

Detecting Math-and-ICT Competence

Maria Shabanova¹ and Borislav Lazarov²

¹*Institute of Mathematics, Information and Space Technologies, M. Lomonosov Northern (Arctic) Federal University, Arkhangelsk, Russia*

²*Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Sofia, Bulgaria*

Keywords: Socratic Style, Dynamic Geometry Software, Van Hiele's Level, Synthetic Competence.

Abstract: Theoretically DGS (dynamic geometry software) environment allows developing students' critical thinking via discovering some properties of the figures by exploring DGS applets. Our goal is to examine how this theory works in teaching a new non-trivial geometry concept. We study which van Hiele's level of geometrical reasoning is covered by the students in two cases. The first testing group worked in Socratic style of teaching without any time limit. The second one was restricted to the regular classroom parameters and the teacher applied instructional mode of teaching. The topic is the introduction of a new locus not included in the standard curriculum but in the students' zone of proximal development. We are interested in the transition of knowledge and skills developed in paper-and-pencil context to the DGS environment. The transition we consider as an indicator of competence of synthetic type. As the outcomes of the study, some important details of incorporating DGS in teaching-learning process in secondary school were clarified.

1 INTRODUCTION

The last pan European initiatives to modernize the secondary school math education refer to so called inquiry based education (Rocard et al., 2007). Key role in the change of the paradigm play the outcomes of the SINUS and SINUS-Transfer projects held in Germany (Baptist et al., 2011) where the Dynamic Worksheets done in dynamic geometry software (DGS) environment occupied the central location in teaching-learning process.

Meanwhile several initiatives to organize more attractive and effective math education based on DGS educational environment are taking place in the Russian Federation (Shabanova et al., 2013). One of them is with cooperation of Bulgarian Institute of Mathematics and Informatics in the frame of the MITE (Methodic and Information Technologies in Education) project and it aims to design educational resources and to prepare staff for implementation of these resources in regular practice (see Note 1).

In general the goal of these movements is the requirements of the key-competences defined in (European Commission, 2004) to be covered after the compulsory education (whatever it means). We adopt the main idea of the key-competence-definition that a key-competence should be a *transferable, multi-*

functional package of knowledge, skills and attitudes (ibid.) Building such kind of packages separately for any particular school subject reduces both its transferability and multifunctionality, this is why we advocate considering the competence as a synthetic concept (Lazarov, 2013). The study we present in this paper is an attempt to clarify some parameters of incorporating DGS in the regular teaching-learning practice aimed to make possible the transfer of knowledge and skills in a new context (*decontextualization*) (see Note 2). More specifically - we are interested in the level of transferability and multifunctionality of some knowledge and skills elaborated in paper-and-pencil technique into DGS environment and the potential advantages that this provides for heuristics and thus for inquiry based approach of teaching-learning.

2 METHOD AND OBJECTIVES

We consider the theory of Van Hieles level of geometrical reasoning (VanHiele, 1984) as the most relevant for describing the student's progress in building a new geometrical concept (a short description of the levels is given in the Appendix). It is important for our purpose the minimal initial van Hiele's level of reasoning to be determined as either 3 or 4. The level 3

guaranties that the student can use DGS as an educational tool but not (only) for fun. Level 4 we accept as the desired educational goal in the secondary school. Our standing point is that *a student has built the math-and-ICT component on a competence of synthetic type when (s)he is able to apply DGS for solving new non-trivial problems*. This is of course in the context of the high secondary school mathematics curriculum (grade 8 and up). Since the education in geometry is based on the traditional paper-and-pencil methodology the ability to present knowledge and skills in DGS format properly ensures their transferability and multifunctionality, i.e. the presence of a competence.

The objectives we stated were to examine the existence or lack of transfer of KSA (which indicates a kind of competence) and to clarify the time needed for elaborating a new concept applying inquiry-based style of teaching-learning. We investigate the reasoning of a new which locus, is close to the basic core of the curriculum but also requires the transfer of the KSA in a non-trivial way. We adopt the following indicators of detecting such kind of competence:

I1. Proper idea about the locus and design of a proper DGS instrument.

