

Angle Measurements during 2D and 3D Movements of a Rigid Body Model of Lower Limb

Comparison between Integral-based and Quaternion-based Methods

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Abstract: Angle measurement system using inertial sensors was developed by our research group, in which lower limb angles were calculated based on the integral of angular velocity using Kalman filter. The angle calculation method was shown to be practical in measurement of angles in the sagittal plane during gait of healthy subjects. In this paper, in order to realize practical measurements of 3 dimensional (3D) movements with inertial sensors, the integral-based and the quaternion-based methods were tested in measurement of 2D movements in the sagittal plane and 3D movements of rigid body models of lower limb. The tested three calculation methods, extended integral-based method, quaternion-based method proposed in this study and simplified previous quaternion-based method, were suggested to measure the 2D movements with high measurement accuracy. It was also suggested that there were no large difference in measurement of 2D and 3D movements between 3 methods. Visualization by stick figure animation of circumduction gait simulated by a healthy subject also suggested that the angle calculation methods can be useful. It is expected to improve measurement accuracies of 3D movements to those of 2D movements.

1 INTRODUCTION

Lower limb motor functions are important to prevent bedridden and to make independence in daily living and social participation. Therefore, motor disabled persons or elderly people with decreased motor function need rehabilitation training of their lower limbs. In that rehabilitation, it is important to evaluate a level of subject's motor function in order to make rehabilitation program and to instruct it.

Generally, therapists perform the evaluation of motor function in rehabilitation by simple manual methods such as watching movements, measurement of the range of motion (ROM) with a manual goniometer, or measurement of time and counting the number of steps in 10 m walking test. Although these simple, manual evaluation methods are effective in limited space and time for rehabilitation training, those evaluation results depend on therapists. On the other hand, for quantitative and objective evaluation of movements, motion measurement system such as a camera-based system or electric goniometers has been used. Rehabilitation program proposed by the quantitative and objective

evaluations with motion measurement system is expected to increase rehabilitation effect and to decrease rehabilitation term. However, those motion measurement systems are mainly used in research works in laboratories, because these systems require large space for setting the system and time-consuming setup process, and are expensive.

Recently, use of inertial sensors (accelerometers and gyroscopes) has been studied in measurement and analysis of movements focusing on its shrinking in size, low cost and easiness for settings. In evaluation of motor functions, segment inclination angles and joint angles have important information for therapists and patients. Therefore, many studies have been performed on measurement of joint angles or segment tilt angles with inertial sensors (Tong and Granat, 1999; Dejnabadi, et al., 2005; Findlow, et al., 2008; Cooper, et al., 2009; Sabatini, 2006; Mazzà, et al., 2012.).

A motion measurement system using inertial sensors has to give joint or segment inclination angles calculating from angular velocities or acceleration signals. In addition, measurement of total lower limb movements such as simultaneous

measurement of hip, knee and ankle joint angles is required for clinical evaluation in rehabilitation support. In our previous study, a joint angle calculation method based on the integral of angular velocity using Kalman filter was applied to all the joint angles of the lower limbs. Measurement of gait with healthy subjects suggested that the method can be used practically in measurement of those angles in the sagittal plane (Saito and Watanabe, 2011; Watanabe, et al., 2011; Watanabe and Saito, 2011).

Angle measurement of 3-dimensional (3D) movements has been required for evaluation of motor function. For measurement of 3D angles with inertial sensors, a method of using attitude angle representation by quaternion was proposed (Sabatini, 2006). However, measurement of Euler angle was tested in that study. On the other hand, the integral of angular velocity can be expanded to measure 3D movements, and it is possible to provide simply angles in the sagittal plane and in the frontal plane.

The question focused in this paper was whether there are any differences in angle measurement between the integral-based method and the quaternion based one or not. Therefore, this paper aimed to evaluate angle measurement accuracy of different calculation methods. For this purpose, a Kalman filtering for angle calculation method using quaternion was developed based on our integral-based method. Then, a previous quaternion-based method was modified to a simplified method for the test. These 2 quaternion-based methods and an extended integral-based method were evaluated in measurements of angles during 2D movements in the sagittal plane, and angles during 3D movements in the sagittal and the frontal planes using rigid body models that represented the lower limb. Finally, a measured 3D movement during walking was tested in recreating stick-figure animation.

2 ANGLE CALCULATION METHODS

In this paper, three calculation methods shown in Figure 1 were evaluated in angle measurement.

2.1 Extended Integral-based Method

Figure 1(a) shows outline of the integral-based method of calculating segment inclination angle. In this paper, the previous integral-based method developed by our group was extended to calculate angles in the sagittal and the frontal planes.

