

# Granular Cognitive Map Reconstruction

## Adjusting Granularity Parameters

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**Abstract:** The objective of this paper is to present developed methodology for Granular Cognitive Map reconstruction. Granular Cognitive Maps model complex imprecise systems. With a proper adjustment of granularity parameters, a Granular Cognitive Map can represent given system with good balance between generality and specificity of the description. The authors present a methodology for Granular Cognitive Map reconstruction. The proposed approach takes advantage of granular information representation model. The objective of optimization is to readjust granularity parameters in order to increase coverage of targets by map responses. In this way we take full advantage of the granular information representation model and produce better, more accurate map, which maintains exactly the same balance between generality and specificity. Proposed methodology reconstructs Granular Cognitive Map without losing its specificity. Presented approach is applied in a series of experiments that allow evaluating quality of reconstructed maps.

## 1 INTRODUCTION

Cognitive maps are abstract soft computing models, which allow describing complex systems flexibly. Cognitive Maps represent knowledge and relationships within knowledge in a form of a directed graph. Nodes represent concepts (units or aggregates of information). Edges between the nodes represent relations between the knowledge gathered in such map. A very important milestone in research on cognitive maps is definition of Fuzzy Cognitive Maps (FCMs) by B. Kosko in 1986, (Kosko, 1986). Fuzzy Cognitive Maps combine cognitive maps with fuzzy sets. They became powerful modeling framework and several practical applications of FCMs have been proposed, (Papageorgiou and Salmeron, 2013), (Papakostas et al., 2008).

Later research on imprecise information representation models has brought further generalizations of knowledge units, (Zadeh, 1997). Granular Computing has emerged as an important branch in information sciences, (Bargiela and Pedrycz, 2003). Information granules generalize units or aggregates of knowledge. Modeling capabilities of a model built

on the grounds of granular information are widened. Granularity elevates existing models, by introducing a controlled balance between specificity of knowledge granules and generality of the described phenomena.

In this paper the authors discuss Granular Cognitive Maps (GCM), a generalization of cognitive maps based on knowledge granules. In the start point of our study we place Fuzzy Cognitive Maps, which get augmented to Granular Cognitive Maps. Most importantly, we propose a methodology for Granular Cognitive Map reconstruction.

The paper is structured as follows. In Section 2 we present methodology of Granular Cognitive Map reconstruction. The proposed approach is applied in Section 3.

## 2 METHODOLOGY

In this paper authors introduce a methodology for Granular Cognitive Map reconstruction. We would like to stress at the beginning that with our procedure one is able to build a Granular Cognitive Map based

only on targets and activations (nomenclature will be discussed in following paragraphs). In this perspective, we may also call our methodology a Granular Cognitive Map construction procedure. We intend to use the term reconstruction, because in our opinion it reflects better character of the methodology. Granular Cognitive Maps describe certain systems (sets of related phenomena). Such system is faithfully modeled with an „ideal” Granular Cognitive Map. Our procedure aims at reconstruction of this „ideal” GCM. Second premise speaking for the term reconstruction derives from the validation scheme, which we use to assess quality of our methodology.

## 2.1 From a Fuzzy Cognitive Map to a Granular Cognitive Map

Let us start the discussion with a drawing that visualizes a cognitive map. Figure 1 shows a cognitive map

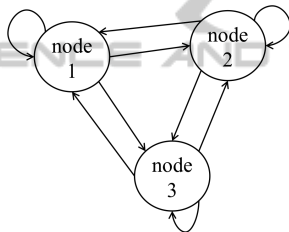


Figure 1: A cognitive map.

consisting of  $n = 3$  nodes. Nodes are connected with directed edges that represent relationships. In Fuzzy Cognitive Maps such relationship may take a value of a real number from the  $[-1, 1]$  interval. Connections are gathered in a weights matrix, which is  $n \times n$  in size and is denoted as  $W$ . Information relevant to FCM states is available in activations. Activations, denoted as  $X$ , are in  $n \times N$  matrix, where  $N$  is the number of observations. Activations are forces that are relevant for  $n$  nodes in  $N$  observations. Activations take values of real numbers from  $[0, 1]$  interval.

Activations together with weights matrix allow to compute map responses in  $N$  observations, according to a general formula:

$$Y = f(W * X) \quad (1)$$

where  $*$  is an operation performed on matrices  $W$  and  $X$ , which produces a matrix  $W * X$  of size  $n \times N$ , and  $f$  is a mapping applied to individually to elements of  $W * X$ . Matrix product is an example of such operation and it is utilized in this study.

Let us denote  $i$ -th row,  $j$ -th column and an element in  $i$ -th row and  $j$ -th column of a matrix  $A$  as  $A_{i.}$ ,  $A_{.j}$  and  $A_{ij}$ , respectively. In order to compute map's response to  $k$ -th activations (response in  $k$ -th iteration),

we apply a formula:

$$Y_k = ftras(W \cdot X_k) \quad (2)$$

and, more specifically,  $i$ -th node response in  $k$ -th iteration is computed by multiplying:

$$Y_{ik} = ftrans(W_{i.} \cdot X_k) \quad (3)$$

where  $ftrans$  is a nonlinear non-decreasing transformation function.  $ftrans : R \rightarrow [0, 1]$ . In this paper we use sigmoid function:

$$fsig(z) = \frac{1}{1 + \exp(-\tau z)}, \quad \tau > 0 \quad (4)$$

We chose the  $\tau$  parameter equal to 2.5 based on experiments.

Map responses are collated with „ideal”, real values, which are called targets ( $TGT$ ). The closer map responses are to targets, the better the map. Quality of a Fuzzy Cognitive Map is assessed by calculating error statistics regarding discrepancies between  $Y$  and  $TGT$ .