I2. The construction is explored and the conjecture is supported by examples; the statement is stated and proved.

I3. Ability for independent work and the level of the teacher's intervention.

We integrate the DGS interface with the common geometrical language of the van Hiele's level 3, forming a kind of metalanguage. It allows more effective communication during the lesson but also more reliable observation on the indicators. For instance measuring the transfer of KSA includes the stability of a particular DGS construction. Meanwhile we keep an eye on the student's zones of actual and proximal development (Vygotsky, 1935) when we evaluate his/her progress.

Data collection was done during and after the lesson: we gathered the sheets and the files of the students; a video record of the lessons was made. However we also used the teacher's observation on each student in a longer period to clarify the van Hiele's level reached in similar matter.

3 TECHNICAL PARAMETERS

3.1 Population

Our target group was 9th grade students of an ordinary school. We organized the experiment in two groups: the first one of 14 students and the second

of nine. All of them were familiar with GeoGebra (they had more than 3 years experience in applying it). The first group is a regular class; the second group is formed by students attending a math circle.

3.2 Teachers

The teachers involved in the study were among the best math teachers, experts in inquiry-based education, authors of the textbooks and having both deep math knowledge and excellent skills in GeoGebra.

3.3 Problems

We designed two problems for the study. Problem 1. Find the locus of points inside a quadrilateral with minimal sum of the distances to the lines containing its sides. Investigate the case:

- a) square;
- b) rhombus;
- c) parallelogram.

Problem 2. Find the locus of points inside a quadrilateral with minimal sum of the distances to its sides. Investigate the case:

- d) square;
- e) rhombus;
- f) parallelogram.

Students were expected to pass through the following steps:

- step 1: the locus is drawn correctly,
- step 2: the conjecture is relevant,
- step 3: the conjecture is confirmed by examples,
- step 4: a proof is made,
- step 5 (in Problem 1): a general proof for the square, the rhombus and the parallelogram is made,
- step 5 (in Problem 2): some particular shapes of the locus are examined,
- step 6 (in Problem 2): a new problem is stated and solved.

3.4 Lesson Plan

Our experiment was held in a regular lesson (2 h 45 minutes) with the second group and an extended lesson without time limit with the first group (2 h 24 min). Both lessons followed the next plan

- (1) Declaring the objectives of the lesson
- (2) Reminding some basic facts
- (3) Stating Problem 1 and the type of the activities
- (4) Individual work on Problem 1
- (5) Solving Problem 1 with tutoring
- (6) Plenary discussion on Problem 1
- (7) Elaborating basic knowledge and skills for Problem 2

- (8) Stating Problem 2
- (9) Individual work on Problem 2
- (10) Solving Problem 2 with tutoring
- (11) Plenary discussion on Problem 2
- (12) Drawing conclusions

Let us note that the concept of distance between point and segment is not introduced in the curriculum. This is why the students were given in advance a homework to elaborate the locus of points at given distance from a segment. The homework includes also problems about the sum of the distances to two parallel segments. The solutions were expected to be written in traditional paper-and-pencil style.

3.5 Benchmarks

The coverage of the indicators was registered as follows.

- I1. The ability to construct a dynamically stable quadrilateral; the examined segments are dynamically stable; the sum of the segments is explicitly noted.
- I2. Progress in the solution of the problems about the square, rhombus and parallelogram: the student gives arguments that support the conclusions up to the rigor proof.
- I3. The student works independently or (s)he manage to obtain the required results after teacher’s reinforcement.

4 LESSON FLOW

The lessons started with a brief review of the homework. For us it was a check of the initial level of reasoning of the new concept: distance between point and segment. Students performed some experimental work with the ready applets. This introductory part showed that in the both groups the DGS language was properly learned.

There were advanced students from Group 1 who managed to perform the standard questions quickly. After doing this, they went further and deeper in the solution of the problems. The unlimited time for individual work did not give any advantages to the others. As a rule, the teacher was more interested in the performance of the advanced students. She performed an individual Socratic inquiry with some of them if needed. After some time the rest of the class was given ready applets (Figure 1) provided with some short instructions.