Basically, a segment inclination angle is calculated by the integral of angular velocity (an output of a gyroscope). Here, the calculated angle is corrected by Kalman filter using angle measured with an accelerometer (Watanabe and Saito, 2011). Joint angles are calculated from 2 inclination angles of the adjacent segments. That is, segment inclination angle $\theta_{inc}(t)$ and joint angle $\theta_{joint}(t)$ are calculated as follows:

$$\theta_{inc}(t) = \int_0^t \omega(\tau) d\tau + \theta_{inc}(0), \quad (1)$$

$$\theta_{joint}(t) = \theta_{inc1}(t) - \theta_{inc2}(t), \quad (2)$$

where $\omega(t)$ shows angular velocity measured with a gyroscope. $\theta_{inc}(0)$ is the initial joint angle calculated from acceleration data. For instance, the angle in the sagittal plane is calculated from acceleration signal, a_x and a_z , by following equation.

$$\theta_{inc}(0) = \tan^{-1} a_z(0)/a_x(0) \quad (3)$$

Kalman filter estimates error in the angle calculated from the output of a gyroscope ($\Delta\hat{\theta}$) by using the difference between angles obtained by a gyroscope and by an accelerometer (Δy). Then, angle ($\hat{\theta}$) is calculated. That is,

$$\begin{aligned} \Delta y(t) &= \theta_{gyro}(t) - \theta_{acc}(t) \\ &= \theta_{inc}(t) - \tan^{-1} a_z(t)/a_x(t) \end{aligned} \quad (4)$$

The state equation and the observation equation are shown by using the error of the angle measured with gyroscopes ($\Delta\theta$) and increment of bias offset for one sampling period (Δb) as follows.

$$\begin{bmatrix} \Delta\theta_{k+1} \\ \Delta b_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta\theta_k \\ \Delta b_k \end{bmatrix} + \begin{bmatrix} \Delta t \\ 1 \end{bmatrix} w, \quad (5)$$

$$\Delta y_k = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta\theta_k \\ \Delta b_k \end{bmatrix} + v, \quad (6)$$

where w and v are errors in measurement with the gyroscope and with the accelerometer, respectively.

Kalman filter repeats corrections (Equation (7)) and predictions (Equation (8)) as follows:

$$\begin{bmatrix} \Delta\hat{\theta}_k \\ \Delta\hat{b}_k \end{bmatrix} = \begin{bmatrix} \Delta\hat{\theta}_k^- \\ \Delta\hat{b}_k^- \end{bmatrix} + \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} (\Delta y_k - \Delta\hat{\theta}_k^-), \quad (7)$$

$$\begin{bmatrix} \Delta\hat{\theta}_{k+1}^- \\ \Delta\hat{b}_{k+1}^- \end{bmatrix} = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta\hat{\theta}_k \\ \Delta\hat{b}_k \end{bmatrix}, \quad (8)$$

where K_1 and K_2 are Kalman gain for $\Delta\theta$ and Δb ,

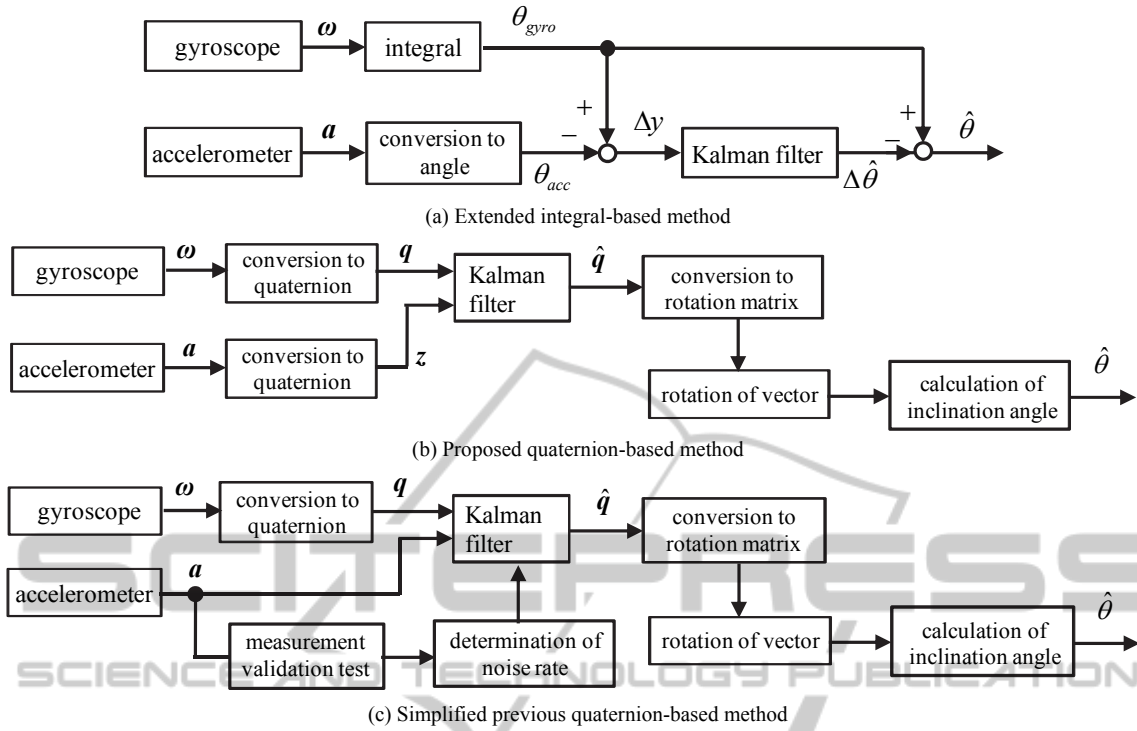


Figure 1: Outline of tested angle calculation methods.

respectively. The hat upon a character and the superscript minus represent estimated value and predicted value, respectively. For the initial state, $\Delta\hat{\theta}_0^-$ was set at zero and $\Delta\hat{b}_0^-$ was set at the value at the last measurement.