In our previous research we have proposed a methodology for Fuzzy Cognitive Map reconstruction. The proposed procedure is based on gradient-based error minimization. Outcome of FCM reconstruction is a weights matrix that describes connections within a map. Weights are adjusted so that differences between  $Y$  and  $TGT$  are the smallest. Let us assume that we have such reconstructed FCM. Thereby, we have optimized weights matrix, denoted as  $W_{fin}$ , activations  $X$  and targets  $TGT$ .

Augmentation to Granular Cognitive Map may occur at two levels: weights and responses. In our experiments we focus on the first approach, where augmentation to knowledge granules concerns weights. Research on methodological details on Granular Cognitive Maps in general and on augmentation from FCM to GCM is in (Pedrycz and Homenda, 2012).

Augmented, granular weights, denoted as  $W_{fin}$ , together with activations  $X$  produce granular outputs:

$$Y = f(W_{fin} * X) \quad (5)$$

where  $*$  is a specific operator, applicable to chosen representation model of knowledge granules.

In Granular Cognitive Maps information (or aggregates of information) are represented with knowledge granules. There are several knowledge description models that may be used to define granules, including:

- intervals,
- triangular fuzzy numbers,
- parabolic fuzzy numbers,
- others.

In this study we use intervals for granules representation.

In the process of Granular Cognitive Maps construction and exploration following elements are relevant: activations ( $X$ ), granular weights ( $\mathbf{W}_{\text{fin}}$ ), granular map responses ( $\mathbf{Y}$ ), additional parameters of granularity and targets ( $TGT$ ).

Elevation to granular information representation model requires adjustment of additional parameters. Specifics of granularity parameters depend on chosen granules representation scheme. In the case of interval-based model it is the length of the interval. The bigger given granule, the more general concept it can describe. Specificity criterion is at variance with generality. Specificity is translated to precision. The more specific knowledge granules, the less fuzzy the description of the corresponding phenomena or relations. In the process of a Granular Cognitive Map reconstruction we manipulate with the length of intervals with the aim to achieve a compromise between generality and specificity.

- size of knowledge granule  $\varepsilon$ ,
- symmetry parameter  $\gamma$ .

Construction of a GCM requires a conceptual settlement of a conflict between specificity and generality. Generality of knowledge granule is directly linked to the  $\varepsilon$  parameter - size of the granule. In the case of interval-based model it is the length of the interval. The bigger given granule, the more general concept it can describe. Specificity criterion is at variance with generality. Specificity is translated to precision. The more specific knowledge granules, the less fuzzy the description of the corresponding phenomena or relations. In the process of a Granular Cognitive Map reconstruction we manipulate with the length of intervals with the aim to achieve a compromise between generality and specificity.

The  $\gamma$  parameter determines symmetry of knowledge granules. Center of a knowledge granule is where feature evaluation is at its peak.  $\gamma = 0.5$  means that the granule is symmetrical. Following formulas allow to elevate represented knowledge from fuzzy to granular model based on intervals:

$$a_i^- = a_i - \varepsilon * \gamma * |range_i| \quad (6)$$

$$a_i^+ = a_i + \varepsilon * \gamma * |range_i| \quad (7)$$

where  $a_i^-$  and  $a_i^+$  are lower and upper limits of the interval that represents the knowledge granule  $a$ . Augmentation from fuzzy to granular model can occur at various levels. In this study we elevate weights towards granular weights. As a result, the maximal length of the interval is 2.

In the case of Granular Cognitive Map quality assessment we are interested in coverage. Concept of coverage is informally illustrated in Figure 2. Granular map responses in Figure 2 are denoted as

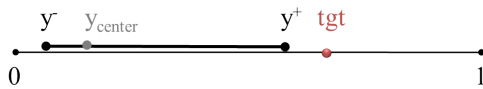


Figure 2: Coverage of a target by a granular map response.

$[y^-, y^+]$ . These are upper and lower limits of the interval, which represents the granule. In the Figure 2 target is not covered by the map response. We aim to construct such a map that produces outputs, which cover as many targets as possible.

A Granular Cognitive Map gives granular map responses that should cover real, observed values - targets. There are many possible definitions of coverage. We would like to discuss two most intuitive kinds of coverage for an interval-based GCM:

- weak coverage,
- strict coverage.

Weak coverage account all targets that fall into map responses. Weak coverage of an  $i$ -th node in  $k$ -th observation is calculated as:

$$covweak_{ik} = \begin{cases} 1, & \text{if } TGT_{ik} \in [y_{ik}^-, y_{ik}^+] \\ 0, & \text{otherwise} \end{cases} \quad (8)$$

Weak coverage for all targets is calculated as follows:

$$covweak = \frac{\sum_{k=1}^N \sum_{i=1}^n covweak_{ik}}{N * n} \quad (9)$$

Weak coverage is averaged by dividing by the number of observations and the number of nodes. It informs, to what extent the Granular Cognitive Map covered a single data point.

In contrast, strict coverage is increased only if all nodes within  $k$ -th observation were covered by map responses.

$$covstrict_{.k} = \begin{cases} 1, & \text{if } TGT_{.k} \in [y_{.k}^-, y_{.k}^+] \\ 0, & \text{otherwise} \end{cases} \quad (10)$$

Strict coverage requires that the whole column of targets is covered by map responses. Strict coverage of all targets is calculated as follows:

$$covstrict = \frac{\sum_{k=1}^N covstrict_{.k}}{N} \quad (11)$$

Strict coverage punishes columns that were covered incompletely. It is coverage criterion much harder to satisfy.

## 2.2 Methodology for Granular Cognitive Map Reconstruction

The developed procedure for Granular Cognitive Map reconstruction aims at finding such a map that maximizes coverage of targets by map responses with given constraints regarding granularity parameters. In this section we discuss the methodology of the GCM reconstruction. Moreover, we present a procedure for validation.