Students of the second group were given ready applets and clear instructions what to do. This approach allowed more uniform movement along the lesson. Moreover, the teacher had direct view on the transfer

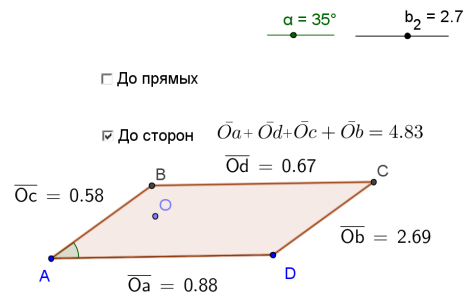


Figure 1: The main experimental applet prepared by the teacher in advance.

of the student’s knowledge build in paper-and-pencil mode to the DGS environment. As soon as students performed the experimental work, she went to the proof of the results.

Very important parts of the lesson in both groups were the plenary discussions when the students presented their ideas to the class. We detected the competence level (or lack of competence) of any particular student more clearly during these discussions. The use of the metalanguage (natural language + math expressions and drawings completed with DGS-interface notation – see Note 3) allowed students to support their arguments better. We consider the ability to use properly such style of communication as existence of some synthetic competence.

5 EXAMPLES

In this section, we present our observations on 3 students. The first two are the marginal cases in Group 1: the most advanced student and the one with modest abilities. The third student is from the more homogeneous Group 2.

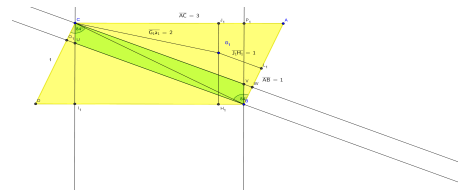


Figure 2: The only DGS construction designed by StudentI during the lesson.

StudentI (Group 1). StudentI’s geometrical reasoning was classified by his teacher as van Hiele’s level 4 for the most topics. StudentI often has nonstandard ideas and gives original solutions. He skipped the first problem as not interesting - he immediately guessed that the sum of the distances to two parallel lines remains the same for any point between them which answers the question in Problem 1.

StudentI designed a single DGS construction (Figure 2) but it was enough to explore the locus in the most general case. He passed the steps 1-3 in a very short time and decided he is ready. After the teacher urged him, StudentI passed step 4 - he proved the assertion without any enthusiasm (Figure 3).

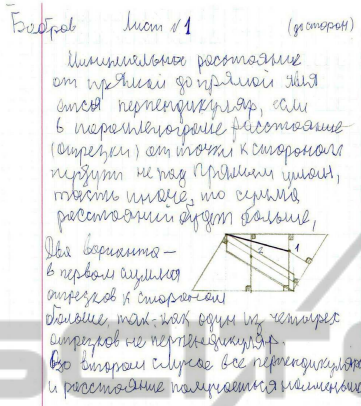


Figure 3: StudentI's P&P "proof".

StudentG (Group 1). The teacher determines StudentG's geometrical reasoning as van Hiele's level 1, but even he managed passing step 1. Further, he used the applet provided by the teacher. StudentG did not show any progress in Problem 2 after the individual work in the DGS environment. He passed step 1 in Problem 2 after some explanations given by the teacher. It seemed to be that Problem 2 is beyond the StudentG's zone of proximal development (ZPD).

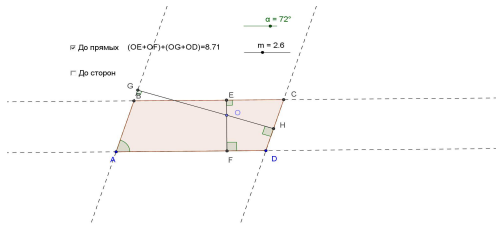


Figure 4: DGS applet provided by the teacher to StudentG with his explorations.

StudentA (Group 2). She was classified by her teacher as reasoning in Van Hiele's level 3. StudentA had some experience in a kind of research activities. She was expected to pass all steps independently. Instead of this she asked the teacher's instructions all the time. But she went further than the other students exploring some more cases. She did not prefer paper-and-pencil drawing and made her proofs using only the DGS applets for supporting her intuition.

