The Uncertainty in the Home Health Care Assignment Problem

Afrae Errarhout[1], Saïd Kharraja[2] and Andréa Matta[3]
[1]*Laboratoire d'Analyse des Signaux et des Processus Industriels, Roanne, France*
[2]*Université Jean Monnet, Saint-Etienne, France*
[3]*Dipartimento di Meccanica, Politecnico di Milano, Milano, Italy*

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Abstract: This paper presents an assignment problem in the home health care structures. In this problem, we search to assign caregivers to patients during a mid-term and long-term planning horizon while considering the caregivers' skills and capacity. Moreover, we take into account the randomness of the patients' demands due to a change in their profiles or to addition of new patients through the planning horizon. As aim we balance the caregivers' workload and secure the continuity of the care. We use the Monte Carlo method in a deterministic way to represent the randomness of the patients' demand in the mixed integer programming model we developed.

1 INTRODUCTION

The home health structures (HHC) were created in order to reduce the hospitals charges (Jones et al.1999). Hence, it provides medical, psychological and social services to the patients at home among their families. During the last decade, the importance grow of the HHC structures allowed them to add more complex and technical cares such as chronic and palliative cares, home chemotherapy and rehabilitation (Tarricone and Tsouros, 2008); (Durand et al., 2010).

Given the standing of the HHC organization, several studies related to the operations management and the organization of the care delivery process were developed. Different works could be found in the literature, some dealing with main processes and decisions related to home care structures for a better understanding of the HHC operations and their management (Matta et al. 2012) and others related to the human resources planning such as: districting problems (Benzarti 2012), routing and scheduling problems (Hertz and Lahrichi, 2009) and the assignment problems (Lanzarone et al., 2012); (Matta et al., 2012); (Yalcindag et al., 2012); (Hertz and Lahrichi, 2009); (Lanzarone and Matta, 2009); (Bredström and Rönnqvist, 2008); (Punnakitikashem et al., 2008).

This paper deals with the assignment problems in the HHC. The works related to the assignment in the HHC structures help the decision maker to know the best way to allocate nurses to patients while securing the nurses' workload equilibrium, which insure the cares' quality and reduce the costs of the nurses' tours. Each patient represents a therapeutic project composed of different cares during his/her treatment. However, while assigning nurses to patients, we should take into account the nurses' capacity. It's important to prevent the nurses' overload which will cause fatigue and stress which could be followed by errors. Thus, workload smoothing would secure the quality of cares and enhance patients' satisfaction.

The HHC assignment problems present a great uncertainty (Lanzarone and Matta, 2012; 2009). In fact, for example, through the planning horizon, there is a great variability in the patients' demand; it may be explained by the admission of new patients or the change or evolution in patients' profile after a short/mid/long period of the treatment beginning. The patients' demand represents the risk factor in these problems as having overloaded nurses and adding some nurses' tours or reducing them which involve additional costs. Hence, we considered that the demand prevision would help to reduce this risk. However, although the prevision provides an overview of the future situation, it doesn't annihilate the uncertainty (Kouvelis and Yu, 1997) in the problem.

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Therefore in this paper, we present an assignment problem. We assign nurses to patient taking into account the variability of the patient demand due to their profile's change or due to new patients' admission through the planning horizon which ensure the care's continuity. The objective is to "master the overload risk" by balancing the nurses' workload in order to reduce the excessive assignment. We assume that the nurses can operate in the whole territory but we limit the number of the districts where they can be assigned and so we reduce their travel workload. The nurses must have the skills required to the care they are assigned to and there're some care that require the presence of two nurses, called synchronized care (Bredström & Rönnqvist 2008).

The article is organized as follow. We introduce in section 2 a literature review about the assignment problems in the HHC. In section 3, we define the problem and present the mathematical model. Then, Section 4 reports results from computational experiment and Section 6 concludes the paper.