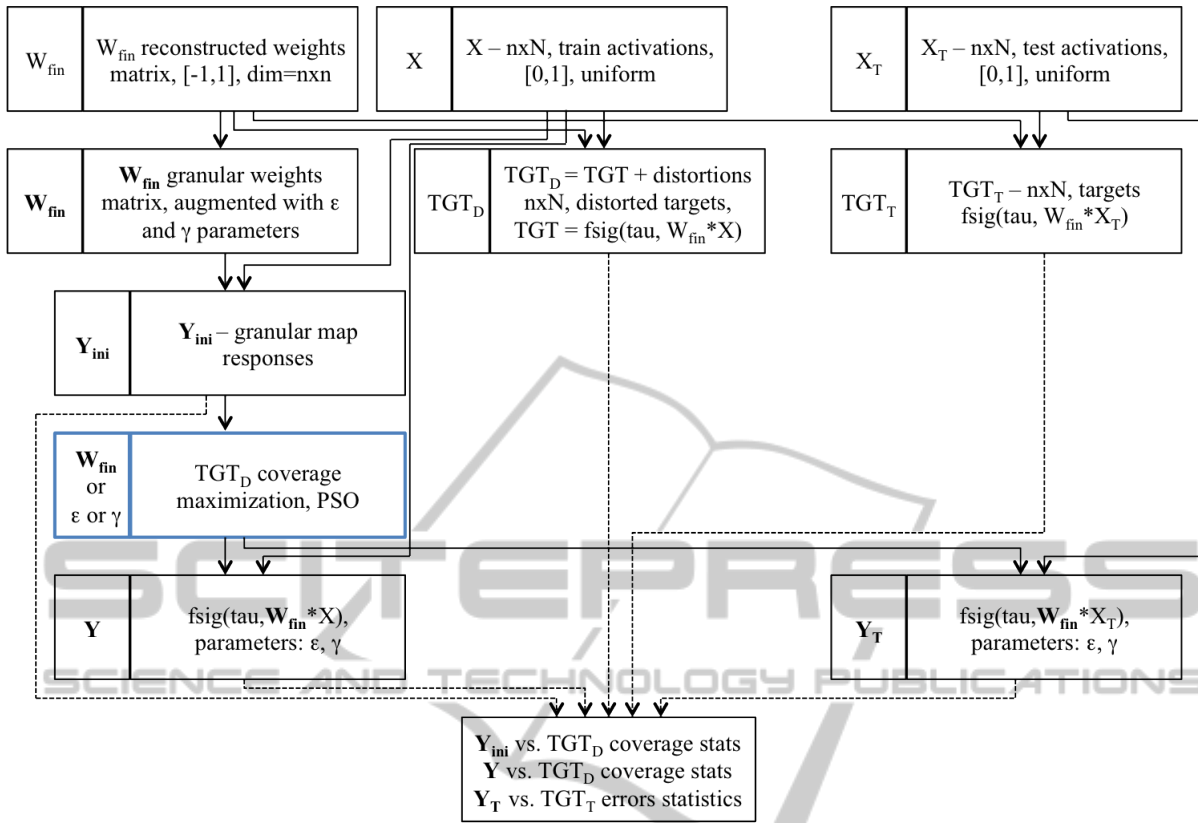


Figure 3: Granular Cognitive Map reconstruction procedure.

Let us recall that information granularity entails certain restrictions. The conflict between granule generality and specificity on one hand limits the model, but on the other hand it balances precision with versatility. In a Granular Cognitive Map based on intervals as granules representation scheme, generality/specificity is controlled with lengths of intervals.

Figure 3 summarizes the methodology of a GCM reconstruction and quality assessment.

The algorithm in Figure 3 in its full state reconstructs a Granular Cognitive Map and assesses its quality based on 3 datasets:

- train not distorted,
- train distorted,
- test.

Dataset denoted as not distorted train is an ideal, perfect, dataset, which is **never** available in real-life information processing. It is only available for experiments on artificially generated data. In this paper we introduce the developed methodology of GCM reconstruction. Therefore, we use the perfect train dataset for model quality assessment purposes.

The perfect data is in real life never available. System modeling methodologies take into account two

kinds of distortions: random and systematic. We assume existence of perfect data, but in fact we operate on distorted dataset. Therefore, the map is trained with respect to the randomly distorted train dataset.

Random distortions are introduced prior to the execution of the GCM reconstruction algorithm. Distortions are added to targets as random values from the normal distribution with standard deviation equal to 0.4. In consequence,  $TGT_D$  contains significant number of 0s and 1s, which cannot be model outputs with sigmoid function as in Formula 4. Due to asymptotic properties of sigmoid function we cannot expect the map to reach targets, which are equal to 0 or 1.

Test dataset is used to assess quality of the map.

At the start point in our algorithm we get pre-trained FCM with weights matrix  $W_{fin}$ . We have also activations  $X$  and distorted targets  $TGT_D$ . We elevate weights matrix  $W_{fin}$  to granular weights matrix, denoted with boldface font:  $W_{fin}$  using initial values of granularity parameters  $\epsilon$  and  $\gamma$ . Granularity of information is propagated, activations with granular weights produce granular map responses  $Y_{ini}$ .

At this point we have a Granular Cognitive Map that is not yet optimized. The optimization aims at coverage maximization and it can be performed

through adjustment of several parameters of the Granular Cognitive Map:

- weights  $\mathbf{W}_{fin}$ ,
- $\epsilon$  - single value for all weights or  $\epsilon$  matrix with adjusted value for each weight,
- $\gamma$  - single value for all weight or  $\gamma$  matrix with fitted value for each weight.

