{c}_1$ and R_2 selects $\boldsymbol{c}_2$. After frame collision occurs, each relay node re-selects one codeword block; R_1 does $\boldsymbol{c}_1$ again and R_2, $\boldsymbol{c}_3$. Upon a reception of $\boldsymbol{c}_1$ from relay node R_1, destination node D aggregates the received $\boldsymbol{c}_1$ and the $\boldsymbol{m}$ in the buffer, and then, decodes $[\ \boldsymbol{m} \mid \boldsymbol{c}_1\ ]$ by a $[2k, k]$ punctured MDS code of $\mathcal{C}$. According to Corollary 1, the message block $\boldsymbol{m}$ can be retrieved if $\boldsymbol{c}_1$ is received with no errors or if the total number of symbol errors in $[\ \boldsymbol{m} \mid \boldsymbol{c}_1\ ]$ is not greater than $\lfloor k/2 \rfloor$. However, it fails in Fig. 4. At this time, destination node D stores two erroneous blocks, $\boldsymbol{m}$ and $\boldsymbol{c}_1$. Subsequently to reception of $\boldsymbol{c}_3$ from R_2, the message block $\boldsymbol{m}$ is successfully recovered by decoding $[\ \boldsymbol{m} \mid \boldsymbol{c}_1 \mid \boldsymbol{c}_3\ ]$ with a $[3k,k]$ MDS code, which can correct up to k errors. Finally, an ACK frame is returned from destination node D. It completes the cooperation phase.

Notice that source node S does not take part in a cooperation phase similarly to PRCSMA (Alonso-Zarate et al., 2009). Furthermore, for $L=1$ the proposed protocol is reduced to the original PRCSMA, since no error-correcting capability is available at destination node D.

5 PERFORMANCE ANALYSIS

5.1 Assumptions and Markov Model

In this section, we analyze the performance in the cooperation phase, based on the Markov model in (Alonso-Zarate et al., 2009). We impose identical assumptions with (Alonso-Zarate et al., 2009). Since we focus on the cooperation phase, it is presumed that destination node D has stored an erroneous message block $\boldsymbol{m}$. We assume that a cooperation phase start with N active relay nodes. We ignore erroneous reception of control frames; ACK frames, and of a header part in each frame. The CW value at each relay node remains constant W all the time, that is, no doubling procedure is carried out even if frame transmission failure occurs, as opposed to the legacy DCF (IEEE Standard 802.11, 1999). All frames involved in collision are to be retransmitted, until the cooperation phase is completed. We assume symmetric channels between relay node R_n and destination node D, that is, the symbol error rates between each relay node and destination node D are identical and independent; $\varepsilon_{\mathrm{R_1D}} = \varepsilon_{\mathrm{R_2D}} = \cdots = \varepsilon_{\mathrm{R}_N\mathrm{D}} = \varepsilon_{\mathrm{RD}}$.

Then, a Markov model with respect to the value of backoff counter at a relay node is quoted in Fig. 5 from (Alonso-Zarate et al., 2009). In Fig. 5, P_{ec} represents the probability that the cooperation phase ends in a slot. Note that a slot duration varies depending on frame transmissions in the slot.

5.2 Equations in Equilibrium

In equilibrium, an in-flow and an out-flow are balanced for every state in Fig. 5. Letting π_w be the steady-state probability of state w for $w = 0,1,2,\ldots,W-1$, we obtain

$$\pi_w = \begin{cases} \pi_{w+1} + \dfrac{1}{W}\left(\pi_0 + \displaystyle\sum_{i=0}^{W-1} P_{\mathrm{ec}}\pi_i\right) \\ \qquad\qquad \text{for } w = 0,1,2,\ldots,W-2, \\ \dfrac{1}{W}\left(\pi_0 + \displaystyle\sum_{i=0}^{W-1} P_{\mathrm{ec}}\pi_i\right) \\ \qquad\qquad \text{for } w = W-1. \end{cases} \tag{4}$$

Solving the recursive expression and the boundary condition in (4) under the normalizing condition $\pi_0 + \pi_1 + \cdots + \pi_{W-1} = 1$, we have

$$\pi_w = \frac{P_{\mathrm{ec}}\{1-(1-P_{\mathrm{ec}})^{W-w}\}}{WP_{\mathrm{ec}} - (1-P_{\mathrm{ec}})\{1-(1-P_{\mathrm{ec}})^{W-w}\}} \tag{5}$$

for $w = 0,1,\ldots,W-1$. Since frame transmission occurs only when the backoff counter reaches to zero, the probability of i-frame collision can be given by

$$q_i = \Pr[i\text{-frame collision}] = \binom{N}{i}\pi_0^i(1-\pi_0)^{N-i} \tag{6}$$

for $i = 0,1,\ldots,N$. Then, a slot is idle with probability q_0, one frame is transmitted in a slot with probability q_1, and frame collision takes place with probability $1-q_0-q_1$.