2 LITERATURE REVIEW

Different fields of studies deal with the assignment problems, such as, the production systems (Bilgin and Azizoglu, 2009), the telecommunication networks (Dell'Amico et al., 2001), the resources planning (Mkaouar et al., 2012), the health care (Volgenant, 2004), etc. The assignment problem was introduced, for the first time, in the fifties by Votaw and Orden (1952). This problem searches to assign one task per agent. Then in 1975, Ross and Soland (1977) introduced the generalized assignment problems (GAP) which allocate many tasks to set of agents while respecting their capacity occurred. From the GAP many other problems emerged depending on the situation to model. The complexity of the GAP is NP-hard (Diaz and Fernandez 2001); (Yagiura et al., 2006); (Woodcock and Wilson, 2010) which conducted to develop many resolution methods exact and based-heuristics approaches. Several methods and heuristic algorithms have been presented in the literature to solve the GAP as the genetic algorithm (Liu et al., 2012), the Tabu search (Diaz and Fernandez, 2001), simulated annealing (Righini 1995), ant colony, local search (Bischoff and Dächert, 2009) and ejection chains (Yagiura et al., 2006). More recently, several researchers develop hybrid heuristics (Woodcock and Wilson, 2010). These are to combine different heuristics or combine elements of exact methods with heuristics.

In the HHC context, as mentioned previously, there're different works related to the human resources planning. The districting problems consider the territory's repartition into districts in order to reduce the nurses' workload and travel workload; Benzarti (2012) developed two mathematical models of the districting problem. The author considers the compactness, the care workload balance and different patient profiles. The visits' scheduling problem search to reduce the nurses' travel during their visits; Ben Bachouch et al., (2008) developed mixed linear programming model of vehicle routing problem with time windows to minimize the total distance travelled by the nurses.

The assignment problem (see table 1) seeks to allocate nurses to patients while considering their skills and workload balance and ensure the continuity care. Lanzarone et al., (2012) proposed different mathematical programming models with the aim to balance the workload of the operators within specific categories. These models consider the care's continuity constraint, operator's skills and the districts where the patients and the operators belong. The patients' demands are considered either in deterministic or stochastic way.

Hertz and Lahrichi (2009) developed two mixed integer programming models. They solved the model with non-linear constraints and a quadratic objective function using a Tabu search algorithm and they used CPLEX to solve the other linear model. By comparing the two solution methods they confirmed the effectiveness of the Tabu search approach. They aim to balance the nurses' workload within different categories.

Yalçindag et al., (2012) coupled the assignment and routing problems in the HHC structures. They focused on the interaction between assignment and routing, where the output of the assignment problem is incorporated as an input into the routing problem, with the assumption of one district.

Lanzarone and Matta (2012) developed a structural policy to assign a newly admitted patient while balancing the operators' workload by minimizing the cost function that penalizes the operators' overtime. They consider that the patients' demands are either deterministic or stochastic.

Bertels and Fahle (2006) present a combination of linear programming, constraints programming and (meta) heuristics for a HHC problem that consider the staff rostering and vehicle routing components while minimizing transportation costs and maximizing satisfaction of the patients and nurses.

Lanzarone and Matta (2009) present an integer programming model for workload balancing among

Table 1: The main literature on HHC assignment problems.

Articles	Objective	Constraint / Creteria
Lanzarone et al (2012)	• Allocating operators to patients • Balancing the operators' workload (visit time)	• The patients' demands are: deterministic or stochastic • Continuity of care • Operators' skills • The districts where the patients and the nurses belong
Hertz & Lahrichi (2009)	• Allocating operators to patients • Balancing the operators' workload	• Continuity of care • Visit load (the weight of each visit) • Travel load (the distance traveled) • Case load (the number of patients assigned) • The operators' capacity
Yalçindag et al (2012)	• Analysing the interaction between assignment problem and routing problem • Balancing the operators' workload (visit & travel time)Minimizing the total distance travelled	• The districts where the patients and the nurses belong • Continuity of care • Operators' skills
Bertels & Fahle (2006)	• Designing rosters with staff rostering and vehicle routing components • Minimizing transportation costs • Maximizing the patients and the nurses' preferences	• Time windows • The qualification for a job • The nurses and patients' preferences

Table 2: Literature about the risk & uncertainty in health care problems through time periods.