We may adjust simultaneously or successively one of the above elements or more. In Figure 3 the step of optimization is in the blue box. In this article we focus on adjustment of the granularity parameters:  $\epsilon$  and  $\gamma$ . We do not interfere with the weights matrix. Instead, we try to explore to the greatest extent the benefits of chosen granular knowledge granules representation model - intervals.

Coverage maximization task is computationally challenging. The optimization procedure has to independently adjust multiple parameters and the maximization criteria (see Formulas 11 and 9) are discontinuous. Therefore, we have applied particle swarm optimization method. PSO (introduced in (Kennedy and Eberhart, 1995) and (Shi and Eberhart, 1998)) does not require that the optimization problem be differentiable. It can search within a very large space of candidate solutions. The drawback of choosing a metaheuristics is that we do not have any guarantee that the optimal solution will be found.

In literature there is a discussion on practical aspects of optimization in Fuzzy Cognitive Maps learning and exploration, for example: (Papakostas et al., 2012), (Stach et al., 2005) and (Stach et al., 2004). The topic of Granular Cognitive Maps and optimization has not been yet researched and documented.

Optimized Granular Cognitive Map gives new, granular responses denoted as  $\mathbf{Y}$ .

The quality of the reconstructed Granular Cognitive Map is assessed on the three aforementioned datasets. We calculate coverage statistics with respect to all three datasets before and after the optimization:

- before optimization: coverage of  $TGT_D$  by  $\mathbf{Y}_{ini(tial)}$ , coverage of  $TGT$  by  $\mathbf{Y}_{ini}$  and coverage of  $TGT_{T(est)}$  by  $\mathbf{Y}_{T(est)ini(tial)}$ ,
- after: coverage of  $TGT_D$  by  $\mathbf{Y}$ , coverage of  $TGT$  by  $\mathbf{Y}$  and coverage of  $TGT_T$  by  $\mathbf{Y}_T$ .

### 3 EXPERIMENTS

In this section authors apply the proposed methodology in a series of experiments. Different approaches to Granular Cognitive Map reconstruction were tested and compared for the same map ( $n = 8$ ,  $N = 24$ , the

same  $X$  and  $TGT_D$  datasets). We reconstruct the GCM by adjustment of granularity parameters:  $\epsilon$  and  $\gamma$  for the interval-based representation of knowledge granules. Optimization procedure maximizes weak coverage defined in Formula 9. In this paper we adjust matrix of  $\epsilon$  and/or matrix of  $\gamma$ . Matrices contain separate parameters for each weight.

Please note that  $\epsilon_{ij} \in [0, 2]$ ,  $i, j = 1, \dots, n$ . 2 is maximal length of the interval for granular weights, for example in the case when granule center is in 0.  $\epsilon$  defines knowledge granule size.  $\gamma_{ij}$  are symmetry parameters and  $\gamma_{ij} \in [0, 1]$ ,  $i, j = 1, \dots, n$ . For  $\gamma = 0.5$  the granule is symmetrical and granule center is in the middle of the interval.

Results presented in this section allow to review the influence on  $\gamma$  parameter with restricted  $\epsilon$  on the coverage. We use common plotting scheme in each subsection. The most important aspect of this section is that as a result of optimization, we substantially increase coverage and maintain the same generality of the model.

In Figures in the following section, in each data point the total specificity of the map before optimization is the same as after the optimization. Thanks to the readjustment methodology we increase coverage and retain the same balance between specificity and generality. Such improvement is performed only by a manipulation with granularity parameters.

The particular GCM reconstruction methodologies applied and presented in this section are based on adjustment of:

- $\epsilon$ ,
- $\gamma$ ,
- $\epsilon$  and  $\gamma$  successively,
- $\epsilon$  and  $\gamma$  simultaneously.

Optimization was performed with PSO in R with default parameters. The number of iterations was set to 4000. Duration of experiments presented in this section was varying. A single experiment course for parallel optimization of 64 variables for 10 values of  $\gamma$  on a standard PC took approximately 15 hours.

The character of the aforementioned datasets is varied. It was already highlighted that the distorted train dataset - the one that is used for GCM training contains 0s and 1s. The model, due to asymptotic properties of the sigmoid function, cannot cover these values. It will be easy to spot that for the distorted train dataset coverage statistics are generally low. Not distorted train dataset is the „ideal” dataset, which describes perfect map responses. Test dataset contains separate values that are not related to training data in any way. Test dataset and „ideal” train dataset are

used to assess map quality. The higher the coverage, the better GCM responses cover targets.

### 3.1 GCM Reconstruction Through Adjustment of $\epsilon$

First possible scenario of optimization is adjustment of the  $\epsilon$  matrix. In this scenario other parameters remain without any modification, we focus on the basic granularity parameter: knowledge granule size. The optimization concerns  $n^2$  values of  $\epsilon$ , separate for each weight.

The optimization procedure was performed 11 times for varying values of the symmetry parameter  $\gamma$ , starting from 0 to 1 by 0.1. As a result we were able to plot results in 3D perspective. The plots are cut and rotated to illustrate outputs in the most convenient way. In each plot coverage is collated with  $\gamma$  and  $\epsilon$ . Values of  $\gamma$  can be read directly from  $\gamma$  axis. Values of  $\epsilon$  are illustrated in less straightforward way.

In the case, when we adjust  $\epsilon$  in 3D plots one can see coverage versus restricted  $\epsilon$ . This restrictions concern two aspects:

- upper limit of individual  $\epsilon$  values:  $\epsilon_{ij} \leq (2 * x)$
- sum of all  $\epsilon$ :  $\sum_{i=1}^n \sum_{j=1}^n \epsilon_{ij} \leq (n^2 * x)$

where  $x$  is on the  $\epsilon$  axis. Introducing named restrictions allows to manipulate with balance between specificity and generality. With the proposed procedure we benefit from the granular information representation model to the greatest extent, by maintaining flexibility of the phenomena description. The sum of all  $\epsilon$  describes generality of the whole map, while restrictions on individual  $\epsilon$  control specificity of a single information granule.