Next, we evaluate the probability P_{ec} of completing the cooperation phase. Destination node D stores an erroneous DATA frame $\boldsymbol{c}_0 = \boldsymbol{m}$, when the cooperation phase starts. The initial probability that the stored message includes e symbol errors is

$$\alpha(e) = \frac{1}{1-(1-\varepsilon_{\mathrm{SD}})^k}\binom{k}{e}\varepsilon_{\mathrm{SD}}^e(1-\varepsilon_{\mathrm{SD}})^{k-e} \tag{7}$$

for $e = 1,2,\ldots,k$. Then, when destination node D receives a non-collided frame; say $\boldsymbol{c}_\ell$, $\ell > 0$ if $L > 1$, aggregating two blocks results in $[\boldsymbol{c}_0 \mid \boldsymbol{c}_\ell]$. The cooperation phase ends, if either of two conditions in Corollary 1 is satisfied. The probability of error-free reception of a block of length k is $(1-\varepsilon_{\mathrm{RD}})^k$. Taking into account the fact that up to $\lfloor k/2 \rfloor$ errors in $[\boldsymbol{c}_0 \mid \boldsymbol{c}_\ell]$ can be corrected, we have the probability of successful decoding at destination node D as

$$P_{\mathrm{succ}} = \begin{cases} (1-\varepsilon_{\mathrm{RD}})^k \qquad \text{for } L = 1, \\ (1-\varepsilon_{\mathrm{RD}})^k \\ \quad + \displaystyle\sum_{j=1}^{\lfloor k/2\rfloor-1}\sum_{e=1}^{\lfloor k/2\rfloor-j}\binom{k}{j}\varepsilon_{\mathrm{RD}}^j(1-\varepsilon_{\mathrm{RD}})^{k-j}\alpha(e) \\ \qquad\qquad \text{for } L \geq 2. \end{cases} \tag{8}$$

In the case of $L > 2$, further gain on P_{succ} can be available when other code word blocks are received. However, we omit it in (8). Finally, we obtain

$$P_{\mathrm{ec}} = q_1 P_{\mathrm{succ}} = n\pi_0(1-\pi_0)^{n-1}P_{\mathrm{succ}}. \tag{9}$$

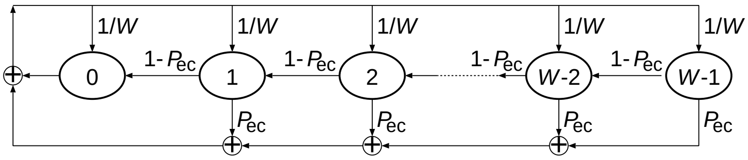


Figure 5: Markov model (Alonso-Zarate et al., 2009).

5.3 Average Duration of Cooperation Phase

Once P_{ec} is provided, it implies that a cooperation phase consists of $1/P_{\text{ec}}$ slots in average, in which the last slot is the only successful one. Hence, the average numbers of idle slots, of slots with 1-frame transmission, and of slots with frame collision can be evaluated by

$$\#[\text{idle}] = \left(\frac{1}{P_{\text{ec}}} - 1\right) \frac{q_0}{1 - q_1 P_{\text{succ}}}, \quad (10)$$

$$\begin{aligned} &\#[\text{1-frame transmission}] \\ &\quad = 1 + \left(\frac{1}{P_{\text{ec}}} - 1\right) \frac{q_1(1 - P_{\text{succ}})}{1 - q_1 P_{\text{succ}}}, \end{aligned} \quad (11)$$

$$\#[\text{frame collision}] = \left(\frac{1}{P_{\text{ec}}} - 1\right) \frac{1 - q_0 - q_1}{1 - q_1 P_{\text{succ}}}, \quad (12)$$

respectively. Then, the average duration of a cooperation phase, given that N active relay nodes collaborate, is given by