2.2 Proposed Quaternion-based Method

Quaternion can be used to represent the attitude of each segment of a rigid body. As shown in Figure 1(b), two quaternions are calculated from acceleration and from angular velocity measured with an inertial sensor. First, attitude angle representation by quaternion is obtained from the angular velocity. Then, Kalman filter was applied to correct error using attitude angle representation by quaternion obtained from the gravitational acceleration.

Using the triaxial angular velocity $\boldsymbol{\omega} = (\omega_x, \omega_y, \omega_z)$, quaternion \mathbf{q} is propagated according to the differential equation (Chou, 1992):

$$\dot{\mathbf{q}} = \frac{1}{2} \begin{bmatrix} 0 & -\omega_x & -\omega_y & -\omega_z \\ \omega_x & 0 & \omega_z & -\omega_y \\ \omega_y & -\omega_z & 0 & \omega_x \\ \omega_z & \omega_y & -\omega_x & 0 \end{bmatrix} \mathbf{q} \quad (9)$$

The state equation is the time integration of Equation (9), where \mathbf{w} is the process noise in measurement with a gyroscope.

$$\mathbf{q}_{k+1} = \frac{1}{2} \begin{bmatrix} 2 & -\Delta t \omega_{xk} & -\Delta t \omega_{yk} & -\Delta t \omega_{zk} \\ \Delta t \omega_{xk} & 2 & \Delta t \omega_{zk} & -\Delta t \omega_{yk} \\ \Delta t \omega_{yk} & -\Delta t \omega_{zk} & 2 & \Delta t \omega_{xk} \\ \Delta t \omega_{zk} & \Delta t \omega_{yk} & -\Delta t \omega_{xk} & 2 \end{bmatrix} \mathbf{q}_k + \mathbf{w} \quad (10)$$

The observation equation is given by the following equation, considering the observation noise \mathbf{v} in measurement with an accelerometer.

$$\mathbf{z}_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{q}_k + \mathbf{v} = \mathbf{I} \mathbf{q}_k + \mathbf{v}, \quad (11)$$

where the observation vector is the quaternion-based attitude representation \mathbf{z} that is obtained from the gravity acceleration. Then, correction and prediction are represented by

$$\hat{\mathbf{q}}_k = \hat{\mathbf{q}}_k^- + \mathbf{K}(\mathbf{z}_k - \hat{\mathbf{q}}_k^-), \quad (12)$$

$$\hat{\mathbf{q}}_{k+1}^- = \mathbf{I} \hat{\mathbf{q}}_k \quad (13)$$

The quaternion-based attitude representation \mathbf{z} can be obtained by the followings (Favre, 2006).

$$z_k = \left[\cos\left(\frac{\theta_k}{2}\right), \sin\left(\frac{\theta_k}{2}\right) \times \left[\frac{A_k}{\|A_k\|} \right] \right], \quad (14)$$

where the angle θ_k and axis of rotation A_k are obtained from the inner and the cross products of a measured acceleration vector a_k and the acceleration vector defined as the initial attitude of the sensor a_0 . That is,

$$\theta_k = \cos^{-1}(a_k \cdot a_0) \quad (15)$$

$$A_k = a_k \times a_0 \quad (16)$$

Using a rotation matrix calculated from the corrected quaternion \hat{q}_k , longitudinal vector of each body segment is rotated. Then, the rotated vector is projected onto the sagittal and the frontal planes of the global coordinate system. Inclination angles are obtained from the inner product of those projected vector and the unit vector of each plane.

2.3 Simplified Previous Quaternion-based Method

The quaternion-based method of measurement of Euler angle during human movements was developed by Sabatini (Sabatini, 2006). In this method, quaternion calculated from angular velocity is corrected by Kalman filter using acceleration signal and magnetic sensor signal. The acceleration and the magnetic sensor signals are used as observation signals directly. In addition, Kalman gain is varied dynamically based on validation test for those acceleration and magnetic sensor signals.

In this paper, the method was simplified removing magnetic sensor as shown in Figure 1(c) in order to compare the method under the same condition using gyroscope and acceleration sensors.

3 EVALUATION OF ANGLE CALCULATION METHODS

3.1 Measurement of 2D Movements

3.1.1 Experimental Method

A rigid body model of a duplex pendulum consisted of steel prop body and L-type aluminium materials corresponding to the thigh and the shank (Figure 2). The hip and the knee joints can be moved smoothly in a plane, while the hip joint position was fixed.

