SKen: A Statistical Test for Removing Outliers in Optical Flow
A 3D Reconstruction Case

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Abstract: The 3D reconstruction can be employed in several areas such as markerless augmented reality, manipulation of interactive virtual objects and to deal with the occlusion of virtual objects by real ones. However, many improvements into the 3D reconstruction pipeline in order to increase its efficiency may still be done. In such context, this paper proposes a filter for optimizing a 3D reconstruction pipeline. It is presented the SKen technique, a statistical hypothesis test that classifies the features by checking the smoothness of its trajectory. Although it was not mathematically proven that inliers features performed smooth camera paths, this work shows some evidence of a relationship between smoothness and inliers. By removing features that did not present smooth paths, the quality of the 3D reconstruction was enhanced.

1 INTRODUCTION

Computer vision is a research area with numerous contributions to the development of 3D reconstruction techniques and it is mainly concerned with the modeling of real world information (Hming and Peters, 2010) (Barbosa, 2006). Advances in this area involve the integration with algorithms for real-time execution (Nistér, 2003), robust statistical approaches (Choi and Medioni, 2009) and dense 3D reconstruction hardware-based acceleration methods (Bouguet, 2000). This way, several challenges still motivate the development of new techniques and the interoperability of these with the existing ones.

When the 3D reconstruction is made from real data, there is an introduction of errors correspondent to the quality of the image acquisition. This procedure depends on parameters such as the image resolution, the camera sensor and light conditions. Once the processing of this image already presents noise, it will be passed on to the next steps. This way, these errors cause more errors to accumulate along the pipeline.

Following, in the tracking phase there are also some inherent difficulties such as the occlusion of part of the scene and false matching. This is due to the image areas with poor textures and gradient with low significance that may also introduce noise in the positioning of features along the trace.

Considering a priori that there are errors arising from both the image acquisition and the tracking stage, it is essential an approach takes into account these errors when calculating the matrix camera and the fundamental matrix (Hartley and Zisserman, 2004).

An optimal reconstruction happens when the data from the tracking stage were obtained without errors, i.e. using the ground truth. Since the actual distribution of the errors is unknown, it is not possible, at first, to filter in the scene which are the reliable features (inliers) and the unreliable ones (outliers).

Numerical errors, the minimization of nonlinear systems and several other limitations of the pipeline make it extremely difficult to avoid the accumulation of past errors from one phase to another. A solution to this problem would reduce the amount of errors in the first stage of the pipeline to its minimum, thereby providing the best possible features to the 3D reconstruction algorithm.

In this context, this paper proposes a methodology based on statistical methods in order to filter features during the tracking phase of a 3D reconstruction pipeline, using only 2D points provided by the tracker. It was developed the SKen, a robust hypothesis test with low computational cost, which aims to reduce the error caused by the tracker without affecting the total execution time.

The remaining sections of this article are organized as follows: section 2 contemplates some techniques on the state of the art, section 3 describes the SKen test, section 4 comprises the validation of the

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test in both synthetic and real scenarios, and finally, section 5 discusses the advantages and some suggestions for improving the methodology.

2 OUTLIER DETECTION

Once the information extracted from real world is already noisy, the 3D reconstruction calculations will not generate flawless results, but a hypothesis instead (an intermediate product). A useful method for evaluating hypotheses is the RANSAC algorithm (Fischler and Bolles, 1987), which consists of an iterative method to estimate parameters of a mathematical model based on a set of observations that contains errors.

The RANSAC performs two steps as follows. A hypothesis is randomly selected and tested with the full universe of data; if the tests confirm the hypothesis through a threshold determined by the user, this assumption is saved as a candidate to the final product. After estimating several hypotheses, the best one is chosen as the final product according to predetermined parameters in the process.

The observations that are consistent with the mathematical model are named inliers and those that do not meet the predetermined parameters are considered outliers. To determine whether an observation is inlier or outlier, the error the hypotheses generate is compared to a threshold determined by the user. The RANSAC chooses as its best hypothesis the one with the highest number of inliers.

