

Multi-objective Optimization for Characterization of Optical Flow Methods

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Abstract: Optical flow methods are among the most accurate techniques for estimating displacement and velocity fields in a number of applications that range from neuroscience to robotics. The performance of any optical flow method will naturally depend on the configuration of its parameters. Beyond the standard practice of manual (*ad-hoc*) selection of parameters for a specific application, in this article we propose a framework for automatic parameter setting that allows searching for an approximated Pareto-optimal set of configurations in the whole parameter space. This final Pareto front characterizes each specific method, enabling proper method comparison. We define two performance criteria, namely the accuracy and speed of the optical flow methods.

1 INTRODUCTION

Optical flow (OF) has been applied widely to quantify motion in computer vision problems. Specific OF algorithms tend to be evaluated (for ranking or for searching of optimal parameters) according to either their accuracy, or their speed. However, when studying the performance and computational requirements of OF methods one can observe that some accurate algorithms are not suitable for real-time applications. For that reason, the evaluation and optimization of optical flow algorithms according to both accuracy and speed at the same time is very important for real world applications, which have a constrained response time or a high-accuracy requirement.

In general, when choosing a computer vision algorithm for a specific application, very often an accuracy-speed trade-off exists. In that case, a researcher may take into account mainly two objectives: algorithm error and execution time. When evaluating the algorithm performance in a fixed image database, the algorithm error and execution time are functions of the algorithm parameters. In the optic flow literature, most papers do not consider the optimal selection of these parameters in a multi-objective manner. They rather fine-tune the parameters manually, usually with the goal of minimizing either the error rate

or the processing time, basically leading to a single-objective optimization of the algorithm. The main disadvantage of the single-objective approach is that the selection or combination of different objectives is arbitrary. Therefore, the only methodology that can give interesting results for the problem of accuracy-speed optimization is multi-objective optimization.

When working with multi-objective optimization, the aim is to improve at least one of the objectives and not to get worse values in any of the other objectives. One extra advantage of multi-objective optimization is that the resulting set of solutions corresponds to an approximation of the Pareto front, which contains information that is much richer than the results of single-objective optimization. First, the Pareto front can be used as a receiver operating curve (ROC) of the optimized algorithms. This curve characterizes each method and allows for comparison of several methods. And second, one run of the optimization algorithm gives information for different applications of the OF algorithm. That information includes the solution-parameter settings and accuracy-speed statistics—for a family of speed-constrained and accuracy-constrained problems.

Searching for the optimal parameter setting represents a large combinatorial problem that can be approached with evolutionary algorithms (Bäck, 1996).

In particular, we employ genetic algorithms (Goldberg, 1989) for this task. Genetic algorithms can solve problems with multiple solutions. They do not require objective function derivatives, thus they are easy to implement and can cope with non-continuous problems. Standard genetic algorithms search the parameter space in an evolutive manner, considering only one objective. To optimize several objectives concurrently we utilize an evolutionary multi-objective optimization (EMO) strategy (Deb and Kumar, 1995). A successful approach for EMO is named NSGA-II (an improved non-dominated sorting genetic algorithm) (Deb and Kumar, 1995; Deb et al., 2002). NSGA-II has a fast approach for non-dominated solution sorting and a smart criterion for diversity preservation. Multi-objective optimization has been applied before to other computer vision tasks, such as segmentation (Everingham et al., 2006), face detection (Verschae et al., 2005), tracking (Benlian and Zhiquan, 2007) and 3D vision (Vite-Silva et al., 2007).

Optical flow is a vector field representing “apparent velocities of movement of brightness patterns in an image” (Horn and Schunck, 1981). Optical flow algorithms tend to be evaluated (for ranking or for searching of optimal parameters) according to either accuracy (Barron et al., 1994)(Baker et al., 2011) or speed (Changming and Sun, 2002)(Bruhn et al., 2005a). By comparing methods in accuracy and speed concurrently, it can be realized that some accurate algorithms are not suitable for real-time applications. Another important observation is the need for specific hardware (graphic processing units) to obtain results in very short execution times.