Two wireless inertial sensors (WAA-006, Wireless Technologies) were attached on each L-type material with double-sided adhesive tapes as shown in Figure 2. The inertial sensor includes a 2-axis gyroscope (ID-400, InvenSense for x and y axes) and a 1-axis gyroscope (XV-3500CB, Seiko Epson for z axis), and a 3-axis accelerometer (H30CD, Hitachi Metals). The inertial sensor communicates with a personal computer using Bluetooth (Ver 2.0 + EDR, Class 2). Markers for the optical motion measurement system (OPTOTRAK, Northern Digital Inc.) were also attached on the L-type materials with double-sided adhesive tapes in order to measure reference angles for evaluation of measurement accuracy.

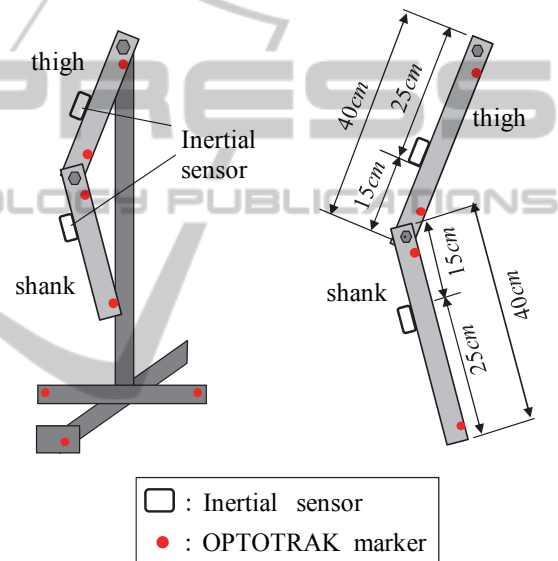


Figure 2: Rigid body model of a duplex pendulum for 2D movement measurements.

In angle measurements, the thigh was moved manually for angular range of ± 15 , ± 30 , ± 45 , ± 60 , and ± 75 deg with cycle period of 2 s (0 deg means the direction of the gravity). The cycle periods were regulated manually by a metronome. The thigh was moved referring to angle gauge, and the shank was moved freely during the thigh movements. Since prolonged measurements did not increase measurement error in our previous tests (Watanabe et al., 2011), the number of measurement trial was increased (10 trials) with reducing measurement time for each trial (35 s) in this paper.

The sensor signals and the marker positions were measured simultaneously with a personal computer at a sampling frequency of 100 Hz. Measured acceleration signals were filtered with Butterworth low-pass filter with the cut-off frequency of 20 Hz in

order to remove high frequency noise. Then, inclination angles were calculated by the 3 methods shown in Figure 1.

3.1.2 Results

An example of measured inclination angles are shown in Figure 3. Although the simplified previous quaternion-based method showed small difference from angle waveforms obtained by other methods, it seemed that angle waveforms obtained by all the 3 calculation method were almost same.

Measured angles were evaluated by root-mean-square error (RMSE) and correlation coefficient (CC) between the measured angle with sensor and its reference signal obtained from camera based motion analysis system. In the evaluation, the difference in position between the sensors and markers were removed by using the measured angle at the beginning of the 1st measurement.

Figure 4 shows average values of the RMSE and the CC of measured angles during 2D movement in the sagittal plane. There were no large differences between 3 calculation methods. However, the integral-based method showed higher accuracy for the thigh movements than the two quaternion-based methods. The simplified previous quaternion-based method increased the RMSE values and decreased

the CC values for the shank movements with angular range of ± 15 and ± 75 deg in comparison to the integral-based method.

3.2 Measurement of 3D Movements

3.2.1 Experimental Method

A wireless inertial sensor (WAA-010, Wireless Technologies) was attached to the rigid body model representing the thigh with the hip joint using a ball joint with double-sided adhesive tapes as shown in Figure 5. The inertial sensor includes a 3-axis gyroscope (IDG-3200, InvenSense) and a 3-axis accelerometer (ADXL345, Analog Devices). The inertial sensor communicates with a personal computer using Bluetooth (Ver 2.0 + EDR, Class 2). Thigh movements of the rigid body model were measured with the sensor and a camera based motion analysis system (OPTOTRAK, Northern Digital Inc.) simultaneously. All the data were measured with a sampling frequency of 100 Hz, and processed as same as that in the previous section.

The original position of the thigh part was in the direction of the gravity. In the measurements, the thigh part was moved repeatedly simulating the circumduction gait. That is, the thigh part was