6 RESULTS

Table 1 represents the progress made by students in solving the problems. The columns a), b), c) stand for the parts of the Problem 1; the numbers in the rows show how many students covered the corresponding step of the solution. Analogous are the columns d), e) and f).

Table 1: The progress in the solution.

Progress in Problem 1	Group 1			Group 2		
	a)	b)	c)	a)	b)	c)
step 1	14	14	14	9	9	9
step 2	8	8	8	9	9	9
step 3	13	13	12	9	9	9
step 4	11	10	6	1	1	1
step 5	1			1		

Progress in Problem 2	Group 1			Group 2		
	d)	e)	f)	d)	e)	f)
step 1	14	14	14	8	8	8
step 2	13	13	13	8	8	8
step 3	13	13	13	8	8	8
step 4	9	9	8	2	2	2
step 5	0	0	2	0	0	1
step 6	1			0		

We refer the steps 1 and 2 to the indicator I1. The success in steps 3 and 4 is related to the indicator I2. The time elapsed by the individual work or working with tutoring on the problems is connected to the indicator I3. These times were as follows:

Group 1.

Individual work on Problem 1 - 40 min.

Individual work on Problem 2 - 30 min.

Solving Problem 2 with tutoring - 17 min.

Group 2.

Solving Problem 1 with tutoring - 27 min.

Solving Problem 2 with tutoring - 17 min.

In our study we use the homework and Problem1 mainly to clarify the metalanguage in which students were going to communicate in Problem 2. The data collected clearly showed that the students were familiar with all needed auxiliary concepts when they started solving the Problem 2. The homework and Problem 1 played another important role for students according to the inductive character of the van Hiele's level - the connections between the components of the figures elaborated during solving these two preparatory problems in paper-and-pencil mode (P&P) were the necessary initial cognitive status for starting Problem 2. We give the observed van Hiele's level (the number of students who achieved the corresponding level) in table 2.

Table 2: The observed van Hiele’s level.

Van Hiele’s level	Group 1		Group 2	
	P&P	DGS	P&P	DGS
I	1	0	0	0
II	3	1	5	2
III	7	5	4	6
IV	3	8	0	2

7 FINAL REMARKS

We will skip advocating the obvious fact that the context of paper-and-pencil is rather different from the context of DGS environment and refer to (Marrades& Gutierrez, 2000) for the review of the research of the types of reasoning. We also agree with the findings that rediscovering nontrivial math facts in classroom practice is impossible even with most advanced students (Lazarov&Sharkova, 2013). So we are trying to evaluate a significant part of the total price that should be paid for incorporating DGS in regular teaching practice - namely the time elapsed for considerable progress in studying a new math concept. Further, after paying the price, do we get the expected quality of the educational product - the competence?

The statistics from table 1 clearly shows that the first indicator is covered by the students (with only one exception) for the most trivial case. The conclusion is that the students "speak the DGS language fluently". There is full-scale transition of the basic concepts from the paper-and-pencil context to the DGS environment. In some intuitive level students feel the matter.

The poor performance in deduction shows that the DGS abilities have little impact on the deeper understanding of the matter. This is not a big surprise if we take into account that *the deductive justification is characterized by the decontextualization of the argument used* (Marrades& Gutierrez, 2000). From our perspective the statistics indicates lack of synthetic competence of 5 students of the Group 1 and 6 of Group 2.

The StudetI’s case shows the existing of KSA transferability from paper-and-pencil to DGS but the lack of it in the reverse way. This means that the step from empirical to deductive justification is not easy even for the students with relatively high math abilities.

The roots of the inquiry based approach are in the Socratic style. This style was applied in the first group after the individual work on Problem 2. The unlimited time and the Socratic style provided some opportunities for creative work to the more able students, but

the tutoring instructional method applied in the second group gave better statistics.

Our initial assumption that the Problem 1 is in the zone of the actual development and Problem 2 after the homework will be in the zone of the proximal (and even actual) development of the students was too optimistic, but our misunderestimation of these very simple problems shows that the desired meaningful inquiry-based approach in math education is nothing more than a fiction for the regular classroom practice. In fact, during a regular lesson in traditional style teachers consider at least 4-5 such simple problems with rigor proofs.