$$\begin{aligned} &\mathrm{E}[\text{duration} \mid N] \\ &= T_{\text{succ}} + T_{\text{slot}}\#[\text{idle}] \\ &\quad + T_{\text{fail}}(\#[\text{1 frame transmission}] - 1 \\ &\qquad + \#[\text{frame collision}]) \\ &= T_{\text{succ}} + \left(\frac{1}{P_{\text{ec}}} - 1\right) \\ &\quad \times \frac{T_{\text{slot}} q_0 + T_{\text{fail}}(1 - q_0 - q_1 P_{\text{succ}})}{1 - q_1 P_{\text{succ}}}, \end{aligned} \quad (13)$$

where T_{slot}, T_{succ}, and T_{fail} are the idle slot duration, the duration of successful message transmission consisting of the DATA and the ACK frames, SIFS and DIFT, and the duration of erroneous reception or frame collision consisting of the DATA frame and ACKtimiout, respectively. They are given as

$$T_{\text{succ}} = T_{\text{DATA}} + T_{\text{SIFS}} + T_{\text{ACK}} + T_{\text{DIFS}}, \quad (14)$$

$$T_{\text{fail}} = T_{\text{DATA}} + T_{\text{ACKtimeout}}, \quad (15)$$

where T_{DATA} and T_{ACK} are DATA frame duration and ACK frame duration, respectively, and other T_x's are the duration of element x.

5.4 Energy Efficiency in Cooperation Phase

Similarly to (13), the average of total energy consumed in a cooperation phase starting with N active relay nodes can be evaluated;

$$\begin{aligned} &\mathrm{E}[\text{energy consumption} \mid N] \\ &= E_{\text{succ}} + E_{\text{idle}}\#[\text{idle}] \\ &\quad + E_{\text{fail}}(1)(\#[\text{1-frame transmission}] - 1) \\ &\quad + \sum_{i=2}^{N} E_{\text{fail}}(i)\#[i\text{-frame collision}] \\ &= E_{\text{succ}} + \left(\frac{1}{P_{\text{ec}}} - 1\right) \frac{1}{1 - q_1 P_{\text{succ}}} \\ &\quad \times \left\{ E_{\text{idle}} q_0 + E_{\text{fail}}(1) q_1 (1 - P_{\text{succ}}) \right. \\ &\qquad \left. + \sum_{i=2}^{N} E_{\text{fail}}(i) q_i \right\}, \end{aligned} \quad (16)$$

where E_{succ} is the total energy consumed by N active relay nodes, source node S and destination node D in a successful slot, E_{idle} is that in an idle slot, and $E_{\text{fail}}(i)$ is that in an unsuccessful slot, given that i-frame collision occurs for $i = 1, 2, \ldots, N$, respectively. Let P_{T}, P_{R}, and P_{S} be consumed power at a node when transmitting, receiving, and sensing the channel, respectively. Then, three states in (16) of the energy consumption in a slot are given by

$$\begin{aligned} E_{\text{succ}} &= P_{\text{T}} T_{\text{DATA}} + P_{\text{S}} T_{\text{SIFS}} + P_{\text{R}} T_{\text{ACK}} + P_{\text{S}} T_{\text{DIFS}} \\ &\quad + P_{\text{R}} T_{\text{DATA}} + P_{\text{S}} T_{\text{SIFS}} + P_{\text{T}} T_{\text{ACK}} + P_{\text{S}} T_{\text{DIFS}} \\ &\quad + N(P_{\text{R}} T_{\text{DATA}} + P_{\text{S}} T_{\text{SIFS}} \\ &\qquad + P_{\text{R}} T_{\text{ACK}} + P_{\text{S}} T_{\text{DIFS}}), \end{aligned} \quad (17)$$

$$E_{\text{idle}} = (N + 2) P_{\text{S}} T_{\text{slot}}, \quad (18)$$

$$\begin{aligned} E_{\text{fail}}(i) &= i(P_{\text{T}} T_{\text{DATA}} + P_{\text{S}} T_{\text{ACKtimeout}}) \\ &\quad + (N + 2 - i)(P_{\text{R}} T_{\text{DATA}} + P_{\text{S}} T_{\text{ACKtimeout}}) \end{aligned} \quad (19)$$

for $i = 1, 2, \ldots, N$, respectively. Finally, we define the energy efficiency η as

$$\eta = \frac{\mathrm{E}[\text{message length in bits}]}{\mathrm{E}[\text{energy consumption} \mid N]} \quad (20)$$

for a cooperation phase starting with N active relay nodes.