A disadvantage of RANSAC is that there is no upper bound on the time it takes to compute these parameters. When the number of performed iterations is limited, the obtained solution may not be optimal; in fact, it may not even be minimally appropriate for the data. In this way, the RANSAC offers the following trade-off: a greater number of iterations increases the probability of conceiving a reasonable model, although the total execution time is also increased. A final disadvantage of RANSAC is that it requires the setting of problem-specific thresholds.

Currently, there are many algorithms that are adjustments or enhancements of the RANSAC: LMedS (Rousseeuw and Leroy, 1987), GASAC (Rodehorst and Hellwich, 2006), StaRSaC (Choi and Medioni, 2009), MSAC (Torr and Zisserman, 2000) and MLESAC (Torr and Zisserman, 2000). These algorithms are supposed to be more robust than RANSAC. Nevertheless, they all present the same drawback: higher computational cost. If the input data could have few outliers initially, the RANSAC alone would generate good hypothesis in the shortest time possible.

In such context, it would be ideal to remove outliers between the tracking phase and the RANSAC execution. Thus, with as few outliers as possible, less iterations are necessary and the pipeline can improve its the performance. To achieve that, a technique for removing outliers should have a computation cost which is practically imperceptible.

3 SKen TEST

Although not formally proven until the present date, it is common sense that inliers features follow smooth paths while outliers do not present smoothness in a camera path. It was not found in the literature a test or technique with low computational cost whose purpose was to quantify the smoothness for a camera path. Thus, in this paper it is presented a hypothesis test capable of evaluating the smoothness of the feature path. The hypothesis test proposed, named SKen, was applied in the context of optical flow, for the paths of features tracked in the scene. The technique presented in this paper proposes a methodology for identifying and ranking features in order of smoothness. An important factor of the proposed methodology is that users do not have to enter any parameters in order to execute the SKen. The result is an automatic and deterministic method.

3.1 Random Variable

Suppose two features a and b in a video sequence. At ten frames, the feature a moved to the coordinates C_a =(1, 8, 12, 14, 13, 8, 4, 5, 8, 12) and the feature b moved to C_b =(1, 8, 12, 10, 13, 8, 9, 5, 8, 12) resulting in two paths which can be seen in figure 1.

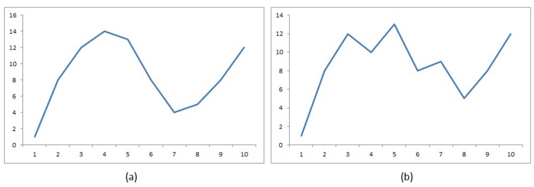


Figure 1: (a) Path of feature a (b) Path of feature b.

The feature a presents a smooth behavior whereas the feature b is clearly noisy. To verify the smoothness, it is used the second derivative of the function that generated the feature path. Once the function is discrete, the derivatives have to be approximated. The derivatives values are shown in table 1.

It can be observed that for the a feature, the second derivative alternate the signal only once due to

Table 1: Values for approximated derivatives of 2nd order for the features a and b.

$f''(a)$	-3	-2	-3	-4	1	5	2	1
$f''(b)$	-3	-6	5	-8	6	-5	7	1

the inflection point, whereas for feature b the second derivative switches 5 times.

In summary, there was only one alternation of signal into eight values of $f''(a)$ against five alternations of signals for the same of values of $f''(b)$. It is necessary a hypothesis test in order to evaluate how many alternations of signs are valid to assure that a path is, indeed, smooth. Thus, the alternating signal of the second derivative is the random variable under investigation. It was applied the nonparametric test SKen for the random variable $K = \sum_i k_i$ so that:

$$k_i = \begin{cases} 1, & \text{if} \quad f''[i+1]f''[i] > 0 \\ -1, & \text{if} \quad f''[i+1]f''[i] < 0 \\ 0, & \quad c.c. \end{cases} \tag{1}$$

where $f''[i]$ is the sequence of second derivatives of the path in question.