Figure 4 illustrates weak and strict coverage before and after adjustment of the  $\epsilon$  matrix. As one may expect, the highest values of coverage are for lenient limits on the  $\epsilon$ . The stronger we limit the generality criterion, the lower coverage we get.

The poorest results are for distorted train dataset. This is because  $TGT_D$  has a lot of 1s and 0s, which cannot be covered by the model.

Second important observation is that highest coverage for the training datasets is for symmetrical granules, around  $\gamma = 0.5$ . For the test dataset optimal value of  $\gamma$  is slightly smaller.

The methodology based on  $\epsilon$  matrix adjustment produces moderately good results. One can see that the optimization produced generally better models, but not in each case. For the „ideal” train dataset (first column) we have increased the coverage in almost each case. Similarly, strict coverage was im-

proved. The worst results are for strict coverage on distorted dataset.

### 3.2 GCM Reconstruction Through Adjustment of $\gamma$

In this approach we reconstruct the Granular Cognitive Map by adjustment of the alternative granularity parameter:  $\gamma$ . In consequence, we allow asymmetrical granular weights  $W_{fin}$ . Sizes of the intervals, which represent knowledge granules remain the same. We look for the optimal center of the granule and move proportionally left and right limits of the intervals.

The results: improvement in coverage after  $\gamma$  adjustment are illustrated in Figure 5. Plots in Figure 5 are not directly comparable with plots in the previous subsection, because optimization procedure allowed to modify the  $\gamma$  matrix.

This approach to GCM reconstruction gave satisfying results. Coverage has improved in each case. Noteworthy is very good coverage on not distorted train dataset and test dataset. Manipulation with symmetry parameters resulted high coverage for rigorous specificity criterion. The initial values of  $\gamma$  (on the horizontal axis) were reset. Chosen optimization algorithm (PSO) is well suited for our approach. Weak coverage for the same value of  $\epsilon$  on the ideal train dataset are similar for each initial value of  $\gamma$ . The procedure is stable and it produces comparable results.

From the theoretical point of view, adjustment of  $\gamma$  in the procedure of Granular Cognitive Map reconstruction plays secondary role. Symmetry parameter does not play any role in the most crucial problem of granularity: balance between generality and specificity. Nevertheless, it was shown, that adjustment of  $\gamma$  improves coverage. Therefore, methodology of Granular Cognitive Map reconstruction should take into account both granularity parameters and take full advantage of granular knowledge representation model.

Based on this premises, in the following subsections we present GCM reconstruction methodologies, that adjust both  $\epsilon$  and  $\gamma$  matrices.

### 3.3 GCM Reconstruction Through Successive Adjustment of $\epsilon$ and $\gamma$

In order to fully benefit from the assumed granular knowledge representation model, we propose an approach to Granular Cognitive Map reconstruction that is based on adjustment of both granularity parameters: granules size  $\epsilon$  and symmetry  $\gamma$ . Figure 6 illustrates differences in coverage prior and after the optimization of weak coverage.