In the optical flow literature, most papers do not consider the optimal selection of method parameters in a multi-objective manner. They fine-tune the parameters manually instead. Some researchers have developed stochastic/statistic methods for optical flow parameter selection (Li and Huttenlocher, 2008)(Krajsek and Mester, 2006)(Heas et al., 2012). They optimize *a posteriori* probability or training loss in order to find the best parameters. Then, they consider just one objective and set aside execution time. An exception is found in (Salmen et al., 2011), where the authors look for highly accurate and efficient OF algorithms. However, they work with non-dense OF methods and define efficiency as the number of flow vectors found per frame. Thus, they are not considering algorithm speed. The present multi-objective methodology is based on the speed-accuracy trade-off observed in computer and biological vision (Chittka et al., 2003).

In this article we explore multi-objective optimization using NSGA-II (Deb et al., 2002) of the

combined local and global (CLG) method proposed by Bruhn *et al.* (Bruhn et al., 2005b), which is a well-known representative of the class of variational OF methods. Nevertheless, our multi-objective optimization strategy can be applied to tune the parameters of any other optical flow method optimally, variational or not. In general, the parameter space of an optical flow method can be very large, which makes the optimization task very challenging.

This work is structured as follows. Section 2 describes the variational optical flow method to be optimized and characterized in this paper. Section 3 reports on the development of genetic algorithm-based multi-objective optimization of optical flow. Section 4 describes the experimental setup, reporting and discussing the results. Section 5 gives some concluding remarks.

2 COMBINED LOCAL-GLOBAL OPTICAL FLOW METHOD

The combined local-global (CLG) OF method (Bruhn et al., 2005b) looks for a flow field $w = (u, v, 1)^T$ on the image space Ω that minimizes the functional

$$E(w) = E_{similarity} + \alpha E_{smoothness} \quad (1)$$

where the term

$$E_{similarity} = \int w^T J_\rho(\nabla_3 I) w d\Omega \quad (2)$$

represents the brightness constancy assumption, based on the motion tensor $J_\rho(\nabla_3 I)$, given by $K_\rho * (\nabla_3 I \nabla_3 I^T)$, a convolution with a Gaussian kernel with parameter ρ , for image spatiotemporal derivatives $\nabla_3 I = (I_x, I_y, I_t)$. Image I is the result of convolution of the original image and kernel K_σ . The second term in the functional, related to requiring a smooth flow field, is

$$E_{smoothness} = \int (|\nabla u|^2 + |\nabla v|^2) d\Omega \quad (3)$$

It is also possible to use more general versions of these two terms, to be able to get discontinuity-preserving optical flow solutions (Bruhn, 2006):

$$E_{similarity} = \int \Psi_D (w^T J_\rho(\nabla_3 I) w) d\Omega \quad (4)$$

$$E_{smoothness} = \int \Psi_S (|\nabla u|^2 + |\nabla v|^2) d\Omega \quad (5)$$

Quadratic penalization $\Psi_D(s^2) = \Psi_S(s^2) = s^2$ gives Equations (2) and (3) as a result. One option for non-quadratic penalization is $\Psi_D(s^2) = \Psi_S(s^2) =$

$\sqrt{s^2 + \epsilon^2}$, a regularized version of the L_1 norm. It behaves as the L_1 norm for large values of s^2 , but has the extra advantage of regularity.

The optimality condition for the minimization problem is described by a system of non-linear partial differential equations. The discretized version of these equations can be solved using, for example, Jacobi, Gauss-Seidel, successive over-relaxation (SOR), or a full multi-grid (FMG) method, taking advantage of the fast high-frequency error removal feature of iterative methods for sparse linear systems. In this work, we use a non-dyadic (image width and height do not need to be a power of 2) linear multi-grid algorithm following (Bruhn et al., 2005a; Briggs et al., 2000).