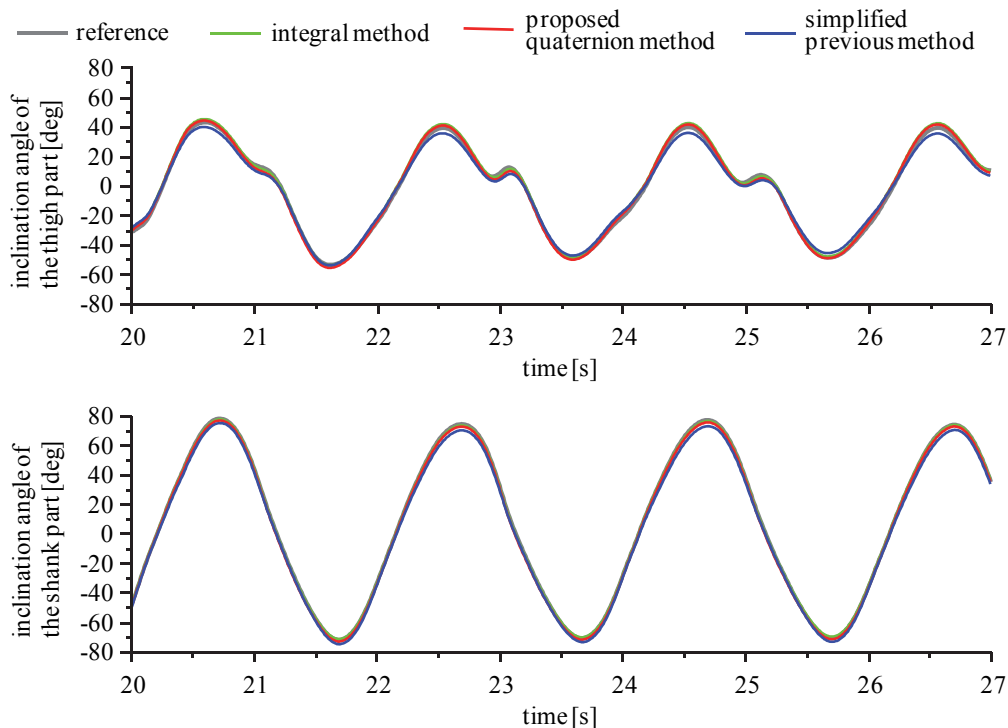


Figure 3: An example of measured inclination angles during 2D movements (± 75 deg).

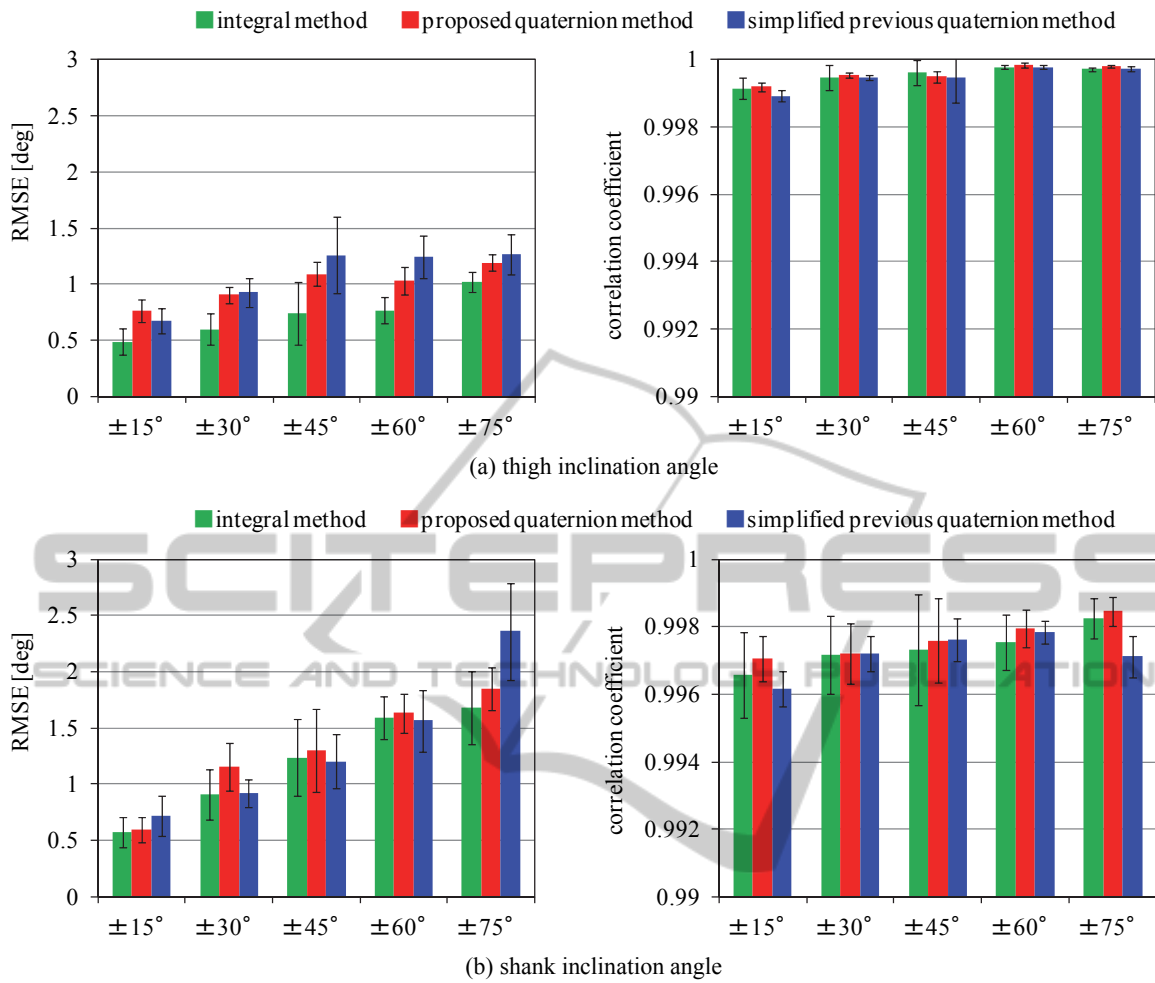


Figure 4: Evaluation of angle calculation methods in measurement of 2D movements.