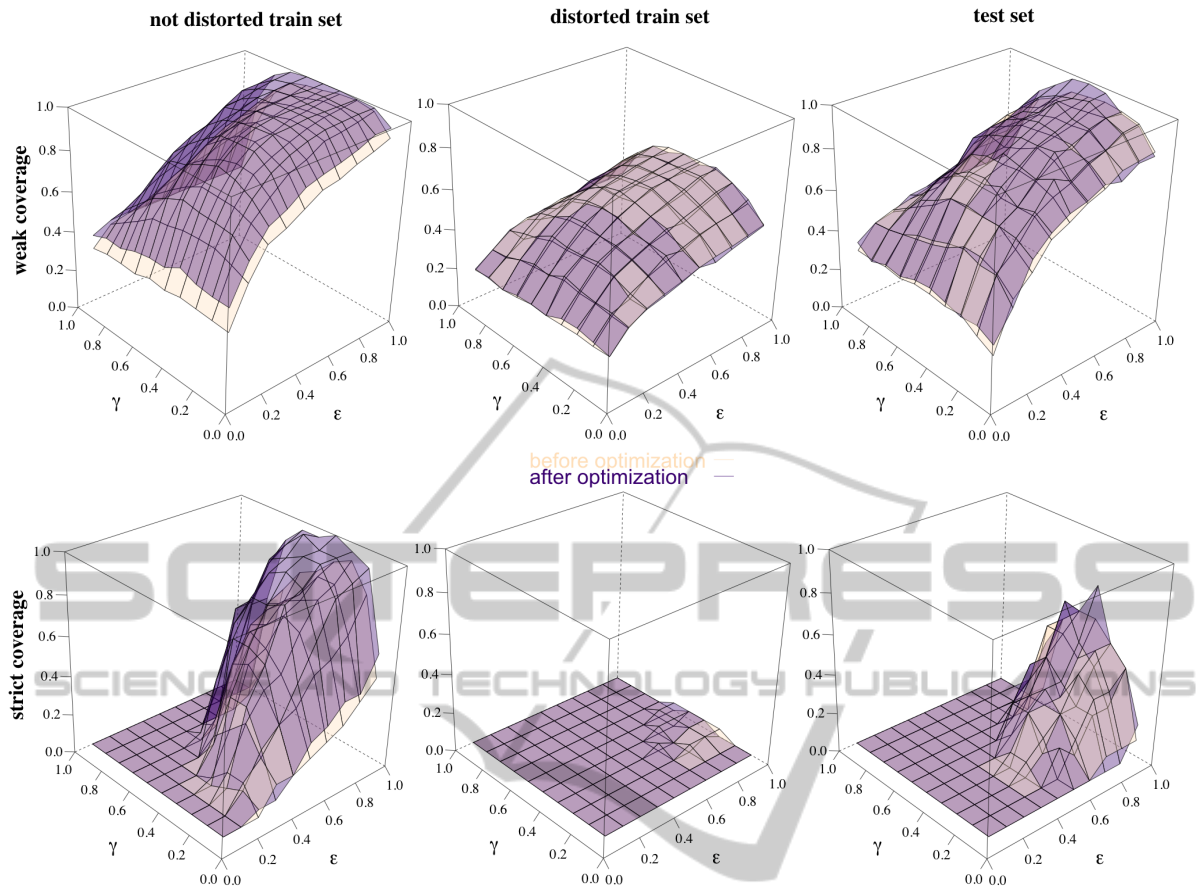


Figure 4: Weak (top row) and strict (bottom row) coverage on not distorted train dataset (first column), distorted train dataset (second column) and test dataset (third column) before and after adjustment of the  $\epsilon$  matrix.

In the GCM reconstruction scheme discussed in this subsection firstly we adjust the size of knowledge granules with respect to restrictions on the sum of all  $\epsilon$  ( $\sum_{i=1}^n \sum_{j=1}^n \epsilon_{ij} \leq (n^2 * x)$ ) and restrictions on the individual value of each  $\epsilon_{ij}$  ( $\epsilon_{ij} \leq (2 * x)$ ). Subsequently, values of  $\gamma$  are adjusted. In other words, in the first step we maximize coverage and maintain the balance between generality and specificity. Next, we tune the map by adjustment of granules symmetry.

Optimization procedure produced more accurate GCMs. Coverage on each dataset was better, than if we adjust only  $\epsilon$  matrix. In contrast, adjustment of only  $\gamma$  parameters gave similar results. For the largest values of granule size, weak coverage on the ideal dataset gets saturated to 1. Most importantly, coverage on the test dataset has improved. Presented approach gives satisfying results.

### 3.4 GCM Reconstruction Through Simultaneous Adjustment of $\epsilon$ and $\gamma$

The last approach to Granular Cognitive Map reconstruction, which we discuss in this article is simultaneous adjustment of both granularity parameters:  $\epsilon$  and  $\gamma$ . Please note that  $\epsilon$  values are limited by restrictions on the sum and on the individual value (as mentioned in the previous subsections). Figure 7 illustrates improvements in coverage provided by the GCM before and after the reconstruction procedure.

The proposed procedure produces better fitted Granular Cognitive Maps that do not loose prior balance between generality and specificity.

The GCM reconstruction strategy based on simultaneous adjustment of  $\epsilon$  and  $\gamma$  is very successful. Weak and strict coverage has improved in each case. On the „ideal” training dataset weak coverage reaches 1 even for strong restrictions on the size of knowledge granules. Weak coverage on the test dataset has improved as well. Strict coverage is much harder to obtain, since it requires all nodes in given observation to

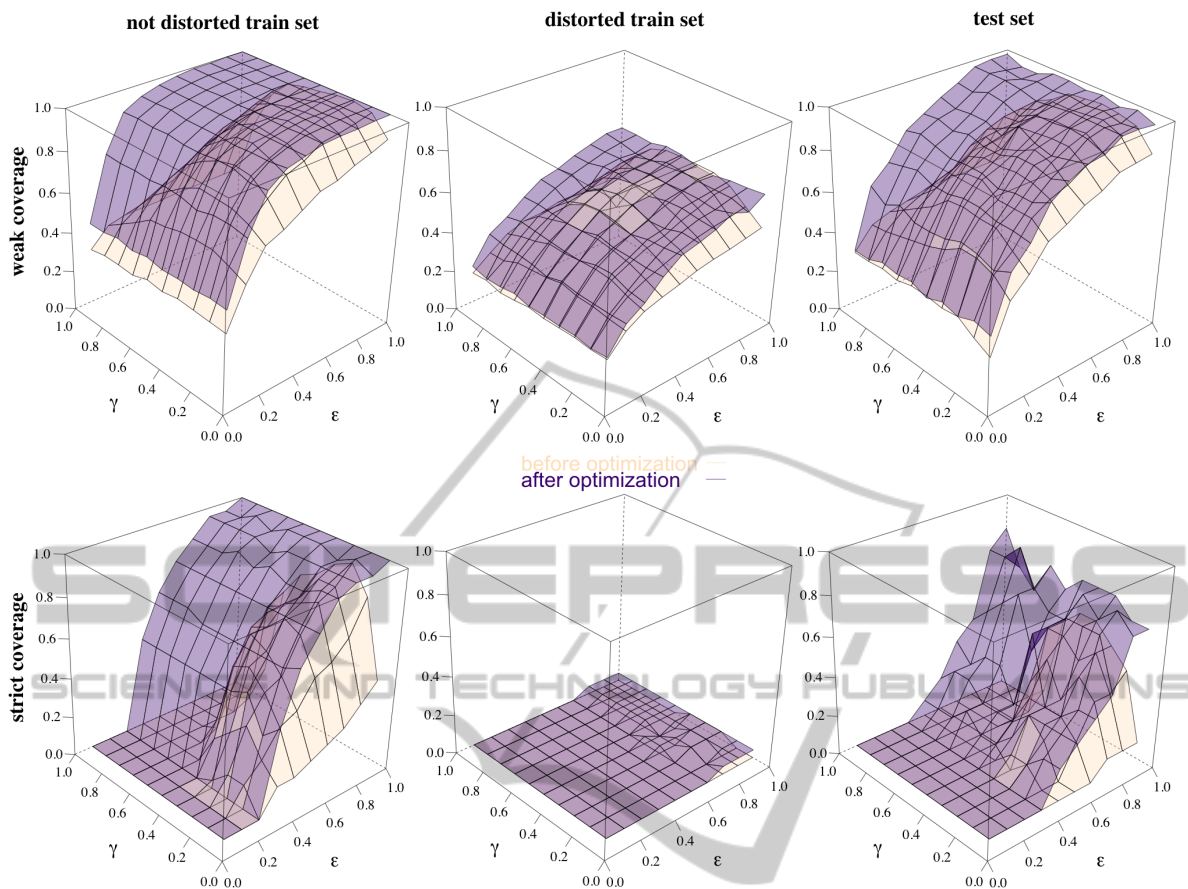


Figure 5: Weak (top row) and strict (bottom row) coverage on not distorted train dataset (first column), distorted train dataset (second column) and test dataset (third column) before and after adjustment of the  $\gamma$  matrix.