3.2 Assumptions

Like in all statistical hypothesis tests, it is necessary to define the assumptions, which must be satisfied in order to guarantee proper precision to the test as well as avoiding entailing wrong decisions. They are:

- ensure that the sequence is derived from a path
- the data must be in temporal order
- the sequence cannot be extremely sinuous

In case a sequence is extremely sinuous, the amount of inflection points may impair the test by making it more rigorous than necessary. In a long and not excessively sinuous sequence, the number of inflection points can be ignored.

3.3 The SKen Statistics

As defined in section 3.1, the random variable in question is the sum of the number of signal transition in the values of second derivatives ($K = \sum_i k_i$). Considering the example given in section 3.1, the value of K for the feature a and b is illustrated in table 2.

It can be noticed that the feature a ($\sum_i k_i(a) = 5$)

Table 2: Values of k_i and K for the features a and b.

$k_i(a)$	+1	+1	+1	-1	+1	+1	+1
$k_i(b)$	+1	-1	-1	-1	-1	-1	+1

has a larger K than the feature b ($\sum_i k_i(a) = -3$), i.e. the feature b has more alternating signs that the feature a, hence a is smoother than b. In other words: the higher the K, the smoother the path.

If the path has n terms, the maximum value of K is $K = n-3$ and the minimum value is $K = -(n-3)$, so the path needs to have at least four terms. The exact probability distribution of K for n terms is presented in Table 3:

Table 3: Exact probability distribution of the variable K for n terms.

K	-(n-3)	-(n-5)	…	(n-5)	(n-3)
$P(K = k_i)$	$\frac{\binom{n}{0}}{2^{n-3}}$	$\frac{\binom{n}{1}}{2^{n-3}}$	…	$\frac{\binom{n}{n-1}}{2^{n-3}}$	$\frac{\binom{n}{n}}{2^{n-3}}$

3.4 Large-sample Approximation

When n increases the calculation of $n!$ becomes computationally costly and this is out of the outline of this paper, so it has to be checked an approximation to the probability distribution $P(K = k_i)$. The central limit theorem (James, 2002) treats the convergence in distribution of normalized partial sums for the standard normal distribution $N(0.1)$. This way, it is assumed that all variances are finite and that at least one of them is strictly positive. The problem lies in finding conditions under which:

$$\frac{S_n - E(S_n)}{\sqrt{Var(S_n)}} \xrightarrow{D} N(0,1) \tag{2}$$

In such way, it is necessary to calculate E(K) and Var(K). Assuming a feature with a path containing n terms, so $n-3$ transitions, the calculation of E(K) and Var(K) are:

$$E(K) = \frac{-(n-3)\binom{n}{0} + \cdots + (n-3)\binom{n}{n}}{2^{n-3}} = 0 \tag{3}$$

$$Var(K) = \frac{(n-3)^2\binom{n}{0} + \cdots + (n-3)^2\binom{n}{n}}{2^{n-3}} = n-3 \tag{4}$$

When using a continuous approximation is attributed to a discrete variable, it is expected that some adjustment must be made. This adjustment is called continuity correction (Bussab and Morettin, 2002). The correction is performed by subtracting 1 to the value of K before calculating the value of the normal. Thus, given a feature with n terms, the approximation of the probability distribution of K to the Gaussian distribution is:

$$\frac{K-1}{\sqrt{n-3}} \xrightarrow{D} N(0,1) \quad (5)$$

It must be highlighted that, due to the characteristic of the equation 5 itself, features with longer paths are considered smoother than features with shorter paths. Whenever n increases, the numerator increases faster than the denominator. For example, suppose a feature with ten coordinates and K equal to ten and another one with twenty coordinates and K equals twenty. Both have all the second derivatives with the same sign, indicating the maximum smoothness. However, the p-value of the first, with ten coordinates, is $3x10^{-4}$ and the second, with twenty coordinates, would have p-value $2x10^{-6}$. Thus the feature with the longest path would be placed first in the order of smoothness than the one with shorter.