Articles	Objective	Constraint / Creteria
Benzarti (2012)	• Partitioning of the area where the HCC structure operates into districts	• The workload balance • The compactness, • The compatibility and indivisibility of basic units • The continuity of care
Lanzarone & matta (2009)	• Evaluating the Expected Value of Perfect Information (EVPI) • Allocating operators to patients in the HHC • Balancing the operators' workload (visit time)	• Patient demand variability • Stochastic demand • Continuity of care • Operators' skills • The districts where the patients and the nurses belong • Sub-periods
Punnakitikashem et al. (2008)	• Assigning nurses to patients in health care	• Shift (time periods) • Nurses qualification • Continuity of care • Direct care • Indirect care (e.g. updates of the patients' condition) • Unanticipated patients
Lanzarone & Matta (2012)	• Assign newly admitted patients • Balancing the operators' care workload (visits time) • Minimizing the operators' overtime penalty	• Continuity of care • Operators' skills • The districts where the patients and the nurses belong • The patients' demands are: deterministic or stochastic
The work presented in this paper	• Assigning nurses to patients in the HHC structures. • Balancing the nurses' workload	• Continuity of care • Uncertainty of the patients' demand • Districts • Synchronized cares • The travel load • The nurses' skills • The nurses' capacity • Sub-periods

nurses, while preserving the continuity of care. They take into consideration the high variability of the patient demand and they assume that the future demand to be stochastic.

Punnakitikashem et al., (2008) develop a stochastic integer programming model to assign nurses to patient, they proposed the Bender' decomposition to solve the problem and they employed a greedy algorithm to solve the recourse sub-problem.

The uncertainty in the future makes the decisions' project, in long term, difficult. Hence, the best way to handle the uncertainty and make decision under it, is to accept it, make strong effort to understand it and structure it, and finally, make it part of the decision making reasoning (Kouvelis 1997). Table 2 summarizes some works about the risk and uncertainty in the health care field.

In this paper, we present an assignment problem where we assign nurses to patients with the objective is to balance the nurses' workload in order to reduce the excessive assignment. The nurses can operate over the whole territory. We consider the variability of the patients demand due to their profile's change or due to new patients' admission through the planning horizon while securing the care's continuity. We use the Monte Carlo method in a deterministic way to represent the randomness of the patients' demand in our integer programming model.

3 MODEL DESCRIPTION

The following assignment model objective is to balance the nurses' workload while considering their skills to give each care and their capacity. During the planning horizon, the patients' profile may progress/regress and/or new patients may be admitted within the HHC structure, which require a dynamic model taking into consideration those changes. Therefore the planning horizon is divided to several sub-periods, weeks in our case. The patients are characterized by the weekly number of visits needed during their treatment, while the nurses are defined by their weekly workload that may change in dependency on the admission or the leave of the patients.

The patients' demands through the planning horizon (sub-periods) are random, which represents an uncertainty factor in our work and may conduct to irrelevant results. On that account, we used a range of estimated values instead of a single guess in order to create more accurate model. Hence, we choose to use the Monte Carlo simulation (Vose 2008) which tells how likely the outcomes are, so, we can have a better understanding of the uncertainty and the risk in the model.

Indeed, we use the Monte Carlo simulation (Vose, 2008) in a determinist way to the mixed integer model. Hence, we add a number of scenarios to the model by considering randomness of the patients' demand for each week.

The mathematical model is presented as follow:

Decision variables:

- $X_{kpji} = \begin{cases} 1 \text{ if the nurse i is assigned to the care j of the new patient p in the district k} \\ 0 \text{ otherwise} \end{cases}$
- $Y_{ik} = \begin{cases} 1 \text{ if the nurse i is assigned to the district k} \\ 0 \text{ otherwise} \end{cases}$
- $W_{i,f}^{h}$ the nurse i workload in the week h of the scenarios f
- $D_{ii'}^{fh}$ the difference between the nurse i and i' workload in the week h for each scenario f.