Table 1: Parameters for numerical results.

(a) Frame format

PHY preamble	96	[μsec]
MAC header (incl. FCS)	34	[byte]
message length	512	[byte]
ACK length	14	[byte]
CFC length	14	[byte]
Block length: k	64	[symbol]

(b) DCF parameters

slot duration: T_{slot}	10	[μsec]
DIFS: T_{DIFS}	50	[μsec]
SIFS: T_{SIFS}	10	[μsec]
ACKtimeout: $T_{\text{ACKtimeout}}$	50	[μsec]
CW: W	16	

(c) Power

Transmission P_{T}	1900	[mW]
Reception P_{R}	1340	[mW]
Channel sensing P_{S}	1340	[mW]

(d) Channel

channel rate (DATA)	54	[Mbps]
channel rate (control)	6	[Mbps]
symbol error rate: ($\varepsilon_{\text{SD}}, \varepsilon_{\text{RD}}$)	$(10^{-1}, 10^{-2})$ $(10^{-2}, 10^{-3})$	

6 NUMERICAL RESULTS

We examine the derived expressions with exhaustive computer simulation and compare the performance of the proposed protocol to that of PRCSMA. The values of parameters employed are shown in Table 1. The frame format and the DCF parameters are basically extracted from (Alonso-Zarate et al., 2009; Predojev et al., 2012) and IEEE 802.11 standard (IEEE Standard 802.11, 1999). The power consumption is identical with (Predojev et al., 2012). Two pairs of the symbol error rates are considered; $(\varepsilon_{\text{SD}}, \varepsilon_{\text{RD}}) = (10^{-1}, 10^{-2})$ and $(10^{-2}, 10^{-3})$. A block length in frame is $k = 64$ symbols and two types of MDS codes $\mathcal{C}$ are considered; a half-rate [128,64] MDS code for $L = 2$, a quarter-rate [256,64] MDS code for $L = 4$. Note that for $L = 2$, a relay node always transmits $\boldsymbol{c}_1$, since a codeword consists of two blocks; $\boldsymbol{c} = [\boldsymbol{c}_0 = \boldsymbol{m} \mid \boldsymbol{c}_1]$. The theoretical results for $L = 4$ are omitted in order to avoid the complexity to derive the probability of successful decoding at destination node D, (8). The simulation program is written in C language and the results are obtained by averaging 10^5 trials of cooperation phases. Recall that a cooperation phase starts with destination node D which has already held $\boldsymbol{m}$ including e errors with probability $\alpha(e)$, (7), for $e = 1, 2, \ldots, k$.

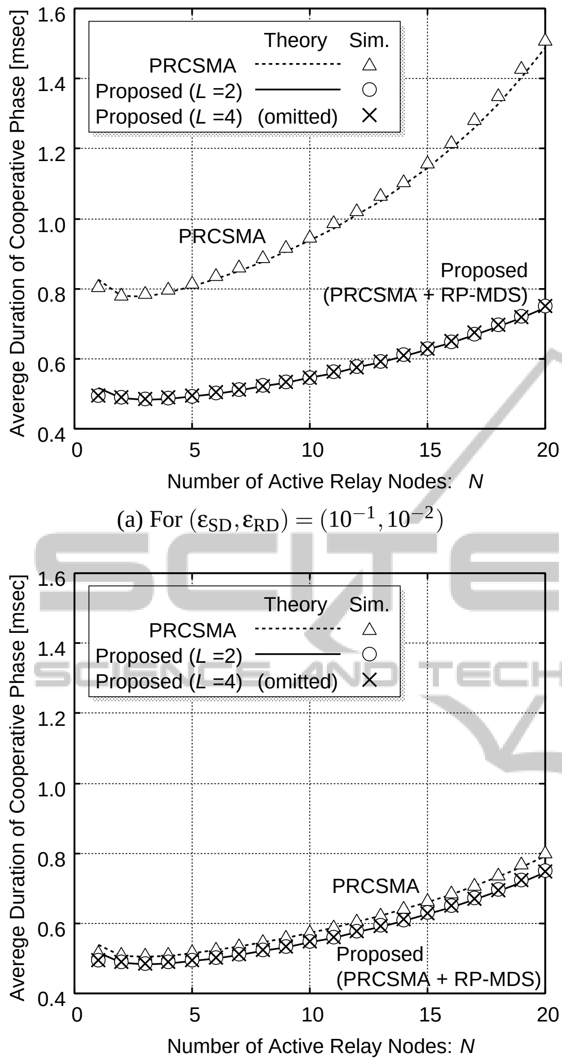
The average duration of a cooperation phase and the energy efficiency in a cooperation phase are presented in Fig. 6 and in Fig. 7, respectively, as a function of the number of active relay nodes N. The agreement between the theoretical and simulation results validates the accuracy of the derived expressions. Evidently, the proposed protocol, PRCSMA+RP-MDS, outperforms the original PRCSMA. In addition, it is revealed from computer simulation that the performance of PRCSMA+RP-MDS for $L = 4$ coincides with that for $L = 2$, so that a half-rate MDS code suffices for PRCSMA+RP-MDS.