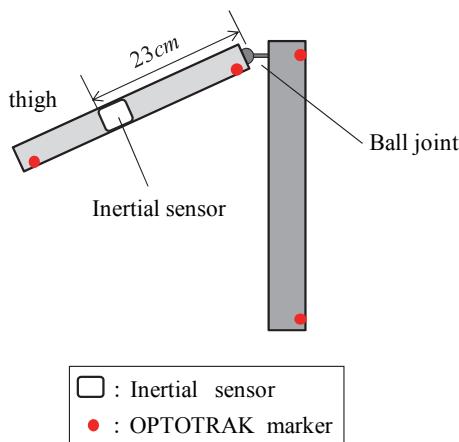


Figure 5: Rigid body model used in measurement of angles during 3D movements.

moved to flexed position of about 45 deg in the sagittal plane through adducted position of about 45

deg in the frontal plane from the original position, and then the thigh part was moved to the original position by extension movement in the sagittal plane. This movement was performed manually with a cycle period of 2s, 4s or 8s. The sensor was facing almost in the frontal plane during the movement. The movement was performed repeatedly in a measurement trial of 35 s. Five measurement trials were performed for each condition of the cycle period.

3.2.2 Results

Figure 6 shows an example of measured inclination angles during 3D movements. Although the inclination angles in the sagittal plane calculated by 3 calculation methods showed similar waveforms, difference between calculated angles and the reference angle was larger than those in measurement of 2D movements. Calculated angles

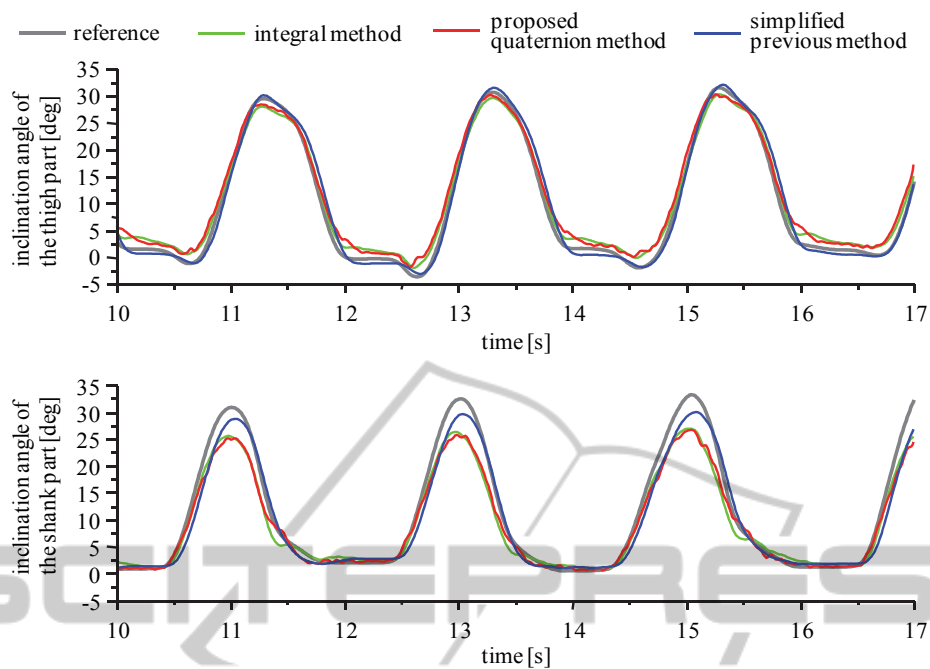


Figure 6: An example of measured inclination angles during 3D movements (2s of cycle period).

in the frontal plane showed larger difference than that in 2D movements and the angle calculated by the simplified previous method showed different waveform from other 2 methods.

Figure 7 shows evaluation results for measurements of 3D movements. In Figure 7(a), average values of all the measurements of the shank during the 2D movements were also shown. For the angles in the sagittal plane, the measurement accuracy of 3D movements decreased compared to the 2D movement measurements. Then, the integral-based method showed higher measurement accuracy than the 2 quaternion-based methods. Values of CC were low for the fast movement (2 s of cycle period).

For the angle in the frontal plane, the 2 quaternion based methods showed higher measurement accuracy than the integral one. The simplified previous method showed the smallest RMSE values. However, variations of CC values of the simplified quaternion method were larger in the slow and fast movements than the proposed quaternion method. In addition, for movements with cycle period of 4 s, values of CC of all the 3 methods decreased, and variations of RMSE were large.