be covered by map response. Nevertheless, strict coverage is also satisfactory. The only dataset that has still rather poor coverage is distorted training dataset. This is no surprise though. Distorted dataset contains 0s and 1s, which cannot be covered by map response.

### 3.5 Comparison of Presented Approaches to GCM Reconstruction

In this subsection we summarize and evaluate applied approaches to Granular Cognitive Map reconstruction. Table 1 compares mean weak coverage on the three discussed datasets.

Table 1: Comparison of proposed approaches to Granular Cognitive Map reconstruction.

approach	mean of weak coverage		
	train ND	train D	test
adjustment of $\epsilon$	0.775	0.413	0.658
adjustment of $\gamma$	0.892	0.466	0.776
adjust. of $\epsilon$ then $\gamma$	0.894	0.456	0.761
adjust. of $\epsilon$ and $\gamma$	0.917	0.461	0.790

Table 1 illustrates differences in mean weak coverage for the 4 different approaches to Granular Cognitive Map reconstruction discussed in this paper. Values in Table 1 are means of weak coverage plotted in Figures 4, 5, 6 and 7. With this statistics we may quantitatively compare applied methodologies.

The most successful strategy of Granular Cognitive Map reconstruction adjusts  $\epsilon$  and  $\gamma$  simultaneously. In the last row in Table 1, mean weak coverages are the highest. The poorest results are when we adjust only the  $\epsilon$  matrix. Quality of Granular Cognitive Maps reconstructed by adjustment of  $\gamma$  matrix and by successive adjustment of  $\epsilon$  and the  $\gamma$  is similar.

Coverage is the worst for the distorted train dataset. This is because of specifics of this dataset (it contains 0s and 1s). We have chosen such training dataset on purpose. As a result, the reconstructed GCMs are not overfitted. The developed methodology for experiments with GCM reconstruction was properly constructed, as it produced high coverages for the „ideal” dataset and for the test dataset.

The proposed approach was focused on Granular



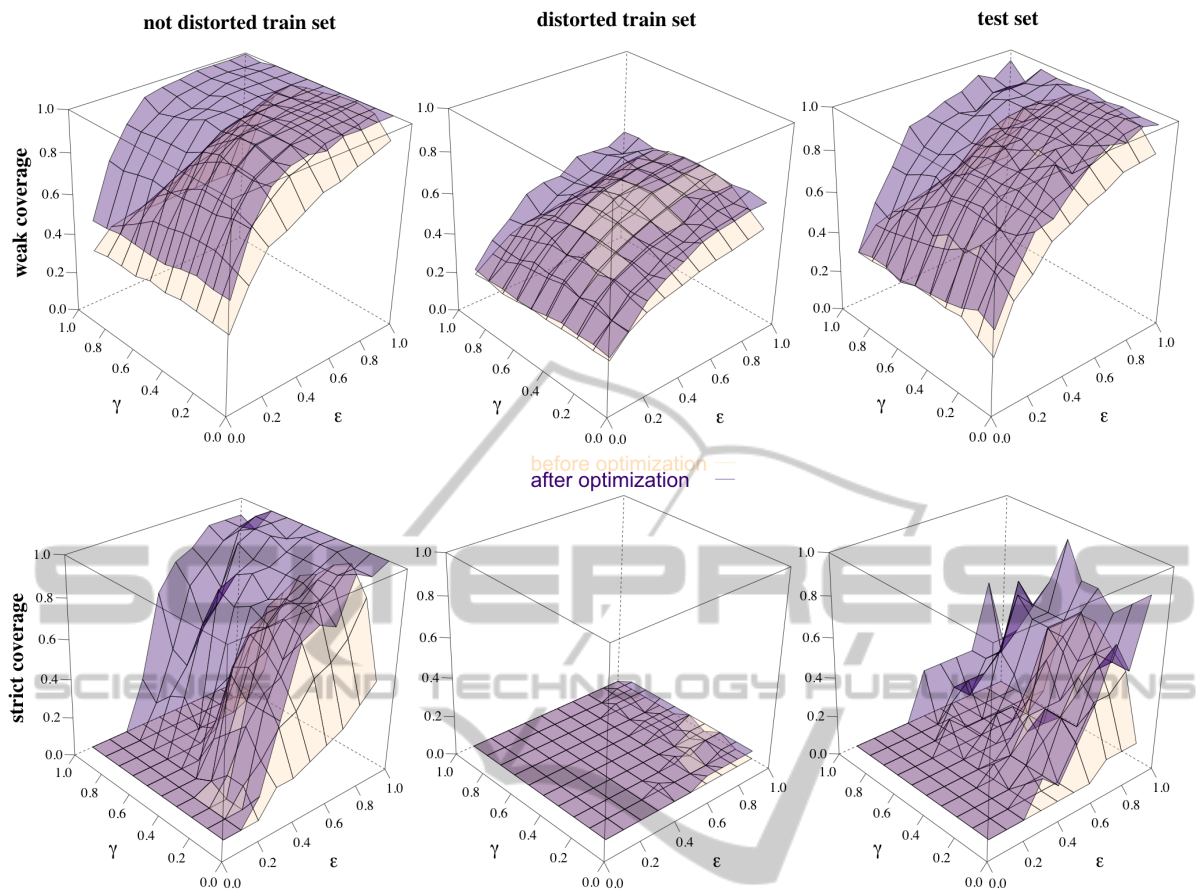


Figure 6: Weak (top row) and strict (bottom row) coverage on not distorted train dataset (first column), distorted train dataset (second column) and test dataset (third column) before and after successive adjustment of  $\epsilon$  and then  $\gamma$ .