3.5 SKen Test Application

The methodology for applying the SKen test is given by the algorithm 1.

Algorithm 1: SKen test algorithm.

1. Track the scene using a generic tracker.
2. Decompose the input given by the tracker in (X,Y). That is, for every feature we have the path it does in X and the path it does in Y.
3. Apply the SKen test in all feature for both coordinates (X,Y) and sort the paths by p-values in ascending order, i.e. in order of smoothness.
4. Verify the requirements of the 3D reconstruction algorithm.
5. Provide as input to the 3D reconstruction algorithm the smoother features that fulfill its requirements. A feature is only considered smooth when is smooth on X and Y.

If the video to be reconstructed is filmed predominantly on a single axis, either X or Y, the methodology can be executed by applying the SKen test only to this axis and thereby decreasing the execution time.

4 RESULTS

This section presents the main results obtained by using the methodology proposed in this paper. It is illustrated with two different scenarios: a synthetic video and a real one. It was decided not to choose a significance level for the SKen test but to rank it by p-values instead. This is due to the lack of precise information on whether the amount of features selected as smooth would be enough to run the 3D reconstruction algorithm.

It is noteworthy that when a feature is not selected by the SKen test, it does not necessarily mean that this feature is not smooth; it just means it is less smooth than the others selected. Likewise, if a feature is selected, it does not necessarily mean that this feature is perfectly smooth, but it is smoother than the other unselected.

The system employed in this work for the 3D reconstruction is the R3D (Farias, 2012). This system uses the 3D reconstruction pipeline described in (Pollefeys, 1999), KLT tracker (Lucas and Kanade, 1981) and the feature detection executed with the GFTT algorithm (Shi and Tomasi, 1994). Each reconstruction algorithm has its peculiarities and the R3D, for instance, needs at least thirty features tracked between the first and second keyframe. In other words, thirty features must exist and be tracked in the first two keyframes.

Applying a significance level α to SKen and selecting the features considered inliers cannot guarantee that the initial conditions of the R3D would be satisfied. For this reason, it is necessary to provide as input to the R3D the smoother features that meet the requirements of such reconstruction algorithm. The SKen test applied to R3D is given by the algorithm as follows 2.

Algorithm 2: Application of SKen to the R3D system.

1. Apply the SKen test in the tracked features.
2. Sort in ascending order, i.e. by smoothness.
3. Select the first thirty features.

while until there is not 30 features that were tracked from first to second keyframe **do** Add to the set the next feature in the order of smoothness.
end while

4. This result is the new input set for the R3D.

In some reconstruction cases, the 3D mesh may be visually harmed, because when the SKen is applied it results in a drastic reduction in the amount of features. In most cases, the 3D mesh is discarded at first, since the most important requirement for 3D reconstruction is a well-estimated camera pose (Farias, 2012). When a good pose estimation is obtained, it can be generated a 3D mesh from dense reconstruction algorithms with better properties than the previous one(Furukawa and Ponce, 2010).

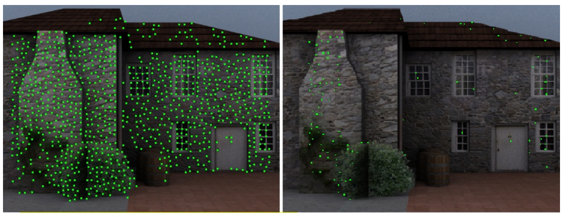
In order to evaluate the quality of a 3D reconstruction, the reprojection error frame by frame must be checked (Hartley and Zisserman, 2004). The finest reconstruction is the one with the lowest average reprojection error frame by frame. This parameter can be calculated either as the average reprojection error of all features in all frames or as the average of the averages of the reprojection error frame by frame; both results are similar (Bolfarine and Bussab, 2005).