3 PROPOSED METHODOLOGY

3.1 Evolutionary Multi-Objective Optimization

Multi-objective optimization is a way of considering many objectives when looking for an optimum, while avoiding arbitrarily combining/weighting them. Furthermore, multi-objective optimization gives the Pareto front for the optical flow method, which is a set of the optimal settings for the given method. Therefore, in our case it is a way of working with different applications of optical flow, both speed- and accuracy-oriented. Choosing execution time and error as objectives, a set of parameter-space solutions can be considered as optima. The concept of Pareto-dominance has been shown to be very useful in order to define that set of solutions.

A solution vector $\mathbf{v} = (v_1, \dots, v_{N_{obj}})$, with v_i the solution for the i -th objective, is said to Pareto-dominate a solution vector \mathbf{w} if $v_i \leq w_i$, for every $i = 1, \dots, N_{obj}$, and for at least one value of i , $v_i < w_i$. A Pareto front is the set of all vectors that are not dominated by any other vector.

A successful approach for EMO is named NSGA-II (an improved non-dominated sorting genetic algorithm) (Deb and Kumar, 1995; Deb et al., 2002). NSGA-II has a fast approach for non-dominated solution sorting and a smart criterion for diversity preservation.

NSGA-II sorts solutions as follows:

1. For each solution p , record the domination count n_p , i.e. the number of solutions which dominate p . Store the set S_p of solutions that p dominates. The set of solutions with $n_p = 0$ is the first non-dominated front (their non-domination rank is 1).

2. For each solution p with $n_p = 0$, reduce the domination count of every solution q in S_p by one. Store solutions q which reached domination count 0 as a new non-dominated front (their non-domination rank is one more than the previous non-dominated front).
3. Repeat step 2 for every new non-dominated front.

This algorithm preserves diversity of solutions. A crowding measure is defined to quantify solution diversity in a non-dominated front:

$$d_{crowd}[i] = \sum_{m=1}^{N_{obj}} (p_{i+1,m} - p_{i-1,m}) / (f_m^{max} - f_m^{min}) \quad (6)$$

with $i = 2, \dots, l-1$, where l is the number of solutions and $d_{crowd}[1] = d_{crowd}[l] = \infty$. Here, $p_{i,m}$ is the i -th solution value for objective m , after sorting solutions using objective m . f_m^{min} and f_m^{max} are the minimum and maximum values of objective m .

3.2 Optimization Methodology for Optical Flow Performance

Given an OF method M , and an image database including N_{seq} image sequences $\{I_{k,l}\}_{k=1, \dots, N_{seq}; l=1, \dots, N_k}$ (total of $N_{DB} = \sum_{k=1}^{N_{seq}} (N_k - 1)$ image pairs), we are interested in the average execution time $T = (1/N_{DB}) \sum_{k,l} f(p, I_{k,l}, I_{k,l+1})$ and the average endpoint error $AEE = (1/N_{DB}) \sum_{k,l} g(p, I_{k,l}, I_{k,l+1})$ for the evaluation of M over all image pairs $I_{k,l}, I_{k,l+1}$, both functions of p , the parameter vector for method M . Using the notation we have described in 3.1, the goal is to find solution vectors $v = (T, AEE)$, $N_{obj} = 2$, that are not Pareto-dominated by any other solution.

Figure 1 shows a diagram for the application of EMO to an OF method. In the diagram, an EMO algorithm (NSGA-II in all the experiments shown here) gives parameter vectors p to an OF method. The OF method processes images I from sequences included in an OF evaluation data set. This data set provides ground truth flow (u_{GT}, v_{GT}) for evaluation of estimated flow (u, v) . The outputs are $\{p_{opt}\}$, a set of Pareto-optimal p -vectors, and accuracy-speed statistics T, AEE .