On the contrary: the Socratic style is attractive and effective way for new concepts to be introduced and elaborated as extracurricular activity. During our study we observed that Socratic style teaching geometry enhanced with DGS environment does not lead to van Hiele’s level 4 automatically, and in some cases even reduces students’ critical thinking.

8 NOTES

Note 1. A frame of didactical technology for the Thales Theorem was among the outcomes of the 3rd meeting of MITE held in October 2006 in Oryahoviza, Bulgaria. During a workshop initiated by the second author an international team designed this frame with elements of ICT, compatible with curriculum in Bulgaria, Russia and Kazakhstan.

Note 2. We borrow the word decontextualization from (Marrades& Gutierrez, 2000) but we do not use it in the same sense.

Note 3. We use the word *metalanguage* to denote an expansion of the common math language used by the students on the DGS icons, operators etc. The general idea follows the description of the *plurilingualism* given in the Common European Framework of Reference for Languages (Language Policy Unit, Strasbourg, www.coe.int/lang-CEFR): *In different situations, a person can call flexibly upon different parts of this (communicative) competence to achieve effective communication with a particular interlocutor.*

The Van Hiele’s level have specific language (Van-Hiele, 1984) which indicates the level. We observed that the students working with DGS use also in specific manner the DGS interface for each level, i.e. they 'communicate' with the computer in a manner that is specific for the Van Hiele’s level reached. The communicative competence in this case is expressed via metalanguage which is specific for the corresponding Van Hiele’s level. A research to clarify the metalanguage is in progress in our team and an interim report

was given at the ITA 2013 Winter Session, Dec 18-19, 2013, Sofia, Bulgaria.

REFERENCES

- Baptist, P. et al. (eds.) (2011). SINUS international. Bayreuth, University of Bayreuth.
- European Commission (2004). Framework for key competences in a knowledge-based society. <http://ec.europa.eu/education/policies/2010/doc/basicframe.pdf> (active in Nov 2013)
- Rocard, M. et al. (2007). Science Education now. European Commission, Directorate-General for Research, Science, Economy and Society.
- Lazarov, B. (2013) Socratic style teaching and synthetic competence building of advanced students in mathematics. DARYN, Astana, 18-19
- Lazarov, B. and Sharkova, I. (2013). Two geometry illustrations of the infinitesimal method. *Education and Technology*. Vol 4/2013, 174-180. (in Bulgarian)
- Marrades, R. and Gutierrez, A. (2000). Proofs produced by secondary school students learning geometry in a dynamic computer environment. *Educational Studies in Mathematics*, 44, 87-125.
- Shabanova, M. et al. (2013) Learning mathematics with using opportunities of GeoGebra. *Pero Pblsh, Moskva*, p 127. (in Russian).
- Van Hiele, P. M. (1984). A child's thought and geometry. In D.Fuys, D.Geddes and R.Tischler (Eds), *English translation of selected writings of Dina van Hiele-Geldof and P.M. van Hiele*, 237-241. Brooklyn College, N.Y. 1984.
- Vygotsky, L. (1935). Dynamics of cognitive development in relation to student learning. *Mental development of children in the learning process*. Moscow-Leningrad, 33-52. (in Russian).

Level 3 (Ordering). Pupils logically order the properties of figures by short chains of deductions and understand the interrelationships between figures (e.g. class inclusions).

Level 4 (Deduction). Pupils start developing longer sequences of statements and begin to understand the significance of deduction, the role of axioms, theorems and proof.

We did not detect the fifth level (Rigor) among the students of our groups.

APPENDIX

We quote the definitions of the first four Van Hiele's levels as given by Michael de Villiers in *Some Reflections on the Van Hiele theory* (Invited plenary presented at the 4th Congress of teachers of mathematics of the Croatian Mathematical Society, Zagreb, 30 June - 2 July 2010).

Level 1 (Recognition). Pupils visually recognize figures by their global appearance. They recognize triangles, squares, parallelograms, and so forth by their shape, but they do not explicitly identify the properties of these figures.

Level 2 (Analysis). Pupils start analyzing the properties of figures and learn the appropriate technical terminology for describing them, but they do not interrelate figures or properties of figures.