4.1 Synthetic Video

The video called Home (Figure 2) is a 640x480 pixels resolution video and it has 102 frames over ten seconds.

Figure 2: Synthetic video - Home.

The initial step of the methodology consists in tracking the scene. The best 1024 features were provided (Figure 3a) according with the GFTT classification. The next step is to decompose the input data provided by the tracker in (X, Y) as shown in the section 3.5. It is important to perceive that in Figure 4, the paths the features made basically follow a horizontal movement, therefore it could be applied the SKen test only on the X axis; it would considerably decrease the processing time. Finally, the last step is to apply the SKen in the 1024 features initially provided by the tracker and to sort the obtained p-values as it increases. The lower the p-value, the smoother is the path.



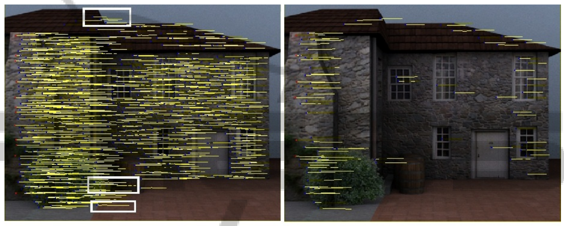
(a) (b)

Figure 3: (a) Frame 1: Synthetic video - Home (b) Frame 1(SKen): Synthetic video - Home

When applying the methodology, the scene is reconstructed using only the first 120 features, which represents an 88.3 % drop concerning all features initially tracked. In other words, the scene was reconstructed using the 120 smoother features and within those 120 features there are thirty features that were tracked between the first and second keyframe. The smoother features evidenced by the SKen test can be seen in Figure 3b.

If visually compared, it can be noticed that throughout the tracked video with 1024 features, some features did not present a smooth movement, as shown in Figure 4a. The white rectangles in the figure emphasize the most relevant non-smooth paths.

When adopted only the 120 features given by the SKen test, these non-smooth features disappeared, as shown in Figure 4b. This is due to the fact that these features had a less smooth behavior, i.e. in the test they presented a p-value higher than the final 120 selected. Another example of non-smoothness also occurs in Figure 5a. These same features do not appear in the video sequence when using the SKen (Figure 5b), they also were classified as less smooth than the selected by the SKen test.



(a) (b)

Figure 4: (a) Frame 11: Synthetic video - Home (b) Frame 11 (SKen):Synthetic video - Home.

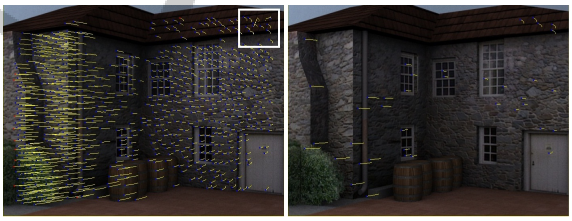


Figure 5: (a) Frame 88: Synthetic video - Home (b) Frame 88 (SKen): Synthetic video - Home.

As one the results of the methodology is the reduction of the number of features and thereby reducing the processing time, the 3D mesh may be less detailed than the one with the points initially tracked. However, as mentioned earlier in this section, the most important requirements are both the reprojection errors frame by frame and a well-estimated camera pose. Nevertheless, even with a restricted number of features, the 3D mesh was not impaired. The mesh and 3D camera path can be seen in Figure 6.

Using the initial 1024 features, the video Home presented an average reprojection error frame by frame of 0.2327 pixels. When executed the SKen test, the reconstructed scene had an average reprojection error of 0.2180 pixels, which means it decreased by 6.3%. The reprojection errors frame by frame can be seen in Figure 7.