Algorithm 1 describes this methodology in more detail. The goal of this algorithm is to find an optimal Pareto front. The initial population is defined randomly, according to parameter ranges specified by the user. Then, each new population is initialized using binary tournament selection, binary recombination (crossover), and binary mutation (Deb and Kumar, 1995; Deb et al., 2002). Previous and current populations are combined and sorted according to

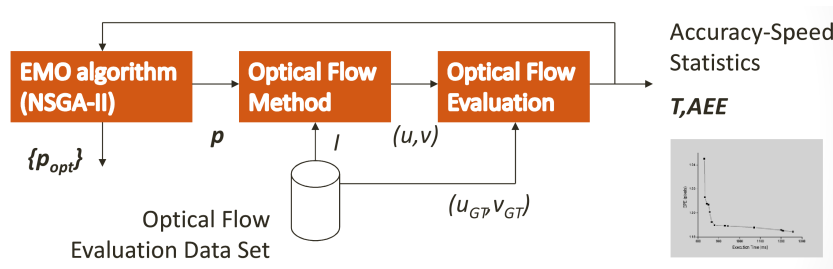


Figure 1: Diagram for Evolutionary Multi-Objective Optimization of Optical Flow Accuracy and Speed.

their non-domination rank and their crowding measure (Deb et al., 2002). Finally the N_{ind} best individuals of the combined population are kept for current generation. The non-domination rank and the crowding measure are determined using the objective values (accuracy-speed statistics in Figure 1, namely, AEE and execution time T for the experiments in this work).

The CLG OF parameters are $\alpha, \rho, \sigma, \epsilon$. The parameters to be varied for optimization are the regularization parameter α , the derivative smoothing parameter ρ , and the image pre-smoothing parameter σ , so for the experiments in this article, $p = (\alpha, \rho, \sigma)$. Parameters ρ and σ affect AEE and the processing time T . Their influence on AEE is clear. They also influence T , because they determine the size of the Gaussian filter used to smooth image intensities and derivatives. The regularization parameter α affects AEE clearly. A fixed value has been assigned to ϵ , following (Bruhn, 2006). For the small set of experiments with the SOR iterative solving method, we will not vary the parameters related to the stopping criteria. We are more interested in multilevel strategies, such as the FMG solving method, which tends to Pareto-dominate iterative methods in our experiments and does not require stopping criteria.

4 EXPERIMENTAL RESULTS

This section describes the experimental setup and results for optimization using sequences and ground truth from the Middlebury data set (Baker et al., 2011) (see Figure 2). The Middlebury data set contains eight synthetic and laboratory sequences with a dense ground truth.

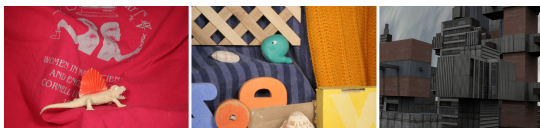


Figure 2: Sample Images from the Middlebury Data Set.

Algorithm 1 : EMO of Optical Flow.

Input: u_{gt} , ground truth optical flow of image sequences in the evaluation data set