From Fig. 6(a) the proposed protocol can achieve approximately 40% reduction in the average duration of a cooperation phase for $(\varepsilon_{\text{SD}}, \varepsilon_{\text{RD}}) = (10^{-1}, 10^{-2})$. The Energy efficiency is also improved by the proposed protocol, as shown in Fig. 7(a). However, it is clear from Fig. 6(b) and Fig. 7(b) that the degree of performance improvement by the proposed protocol decreases, as the channel quality is enhanced, since the opportunity to take advantage of the error-correcting capability of the MDS code decreases at destination node D. For the values of parameters given in Table 1, the probability of error-free reception of a frame is

$$(1 - \varepsilon_{\text{RD}})^k \approx \begin{cases} 0.526 & \text{for } \varepsilon_{\text{RD}} = 10^{-2}, \\ 0.938 & \text{for } \varepsilon_{\text{RD}} = 10^{-3}. \end{cases} \tag{21}$$

It implies that destination node D requires to receive a frame approximately $1/0.526 \approx 1.90$ times and $1/0.938 \approx 1.07$ times in average before the message $\boldsymbol{m}$ be successfully recovered for $\varepsilon_{\text{RD}} = 10^{-2}$ and $\varepsilon_{\text{RD}} = 10^{-3}$, respectively. On the other hand, since destination node D can receive a frame other than $\boldsymbol{m}$ in the cooperation phase in the proposed protocol, the error-correcting decoding for a half-rate $[2k, k]$ MDS code can be carried out. In this case, at most $\lfloor k/2 \rfloor$ symbol errors can be corrected. Then, the probability of decoding failure is given as

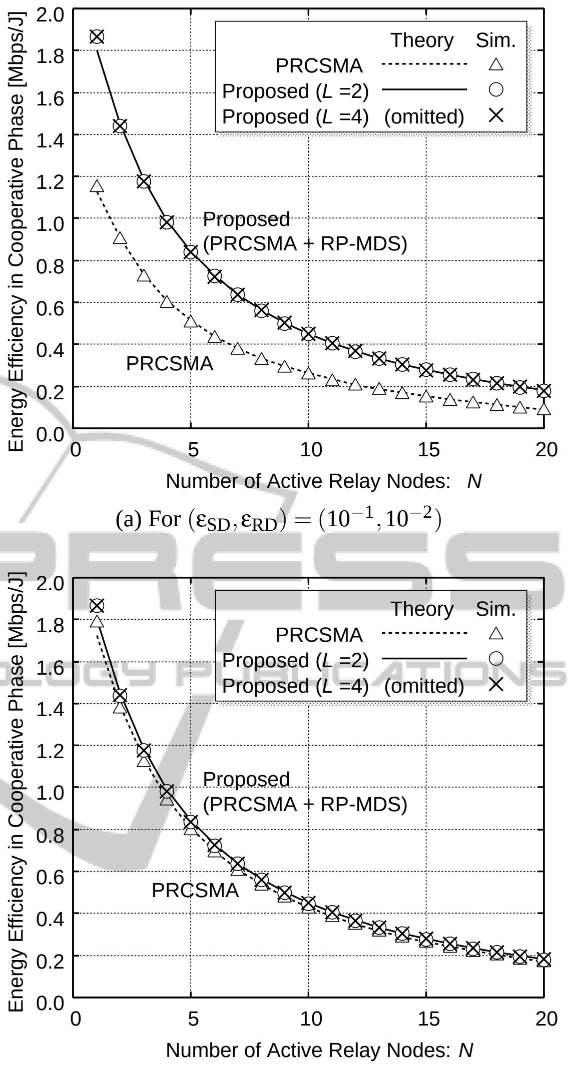
$$\sum_{i=\lfloor k/2 \rfloor+1}^{2k} \binom{2k}{i} \varepsilon_{\text{RD}}^i (1 - \varepsilon_{\text{RD}})^{2k-i}$$
$$\approx \begin{cases} 1.70 \times 10^{-36} & \text{for } \varepsilon_{\text{RD}} = 10^{-2}, \\ 3.92 \times 10^{-96} & \text{for } \varepsilon_{\text{RD}} = 10^{-3}, \end{cases} \tag{22}$$