Once the average reprojection error of all features and the average error between features used by the

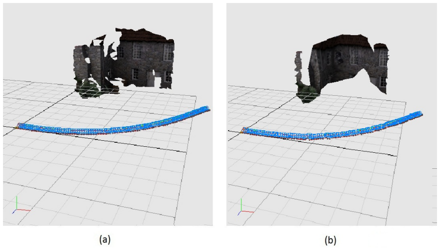


Figure 6: (a) 3D Reconstruction: Synthetic video - Home (b) 3D Reconstruction (SKen): Synthetic video - Home.

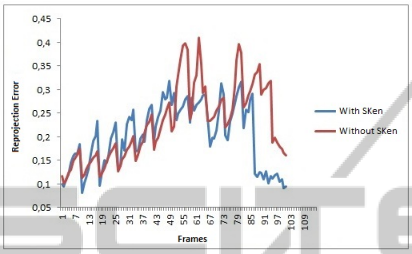


Figure 7: Synthetic video - Home: Reprojection error frame by frame.

SKen were very close to each other (only 6.3 % difference), it was performed the Wilcoxon test (Hollander and Wolfe, 1999) for comparing these means. This test was selected because the generated reprojection error frame by frame, in this case, does not satisfy aspects of normal distribution (Hollander and Wolfe, 1999). With the Wilcoxon test it was obtained a p-value of 0.3764, which indicates that there is no significant difference between the mean errors, i.e. the reconstructed scene, when using the SKen test, did not affect the reprojection error.

It must be highlighted that the Home video is a synthetic composition; therefore it presents a low-noise sequence. The main gain in this scenario is due to a significant restriction in the amount of features used, thereby also reducing the processing time of the RANSAC. Even with an 88% decrease in the features, it was statistically maintained the same reprojection error.

The test was executed in an Intel(R) Core(TM) i3-2120 CPU with 3.3GHz. The average execution time of the SKen in C was approximately 0.4 milliseconds. That is, the increment in runtime in C is practically imperceptible.

4.2 Real Video

The second experiment used a real video called Pineapple, which has been recorded with a 960x544 pixels resolution and composed of 164 frames over five seconds (Figure 8).

As well as in previous case, the initial step of the

Figure 8: Real Video - Pineapple.

method consists in tracking the scene. The best 2000 features were provided (Figure 9a) according to the GFTT classification. The next step is to decompose the input data given by the tracker (X, Y) as shown in section 3.5.

It is important to notice that, in the Figure 10, the paths the features made basically follow vertical movement; therefore it could be applied the SKen test only on the Y axis. Again, the last step is to apply the SKen in the 2000 features initially provided by the tracker and to sort the obtained p-values as it increases.

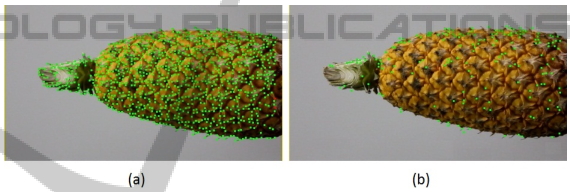


Figure 9: (a)Frame 1: Real Video - Pineapple (b) Frame 1 (SKen): Real Video - Pineapple.

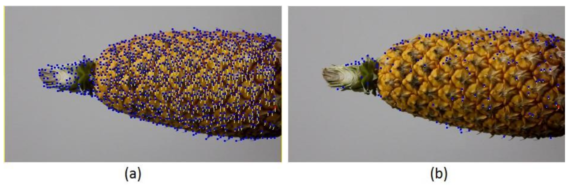


Figure 10: (a) Frame 12:Real Video - Pineapple (b) Frame 12 (SKen): Real Video - Pineapple.

According to the Algorithm 2, the scene might be reconstructed using only the first 153 features, which represents a 92.3% decrease regarding the total features that were initially tracked. In other words, the scene was reconstructed using the smoother 153 features and within those 153 features there are thirty features that were tracked between the first and second keyframe. The smoother features evidenced by the SKen test can be seen in Figure 9b.