Input: N_{gen} , number of generations

Input: N_{ind} , number of individuals in a generation

Input: N_{seq} , number of image sequences in the evaluation data set

- 1: Initialize a parent population $P_0 = \{p_j\}_{j=1, \dots, N_{ind}}$
 - 2: Create an offspring population Q_0 (tournament selection, recombination, and mutation)
 - 3: **for** $i = 1, \dots, N_{gen}$ **do**
 - 4: Create a combined population $R_i = P_i \cup Q_i$ including the parent population P_i and the offspring population Q_i . Thus, population R_i has $2N_{ind}$ individuals
 - 5: Sort individuals from R_i according to non-domination rank (described in Section 3.1). The result is the set $F = \{F_1, F_2, \dots\}$ of all the non-dominated fronts in R_i
 - 6: Set $P_{i+1} = \emptyset, f = 1$
 - 7: **while** Parent population is not yet filled, $|P_{i+1}| + |F_f| \leq N_{ind}$, with $|P|$ the number of elements in set P , **do**
 - 8: Include f -th non-domination front in the parent population, $P_{i+1} = P_{i+1} \cup F_f$
 - 9: Increment front index, $f = f + 1$
 - 10: **end while**
 - 11: Sort next front F_f according to crowding measure (described in Section 3.1).
 - 12: Choose the best N_{ind} individuals, $P_{i+1} = P_{i+1} \cup F_f[1 : (N_{ind} - |P_{i+1}|)]$
 - 13: Initialize population Q_{i+1} (selection, mutation, and crossover)
 - 14: **for** $j = 1, \dots, N_{ind}$ **do**
 - 15: OpticalFlowEvaluation(p_j), Algorithm 2
 - 16: **end for**
 - 17: **end for**
- Output:** Global accuracy-speed (error-execution time) statistics
- Output:** Pareto-optimal p -vectors $\{p_{opt}\}$
-

Algorithm 2 : OpticalFlowEvaluation(p).**Input:** \mathbf{u}_{gt} , ground truth optical flow of image sequences in the evaluation data set**Input:** N_{seq} , number of image sequences in the optical flow evaluation set**Input:** \mathbf{p} , parameter vector for the optical flow method being studied

- 1: **for** $k = 1, \dots, N_{seq}$ **do**
- 2: Calculate optical flow for image sequence k with parameters \mathbf{p} , measure execution time.
- 3: Evaluate performance for image sequence k using \mathbf{u}_{gt}
- 4: **end for**
- 5: Calculate average error (*AEE*) and execution time (*T*) for the N_{seq} image sequences

Output: Accuracy-speed (error-execution time) statistics

Efficient genetic optimization requires user-knowledge about the optimization problem. As a way of providing that knowledge about multi-objective optimization of optical flow, we chose a wide range for the parameters, containing parameter settings that proved to perform well in preliminary experiments of CLG optical flow. Minimum and maximum values for discretization of parameters for the whole Middlebury data set were configured so that $\alpha \in [0, 10000]$, $\rho \in [0, 6]$ and $\sigma \in [0, 6]$. Smaller parameter ranges were chosen for previous experiments; as an example, for some preliminary experiments with the Middlebury data set, we selected $\alpha \in [650, 1100]$, $\rho \in [0.1, 4.5]$ and $\sigma \in [1, 4]$.

Preliminary experiments were conducted with 32 and 64 individuals per generation. Results were similar, so all experiments shown were run with populations of 32 individuals. Probability of crossover was set to 0.9 and probability of mutation to 0.33 ($1/n$, with $n = 3$ the number of decision variables for real-coded genetic algorithms)(see (Deb et al., 2002)). Distribution indexes for crossover and mutation were set to 20.

Two different sets of experiments were performed. First, short experiments of 10 generations compared successive over-relaxation (SOR) and full multi-grid (FMG) solvers for the CLG optical flow method, and then long experiments surveyed the convergence of the optimization methodology when optimizing CLG+FMG, the best performing (best accuracy and speed) OF algorithm and solver, and tested the effect of a larger parameter range.

Experiments were run on linux based PC's with Intel Core i7 CPU. Single-threaded optical flow methods were implemented in C/C++. However, in or-

der to take advantage of the multicore architecture we had, genetic algorithm individuals were run in several parallel threads, using OpenMP. A version of NSGA-II C/C++ library (Deb et al., 2002) was modified to run in parallel and used for EMO.