4 DISCUSSIONS

The evaluation of measured angles with the rigid

body model showed that all the 3 methods measured the angles in the sagittal plane during 2D movements with average RMSE values less than 2.5 deg and with average correlation coefficients larger than 0.996. For angles in the sagittal plane during 3D movements, average RMSE values were less than about 3 deg and average CC values were larger than 0.983. However, for angles in the frontal plane during 3D movements, average RMSE values were less than about 4 deg and its standard deviation was less than about 1 deg. Although the measurement accuracy was not so high for angles in the frontal plane, it can improve manual measurement with goniometer that is used with resolution larger than about 5 deg, and it makes possible to measure angles during movements in addition to measurement of range of motion (ROM). These suggest that all the tested 3 angle calculation methods can be practical in measurement of movements. However, it is expected to improve measurement accuracy for 3D movements to those of the 2D movements.

Measurement accuracies of 2 quaternion methods were lower than the integral-based method for angles during the 2D movements in the sagittal plane and for angles in the sagittal plane during 3D movements. The simplified previous method showed good accuracy for angles in the frontal plane in 3D movement measurement. However, the differences in the accuracy between calculation methods were not so large. One of differences of the simplified

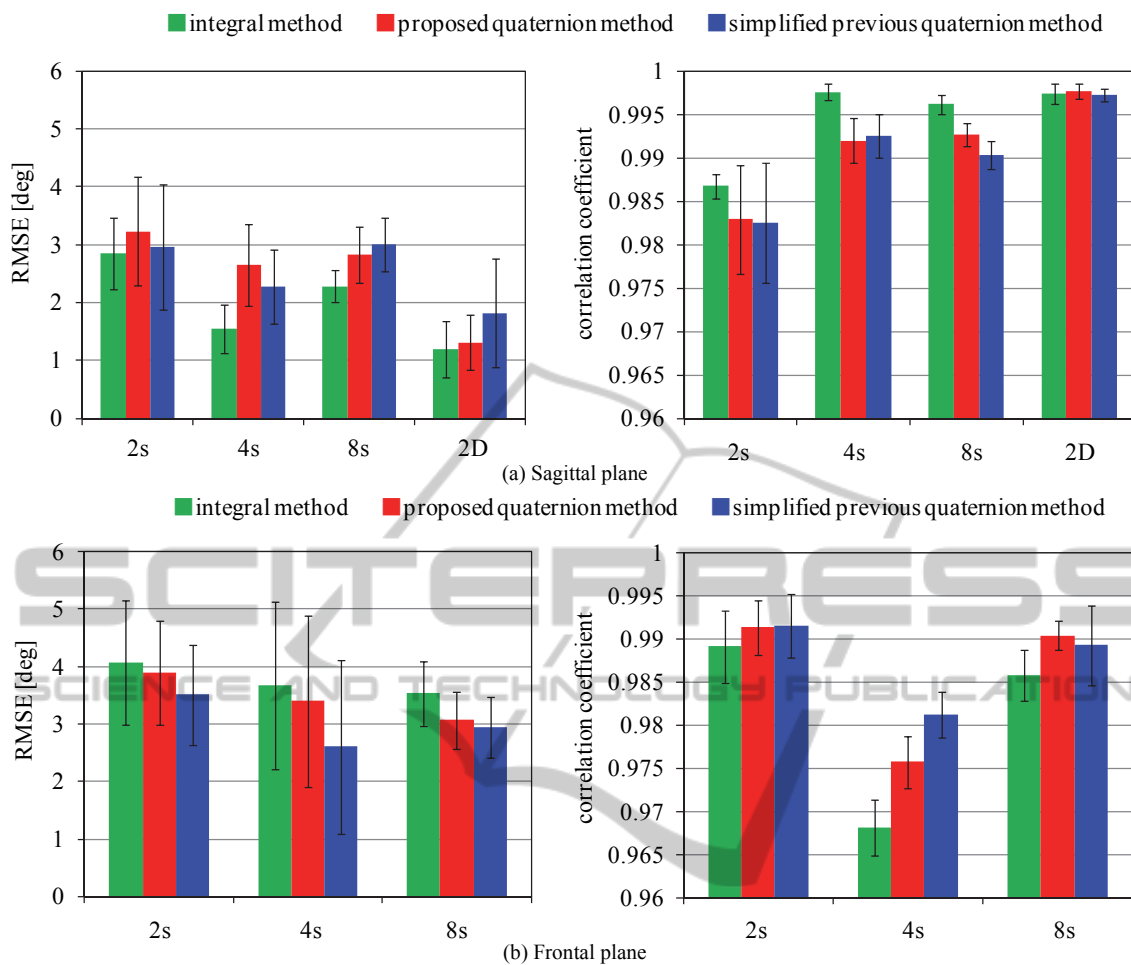


Figure 7: Evaluation of angle calculation methods in measurement of 3D movements.

previous quaternion method from other 2 methods is to determine Kalman gain through measurement validation test of acceleration signals. Although Kalman gain can be varied dynamically during measurement by this method, it is necessary to test the validity of adjusting Kalman gain based on measured acceleration signals. In our study, a method of changing Kalman gain dynamically based on magnitude of acceleration signal was not so effective with the integral-based method (Teruyama and Watanabe, 2013).