The resulting 3D mesh with the points selected by the SKen test has shown slightly different changes even though it used only 7.6% of the initial features. The mesh and the 3D camera path can be seen in Figure 11.

Using the initial 2000 features, the Pineapple

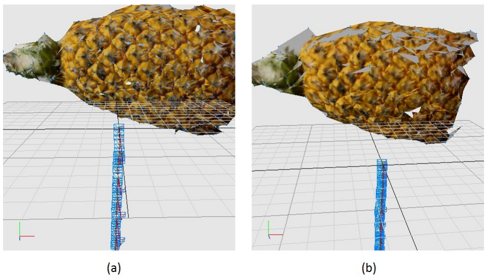


Figure 11: (a) 3D Reconstruction: Real Video - Pineapple (b) 3D Reconstruction (SKen): Real Video - Pineapple.

video presented an average reprojection error frame by frame of 1.689 pixels. When used the SKen test, the reconstructed scene had an average reprojection error of 0.90 pixels, which means a significant 46.9% drop. The reprojection errors frame by frame can be seen in Figure 12.

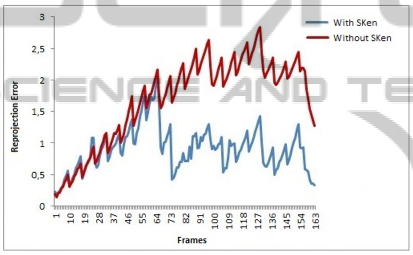


Figure 12: Real Video - Pineapple: Reprojection error frame by frame.

Although the difference between the reprojection errors was considerably high, and shown in Figure 12 (mainly from the frame 70 onward), it was performed the Wilcoxon test (Hollander and Wolfe, 1999) for statistical confirmation of this difference. As well as in the previous case, this test was adopted because the reprojection error frame by frame produced in this case does not meet the conditions of the normal distribution (Hollander and Wolfe, 1999). With the Wilcoxon test it was obtained a p-value smaller than 10^{-5}, which indicates that there is significant difference between the mean errors, i.e. the reconstructed scene reduced the average error of reprojection with the SKen test.

The same experiment setup was defined, as well as the hardware specifications. When implemented in C the average processing time was approximately 0.5 milliseconds. That is, the increment in execution time in C is practically imperceptible.

Considering this last scenario, applying the SKen caused a 92.3% drop in the number of features. It practically did not change the pipeline total runtime and decreased by 46.9% the reprojection error frame by frame.

5 CONCLUSIONS

This work presented important results concerning a 3D reconstruction pipeline by adopting a new technique called SKen. Although this method may reduce the amount of features during the tracking stage, it is not its objective; this is a consequence. The focus of the SKen is the selection of the best features. Thereat, the pipeline calculations can be more accurate and thus more reliable scene reconstructions scenes will be produced.

The two case studies present evidence that features that performed smooth paths are inlier features. This correlation can be confirmed by analyzing the reprojection error behavior; with the SKen selected features, reprojection errors either decreased or remained the same. Furthermore, it is also shown that it was not necessary to use the total amount of features initially tracked, since the same reconstruction results were achieved with less features and minor errors.

While in the synthetic and less noisy case the features selected by the SKen test achieved the same average reprojection error than the features originally tracked, this result was obtained by using only 11.7% of features provided by the tracker. For the real case with higher noise, the average reprojection error decreased by 46.9% and this result was obtained using only 7.7% of features provided by the tracker.

The main challenge of this investigation was to develop a methodology for obtaining results in real-time as well as it should not compromise the performance of the 3D reconstruction pipeline. The implementation of the SKen in C obtained an approximate average execution time of 0.5 milliseconds for a scene with 2000 features. It represents only 1.5% of what is necessary for meeting real-time requirements (approximately 33.3 milliseconds).

Finally, one of the most important features of the proposed methodology is that the user does not need to enter any parameters. Thus, this paper presented an automatic and deterministic method in a way that it does not need user intervention in order to remove outliers.

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