4.1 Middlebury Subset

For preliminary experiments, three image pairs were selected: images 10 and 11 from sequences Dimetrodon, Rubberwhale and Urban3 (see Figure 2), and the proposed methodology was applied to that set. Figure 3(a) shows final Pareto fronts resulting from EMO. Each Pareto front gives a characterization of its corresponding method and the concept of Pareto dominance can be applied for further analysis of these results. Solutions related to the FMG solver Pareto-dominate every SOR solution in the graph. Then, the global Pareto front for both methods is the FMG curve. Increasing the number of SOR iterations would presumably give lower AEE values, and perhaps a non-dominated solution, but with a very long time spent in every execution. As a conclusion for this figure, when working with images that are similar to those in the database used, FMG solving methods would be preferred over SOR.

Figure 3(b) presents two NSGA-II experiments for the same OF algorithm and parameters. The chosen OF algorithm is the best performing one in the previous experiment (shorter experiments), CLG+FMG. Both experiments were conducted with 60 generations and the same NSGA-II parameters. Despite the randomness of genetic algorithms, most of the measures converged to the same value or to quite close values.

Figure 3(b) shows final Pareto fronts (accuracy-speed plots) for both NSGA-II experiments. The curves are very similar. The only slight difference is a shift of less than 5 ms in two solutions. When comparing these curves with Figure 3(a), an improvement can be observed. These new curves dominate the previous 10-generation experiment: every solution in these new experiments is faster (less execution time) or equally as fast as all solutions in the previous run, and the minimum AEE obtained is less than the previous minimum value. Although evident, the difference between the results of the previous experiment and these new ones is small enough and does not suggest a change in the relative position of different OF solving algorithms.

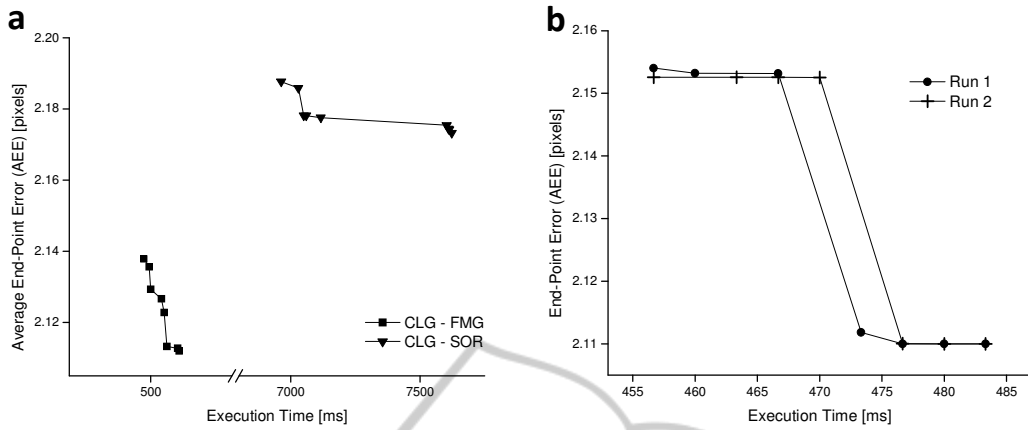


Figure 3: Comparison of two different experiments of NSGA-II for the CLG OF algorithm with FMG solving method, bounds for parameters as described in the text of the article. (a) Final Pareto front for CLG OF with FMG and SOR solvers, varying the global OF regularization (α), derivative smoothing (ρ) and image pre-smoothing (σ), see Section 2. The execution time axis was broken for better visualization. (b) Two experiments for the CLG OF algorithm with FMG solving method. Both experiments were run with 60 generations of 32 individuals and the same NSGA-II parameters. Final Pareto fronts are shown. Curves for one experiment are marked with plus signs (+) and curves for the other experiment use dots (.) as markers.