Parameters:

- m the number of nurses I = {1,.., m}
- n the number of cares J = {1,…,n}.
- v the total number of the patients during the planning horizon P = {1,…, v}. This set is the sum of the groups of patients during each week t.
- z the number of districts Z = {1,…, z}
- t the number of weeks in the planning horizon T= {1,…, t}
- s the number of scenarios F={1,…,s}
- P_{kpj}^{fh}= 1, if the patient p of the district k needs the care j during the week h for each scenarios f, otherwise 0.
- S_{kpj}^{fh} = 1, if the patient p of the district k needs a care j that requires one nurse during the week h for each scenario f.
- S_{kpj}^{fh} = 2, if the patient p of the district k needs a care j that requires two nurses during the week h for each scenarios f.
- N_z, the limited number of district where a nurse can be assigned.
- $\propto_{kpj,f}^{h}$, the frequency of the cares of the patient p of the district k during the week t for the scenarios f.
- q_{kpji}= 1, if the nurse i has the skill to give the

care j to patient p of the district k, otherwise 0.

- b_i, the capacity that defines the nurse availability.
- $\tau \in [0,1]$ which represents the travel time ratio in the nurses' availability b_i
- a_{kpji}, the time required by the care j of the patient p of the district k from the nurse i.
- t_{jp}, is the average time for the care j to be provided to patient p
- M, a great constant

The mathematical formulation of the problem is given below:

Objective functions:

The model allows balancing the workload among the nurses in the planning horizon by minimizing the difference between the nurses' weekly workload.

Minimize

$$1/_{s}\sum_{f=1}^{s}\sum_{h=1}^{t}\sum_{i=1}^{n}\sum_{\substack{i'=1\\i'\neq i}}^{n} D_{ii'}^{fh} \tag{1}$$

The constraints' set:

$$\sum_{i=1}^{m} P_{kpj}^{fh} q_{kpji} X_{kpji} = S_{kpj}^{fh} \quad j \in J, p \in P, k \in Z, h \in T, f \in F^{s} \tag{2}$$

$$X_{kpji} \leq q_{kpji} x\, P_{kpj}^{fh} \quad i \in I, j \in J, p \in P, k \in Z, h \in T, f \in F^{s} \tag{3}$$

$$W_{i,f}^{h} = \sum_{k=1}^{z}\sum_{p=1}^{v}\sum_{j=1}^{n} P_{kpj}^{h} \propto_{kpj,f}^{h} a_{kpji} X_{kpji} \quad i \in I, h \in T, f \in F^{s} \tag{4}$$

$$\sum_{h=1}^{t} W_{i,f}^{h} \leq (1-\tau) * b_i \qquad i \in I, f \in F^{s} \tag{5}$$

$$D_{ii'}^{fh} \geq W_{i,f}^{h} - W_{i',f}^{h} \qquad i \in I, i' \in I, h \in T, f \in F^{s} \tag{6}$$

$$D_{ii'}^{fh} \geq W_{i',f}^{h} - W_{i,f}^{h} \qquad i \in I, i' \in I, h \in T, f \in F^{s}) \tag{7}$$

$$\sum_{k=1}^{z} Y_{ik} \leq N_z \qquad i \in I \tag{8}$$

$$\sum_{j=1}^{n} X_{kpji} \geq 0 \quad i \in I, p \in P, k \in Z \tag{9}$$

$$\left\lceil \frac{\sum_{p=1}^{v}\sum_{j=1}^{n} X_{kpji}}{M} \right\rceil = Y_{ik} \quad i \in I, k \in Z \tag{10}$$

The constraints (2) and (3) assume respectively that a same care can be given by one or two nurses and that a nurse should be qualified to give the care. Constraints (4) define the workload of each nurse during the week t for each scenario. The constraint (5) secures the nurses availability. Constraints (6) and (7) define the difference between the nurses workload to be minimized in the objective function. Finally, constraints (8), (9) and (10) limit the number of districts for the nurses.

4 EXPERIMENTATION AND COMPUTATIONAL RESULTS

To test our integer programming model, we used the optimization software ILOG CPLEX 12.5 and the experiments were implemented on 3.39 GHz, 3GB RAM computers.