(a) For $(\varepsilon_{SD}, \varepsilon_{RD}) = (10^{-1}, 10^{-2})$

(b) For $(\varepsilon_{SD}, \varepsilon_{RD}) = (10^{-2}, 10^{-3})$

Figure 6: Average duration of cooperation phase.



(a) For $(\varepsilon_{SD}, \varepsilon_{RD}) = (10^{-1}, 10^{-2})$

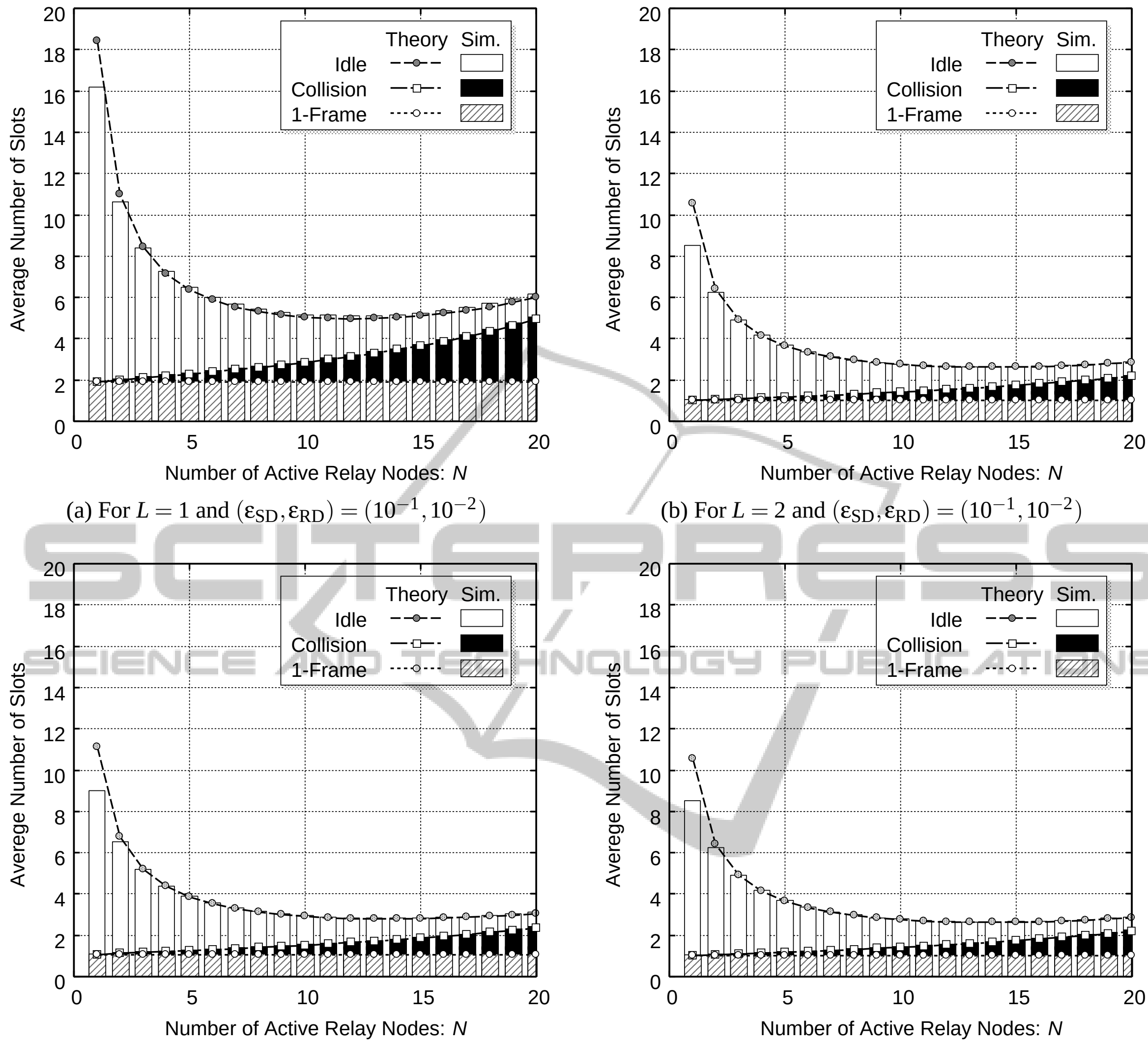
(b) For $(\varepsilon_{SD}, \varepsilon_{RD}) = (10^{-2}, 10^{-3})$