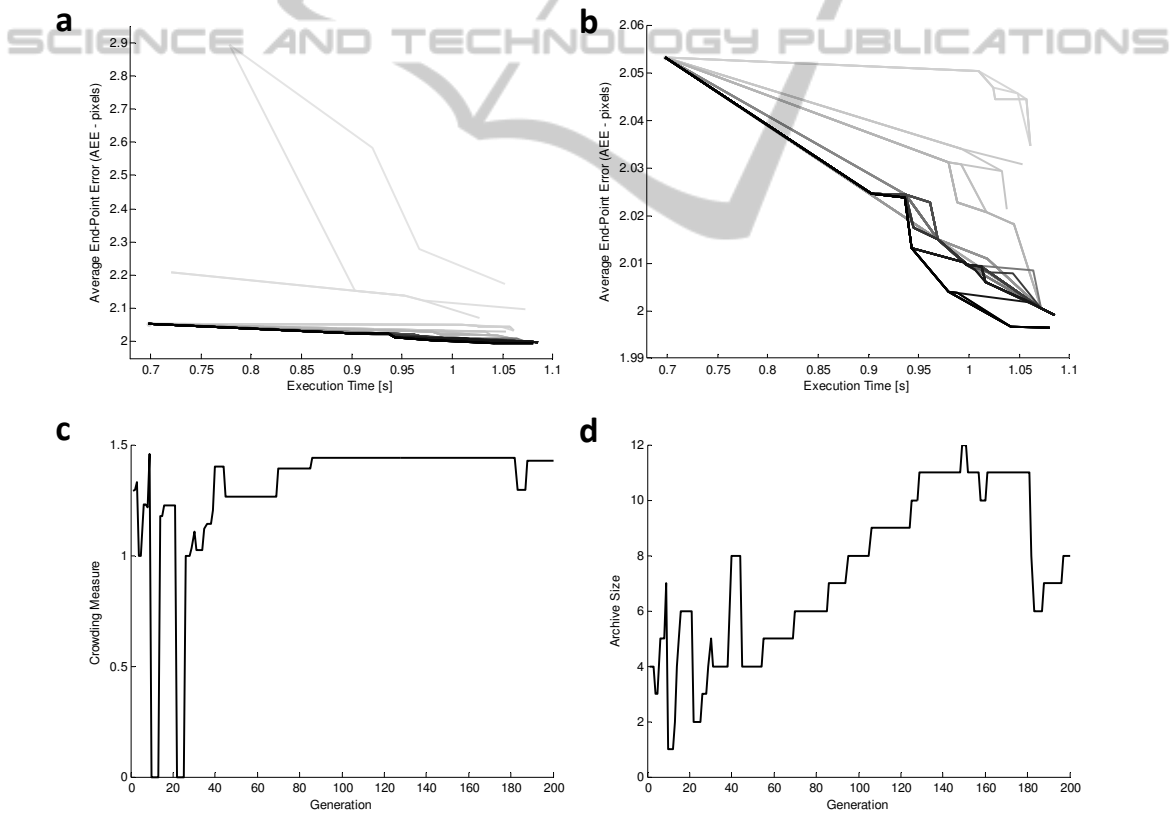


Figure 4: Evolution of execution time and error for the whole Middlebury database. (a) Evolution of the Pareto front in the objective space. Black lines represent the final Pareto fronts. Solutions in each Pareto front are connected by line segments for viewing purposes. The darkness of the gray levels decreases for earlier generations. (b) Detail from (a). (c) Crowding measure evolution. (d) Archive size evolution.

4.2 Whole Middlebury Data Set

Figure 4 presents the evolution of solutions and NSGA-II measures for the whole Middlebury data set and for a long experiment (200 generations). Figures 4(a) and (b) show the Pareto fronts for even numbered generations. The gray level intensity is high (light gray) for early generations and low (black) for late generations. Solutions are connected by line segments to facilitate visual analysis. Many small and a few large changes between generations can be observed. Both the minimum execution time and AEE were reduced. Figures 4(c-d) present the evolution of NSGA-II measures. Every measure remained very stable for the last tens of generations. Even the average AEE in the Pareto front (not shown) was reduced, while preserving the crowding measure and increasing the archive size.