4.1 Data Generation

Our model takes into account the uncertainty of the patients' demand. So, we used a real life data from the health home care benchmarks (Howard, 1997). We have 75 patients through the horizon planning (8 weeks) and 6 nurses. The HHC proposes 10 cares that request a time varying from 25 to 45 minutes and each nurse is qualified to give at least 4 types of cares. The travel time is about τ=30% of the whole time capacity of the nurses (each nurse works 8 hours per day and a week =5 days). The patients' demand is represented by the frequency of the cares they need, 3 to 8 visits per week. For each patient the frequency may change from a week to another over the scenarios.

In order to generate scenarios, we suppose that the patients' demand follow a normal distribution (i.e. we consider the deterministic demand as the mean and the standard deviation is represented by the root square of the deterministic demand (Verweij et al. 2003), see figure 1).

4.2 Computational Results

In order to test the efficiency of our model, we generated 3 tests of random demands starting with 5 scenarios, 10 then 20 (i.e. each scenario represents a number of the patients' demand that was generated from the normal distribution, see fig. 1) and we compared their results with the results from the deterministic demands. For all the tests the program runs over 3 hours before running out of memory without finding new solutions. So, we terminated the program after 3 hours of running to get the latest solution found. We report in the follow table the results found for all the tests (see table 3).

Table 3: The results of the experiments showing the average workload per scenario and per nurse, and the Minimum and maximum workload.

Tests	Avg. workload	Avg. nurses workload	Min. workload	Max. workload
5 scenarios	68725	11454	9479	12722
10 scenarios	69201	11534	11259	11618
20 scenarios	69004	11501	11286	11687
Deterministic	-	11114	11080	11155

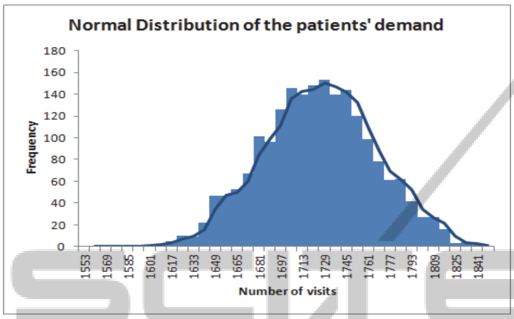

Figure 1: The normal distribution of the patients demand.

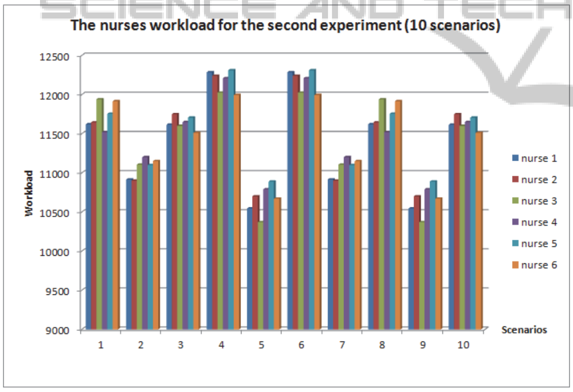

Figure 2: The nurses' workload for the second experiment "10 scenarios.

The results obtained by the experiments are different from each other and we notice that these results converge while increasing the numbers of scenarios. So in order to get better result it's more appropriate to use a large number of scenarios. These scenarios may represent the worst case, the best case and the most likely case. Hence, we notice that the workload from the deterministic demand to the 5, 10 and 20 scenarios increases. Moreover, the comparison between the results obtained by using different scenarios and the results of the deterministic demand shows a great difference, which concludes that the resulting assignment is far away from being accurate (see table 1). Furthermore, the experiments show that the nurses' workload is balanced over all the scenarios (see fig. 2).

5 CONCLUSIONS

In this paper, we modeled the assignment problem in the HHC structures. The main objective is to balance the nurses' workload and define the variability in the problem that's caused by the randomness of the patients demand and the changes in the patients profile every week represent an uncertainty factor. Hence, in order to represent the variability in our model we used the Monte Carlo method which consists of generating a great number of scenarios that represent the randomness of the patients' weekly demand. We tested our mixed integer model on CPLEX with different experiments that use a number of scenarios and the results shows a big difference between the test using the deterministic demand and the ones using a random demand. This difference illustrates the uncertainty in the assignment problem and gives more accurate solutions.

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