Cognitive Map reconstruction with given restrictions on the generality/specificity balance criterion. The aim of the discussed strategies of optimization was to produce better-fitted Granular Cognitive Maps that do not lose prior balance between generality and specificity. Such requirements were implemented through explicit limitations on values of individual  $\epsilon$  and on sum of all  $\epsilon$ . As outcome, reconstructed maps cover more targets, but maintain required balance that determines their precision.

## 4 CONCLUSIONS

In the article authors have proposed a general methodology for Granular Cognitive Map reconstruction. Presented approach has been concertized with 4 distinct GCM optimization schemes. We have discussed and compared Granular Cognitive Map reconstruction procedures that adjust granularity parameters. Theoretical assumptions of the proposed methodology focus on most important indicators of granularity:

balance between generality and specificity.

The paper is supported by a series of experiments, which illustrate results of the proposed Granular Cognitive Map reconstruction procedures. We showed that proper optimization schemes allow increasing coverage and maintaining the same generality of a whole map. GCM reconstruction strategy that produces the most accurate model - a map that covers the greatest number of targets is simultaneous adjustment of both granularity parameters:  $\epsilon$  and  $\gamma$ . Discussed approaches of  $\epsilon$  and/or  $\gamma$  optimization take full benefit of granular knowledge representation model.

The objective of the research discussed in this article is to propose appropriate methodologies for Granular Cognitive Maps training and exploration. In future research authors plan to investigate other knowledge granules representation models, most importantly fuzzy numbers and bipolar knowledge representation schemes.

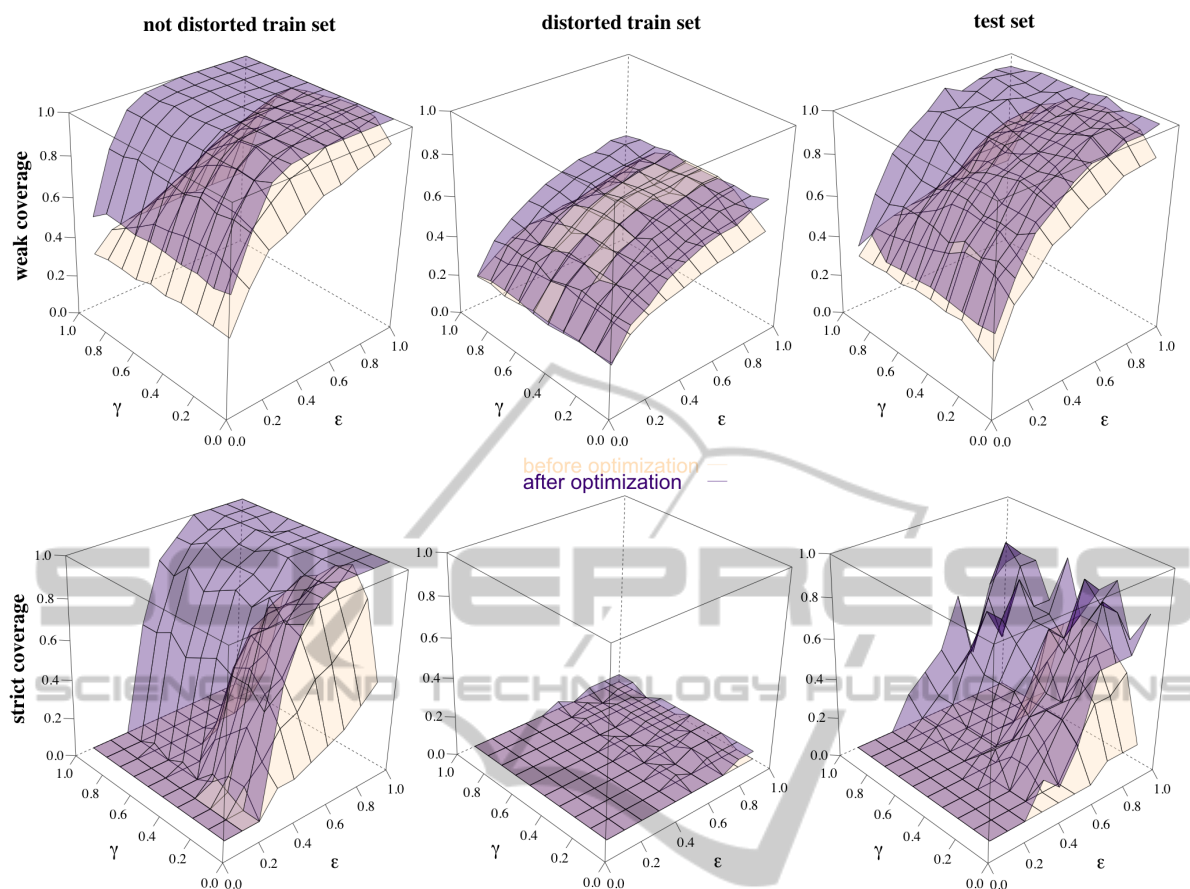


Figure 7: Weak (top row) and strict (bottom row) coverage on not distorted train dataset (first column), distorted train dataset (second column) and test dataset (third column) before and after simultaneous adjustment of  $\epsilon$  and  $\gamma$ .

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