Figure 5 shows the evolution of the Pareto fronts for three combinations of optical flow functionals and solving techniques. This figure gives an example of the application of the proposed methodology to the characterization and comparison of optical flow methods. A global Pareto front can be defined as the Pareto front for all the solutions in this plot, even when these solutions are related to different optical flow methods. The global Pareto front for these algorithms includes the Pareto front for CLG+FMG and part of the Pareto front for RegL1CLG+SOR, a regularized L_1 version of the CLG functional and SOR solving method, see Section 2. Then, the CLG+SOR option should not be chosen for any application. Depending on the accuracy-speed requirements of each particular application, an operating point in the global Pareto

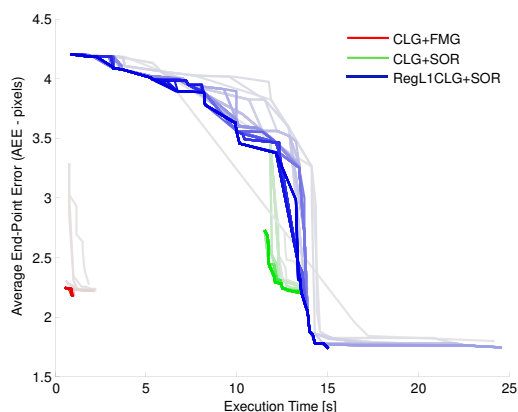


Figure 5: Evolution of the Pareto front for three combinations of optical flow functionals and solving methods, when applied to the whole Middlebury data set. The evolution of the Pareto fronts for three different variants of optical flow methods and solving techniques (CLG+FMG, CLG+SOR, RegL1CLG+SOR) were given different colors, as shown in the legend, and displayed on the same axes.

front should be chosen, either in the CLG+FMG or in the RegL1CLG+SOR variants.

The overall running time of one optimization experiment (200 generations, whole Middlebury database, FMG solving method) was about three hours. That experiment requires 6400 optical flow evaluations. The resolution was 256 levels per parameter, then the equivalent brute force search would have needed 16 million optical flow evaluations, more than three orders of magnitude slower than our approach.

5 CONCLUSIONS

This article describes a strategy for optimizing the parameter setting of any optical flow method focusing on two performance criteria, namely, the accuracy and speed. The proposed methodology is based on evolutionary multi-objective algorithms.

When choosing and adjusting an optical flow method to a specific application, the design requirements for accuracy and speed are the keys to finding the right method and its parameter configuration in a graph of execution time versus error, showing operating points for different methods. A straightforward way to do it would be finding a global Pareto front of all methods and looking for those operating points that fulfill the design requirements. In general, the Pareto front of a first method/implementation dominating the Pareto front of a second method—understood as every point in the second curve being dominated by at least one point in the first curve—means the first method is better because it can achieve lower error or execution time than any operating point of the second method.

We have shown this analysis for two solvers of the *combined local and global* (CLG) variational optical flow method of Bruhn *et al* (Bruhn *et al.*, 2005b). Nevertheless, the proposed methodology can be utilized to compare different optical flow methods and to find their optimal operation points, i.e. parameter settings.

This work shows experiments for the Middlebury optical flow evaluation data set. When analyzing the results, the following conclusions can be reached: 1) The Pareto fronts for a multi-grid solving method dominate the fronts related to the SOR solving method. 2) In spite of the randomness of genetic algorithms, our tests show that the method converges. The convergence time was a few orders of magnitude faster than a brute force search. 3) The method effectively reduces execution time and error, and gives a receiver operating curve, where every operating point is associated with a parameter setting.

Finally, we would like to state that multi-objective optical flow parameter optimization and characterization are needed for further development of optical flow applications. It is perfectly reasonable to think about Pareto-based optical flow rankings, assuming some rules for fair result comparison. One solution could be to allow researchers to run their experiments on a common hardware platform. Current web-based rankings can easily provide a graphical representation of several objectives.

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