In order to test if current measurement accuracy can be used for visualization of measured movement or not, circumduction gait simulated by a healthy subject was measured with the wearable inertial sensor system developed by our research group (Watanabe and Saito, 2011). The system consisted of seven wireless inertial sensors (WAA-010, Wireless Technologies) and a notebook computer (Figure 8). Each sensor was attached on the body with a stretchable band with hook and loop fastener

and a pocket for the sensor. The sensors are put inside of the pocket and attached with the bands on the feet, the shanks and the thighs of both legs, and lumbar region as shown in Figure 8. Acceleration and angular velocity signals of each sensor were sampled with a frequency of 100Hz, and transmitted to the PC via Bluetooth network and recorded. Figure 9 shows an example of screenshots of stick figure animation obtained from the software developed by our research group (Watanabe and Saito, 2011), in which the 3D angle measurements in this study was implemented (simplified previous quaternion-based method). By using angles in the sagittal and the frontal planes, the stick figure animation could represent the subject's gait movement appropriately. Although it is necessary to evaluate measurement accuracy in human gait measurement, it is expected that the calculated angles can be effective for visualizing gait movement measured with wearable inertial sensors.

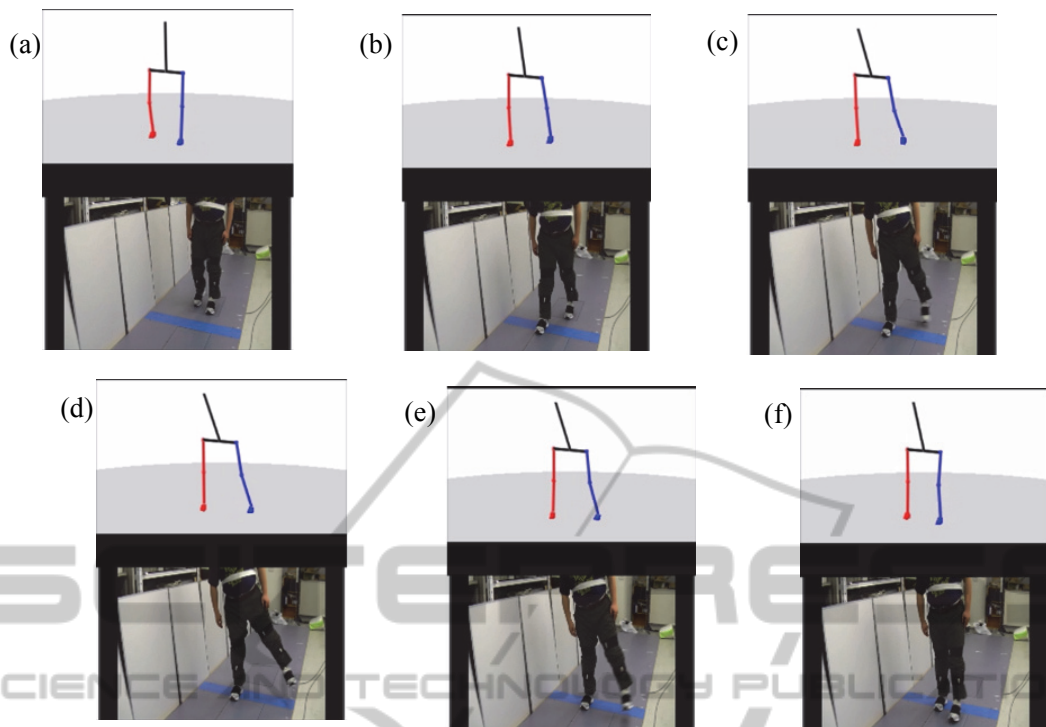


Figure 9: Screenshots of stick figure animation recreated by angles in the sagittal and the frontal planes. From (a) to (f), the subject walked forward.

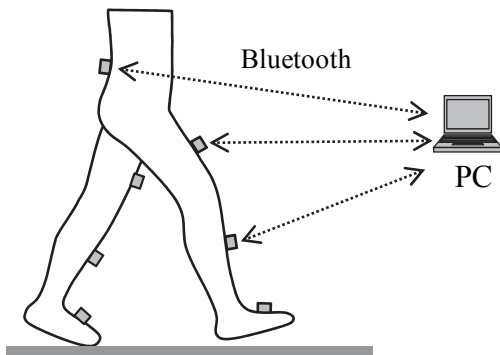


Figure 8: Outline of the wearable inertial sensor system developed by our group.

5 CONCLUSIONS

In this paper, the integral-based and the quaternion-based angle calculation methods were compared in angle measurements with the rigid body model. Measurement of 2D movements in the sagittal plane suggested that all the 3 methods can measure angles with high measurement accuracy. For 3D movements, although measurement accuracies of those methods decreased compared to the measurement of 2D movements, there were no large

difference in measurement accuracy between the 3 methods. Since the stick figure animation using angles in the sagittal and the frontal planes showed appropriately the measured circumduction gait, it would be effective to use these angles calculation methods. It is expected to improve measurement accuracies for 3D movements to those of the 2D movements.

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