Figure 7: Energy efficiency in cooperation phase.

which is negligibly small, so that one frame reception other than $\boldsymbol{m}$ suffices for destination node D to recover the message block $\boldsymbol{m}$ in most cases. Therefore, the performance of the proposed protocol is independent of the value of $L \geq 2$.

Another observation from Fig. 6 is that the average duration slightly decreases for $N \leq 3$ and then it turns to increase. For $N \leq 3$, frame collisions are rare events. In addition, the more active relay nodes exist, the sooner the first transmission at a relay node takes place in a cooperation phase. These observations decrease the average duration with or without the use of RP-MDS. However, for $N \geq 4$, the probability of frame collisions can not be negligible and frame collisions add another backoff interval and frame retransmission. Hence, the average duration of a cooperation phase increases.

Next, as shown in Fig. 2 and Fig. 4, a cooperation phase consists of consecutive and synchronized slots. These slots are classified into three categories; idle slots of duration T_{slot}, slots with 1-frame transmission of duration of T_{succ} or T_{fail}, and slots with frame collisions of duration of T_{fail}. Clearly, one slot in slots with 1-frame transmission is a successful slot of duration of T_{succ} which is the last slot in a cooperation phase. Fig. 8 shows the average number of slots in a cooperation phase, classified into the three categories. The average number of these slots are theoretically evaluated as (10)–(12). Predictably, the average number of slots with frame collision monotonously increases in proportion to increment of the number of active relay nodes. The average number of idle sots decreases



(a) For $L = 1$ and $(\varepsilon_{\mathrm{SD}}, \varepsilon_{\mathrm{RD}}) = (10^{-1}, 10^{-2})$

(b) For $L = 2$ and $(\varepsilon_{\mathrm{SD}}, \varepsilon_{\mathrm{RD}}) = (10^{-1}, 10^{-2})$

(c) For $L = 1$ and $(\varepsilon_{\mathrm{SD}}, \varepsilon_{\mathrm{RD}}) = (10^{-2}, 10^{-3})$

(d) For $L = 2$ and $(\varepsilon_{\mathrm{SD}}, \varepsilon_{\mathrm{RD}}) = (10^{-2}, 10^{-3})$

Figure 8: Slot distribution in cooperation phase.

on the contrary. The incorporation of RP-MDS successfully facilitates the completion of a cooperation phase. Therefore, the average number of slots with 1-frame transmission can be reduced by the use of the proposed protocol. Particularly, the use of RP-MDS can approximately halve the average number of slots for $(\varepsilon_{\mathrm{SD}}, \varepsilon_{\mathrm{RD}}) = (10^{-1}, 10^{-2})$, comparing Fig. 8(a) to Fig. 8(b).

7 CONCLUSIONS

We have proposed incorporation of RP-MDS, which has been proposed for multi-hop cooperative relay networks (Sakakibara et al., 2011), to PRCSMA over noisy channels. The proposed protocol elaborately takes advantage of the powerful error-correcting capability of MDS codes into cooperative communication systems and introduces the incremental redundancy concept to PRCSMA. A destination node can reinforce the error-correcting capability when it receives a new frame. Assuming symmetric relay channels, we have analyzed the performance of the proposed protocol in terms of the average duration of a cooperation phase and the energy efficiency in a cooperation phase. The accuracy of theoretical results has been validated by means of computer simulation. Numerical results have indicated that the proposed protocol can improve the performance, compared to the original PRCSMA, particularly over severe noisy channels. It is also revealed that the use of a half-rate MDS code suffices in the proposed protocol.

Further study includes, for example, the consideration of header errors and feedback errors, and the extension to bidirectional communication systems and to the use of network coding.

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