Optimal Product Line Pricing for Two Customer Segments with an Extension to Multi-Segment Case

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Abstract: In this paper we consider the problem of determining optimal prices for a product line. Items are distinguished by a single attribute which we call quality and which is proportional to the cost of the item. Demand for an item is dependent on the price differential between the item and the next item with higher cost. Customers can be grouped into two segments based on the lowest features acceptable and the maximum acceptable price. We develop an algorithm to determine the optimal pricing to maximize profit. We also consider assortment decisions to add or drop items based on regularity conditions and optimality considerations.

1 INTRODUCTION

The existence of consumers with vast heterogeneity in tastes has made it very common for firms to offer multiple items with correlated demand, often called a product line ((Shugan(1984))). A product line is a set of items that cater to essentially the same customer need, but that differ from each other due to either the existence or non-existence of a feature; or variation in performance with respect to some measure. A product line consists of many individual items, which are referred as variants or items or products. For consistency, we use the term 'items' throughout this paper.

(Monroe(1990)) states that within the domain of pricing strategy, product line price setting is the most complicated decision area. Product line decisions are difficult to make because the items in the line are not usually independent. Substitution patterns of items in a product line play an important role in this regard.

Some mathematical models for pricing product lines that take the inter-item dependencies into consideration have been developed like the model by (Shugan and Desiraju(2001)). The authors assume that the product line attracts a homogeneous set of customers where all of them treat the product line assortment alike. But usually, within a product line, different items attract different classes of customers based on customer preference. These heterogeneity among customers can be better modeled by considering several customer segments.

In this paper, we extend the work of (Shugan and Desiraju(2001)) to solve the optimal product line pricing problem with multiple customer segments. The presence of multi-customer segments adds many levels of complexity to the mathematical analysis. First, we present analysis for two customer segments and devise an optimal algorithm for pricing. This is then extended to multiple customer segments. We develop important managerial insights on the 'best' items to add to a product line and 'best' items to drop from a product line.

2 LITERATURE REVIEW

Marketing literature in the managerially important area of product line pricing strategies is relatively sparse due to the interdependencies of the optimal prices and demand of items in a product line. In his seminal work on interdependencies in a product line, (Urban(1969)) develops and tests a mathematical model encompassing the major factors and market phenomena affecting the problem of finding the best marketing mix for a product line. (Palda(1969)) was amongst the first to consider individual item prices simultaneously and his model used interrelated demand functions. (Little and Shapiro(1980)) were the first researchers to demonstrate the necessity of a nonlinear sales 'response' function in the context of pricing a product line in supermarkets. Cross-elasticity terms were explicitly considered while pricing each item in the line by (Reibstein and Gatignon(1984)). (Lilien et al.(1992)Lilien, Kotler, and Moorthy) and (Yano

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and Dobson(1998))present comprehensive discussion of marketing models for product line selection.

(Blattberg and Nelsin(1990)), (Levy and Weitz(2004)), (Shugan(1984)) and (Zenor(1994)) have emphasized the importance of an item's price on its item's profits and the profits of other items. (Blattberg and Wisniewski(1989)) show that high-priced brands compete among themselves and with low-priced brands. Within retail product lines, high quality/price brands tends to steal sales from low quality/price brands but converse is not true ((Mulhern and Leone(1991)), (Sivakumar and Raj(1997))). Considering these results (Shugan and Desiraju(2001)) developed a mathematical pricing model when items exhibit either symmetric or asymmetric competition and discuss the implications of asymmetry. They also provide guidelines for changes in pricing strategies when costs or line composition changes. (Moorthy(1984)) showed that the effect of customer self selection leads to competition within the firm's own product line such that, the optimal product and price cannot be determined separately for each segment. Our work seeks to create such a methodology that considers product-line pricing in the context of multiple customer segments.

3 PROBLEM DESCRIPTION

In this paper we consider a vertically differentiated product line. The demand for any item in the line follows the distribution function given by (Shugan and Desiraju(2001)) who consider a product line with V items and single customer segment. Let i = $1, 2, \ldots, V$ denote the items which cost the firm $c_1, c_2, \ldots, c_V$ such that $c_i < c_j$ for all items $i < j$. Then the demand of the i^{th} item is given by

$$D_i = \begin{cases} M\ (p_{i+1} - p_i) & 1 \leq i < V, p_i < p_{i+1} \\ M\ (\theta - p_i) & i = V, p_i < \theta \end{cases} \quad (1)$$

where, p_i is the price of the i^{th} item, M is a measure of aggregate demand and θ can be interpreted as the maximum reservation price of the customer segment of this product line. The reservation price is the maximum amount any customer is willing to pay.

Since positive demand requires that $p_i < p_j$ for all items $i < j$ they propose a sufficient condition, which they call regularity condition. Regularity requires: $c_i < A\ \forall\, i = 1, 2, ..., V$, where A is the adjusted average cost of the line given as

$$A = \frac{\sum_{k=1}^{V} c_k + \theta}{V + 1} \quad (2)$$

Then the optimal price p_i^* for item i is given by $p_i^* = \sum_{k=0}^{i-1} (A - c_k)$, where $c_0 = 0$.

In this model, an item competes for customers with the items that are priced immediately above it. The demand for a particular item along the product line can then be said to be driven by its price difference with respect to the next higher priced item.

Next consider two customer segments that are distinguished by two parameters. First, there is a segment specific reservation price that limits the items that customers in that segment can purchase. Second, each segment has minimum quality/attribute requirements that limits from below the items that customers in that segment are willing to purchase. The potential consideration set for customers in each segment is, therefore, bracketed from below and above. The actual consideration set for each segment, of course, depends on the prices that the firm sets. Let,

N_1, N_2: customer population of the two segments
u_1, u_2: index of the lowest acceptable item for the two segments
H_1, H_2: consideration set for the two segments without price
θ_1, θ_2: reservation price of the two customer segments such that $\theta_2 > \theta_1$
a: costliest item available to customer segment 1 for purchase or reservation price boundary item for customer segment 1
$V_1 = \underset{i=1,V}{\text{argmax}}\{c_i < \theta_1\}$

Then the potential consideration sets for the two customer segments are given by $H_1 = \{u_1, \ldots, V_1\}$ and $H_2 = \{u_2, \ldots, V\}$.

Consider the most general case in which $u_2 < V_1$. Also let $W(a) = \{u_1, \ldots, a\}$ be the set of items that the firm, through pricing, makes available to segment 1. Then the pricing structure for the product line can be represented as shown in figure 1.

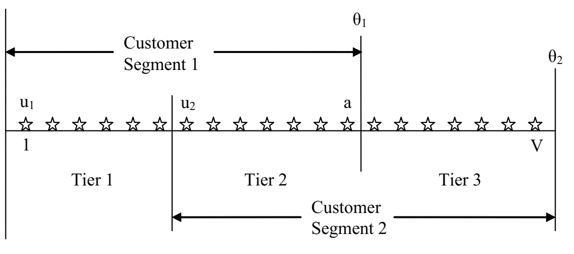

Figure 1: Schematic representation of product line Pricing for Two Customer Segments .

The solution of the optimal product line pricing problem breaks up into two parts: the identification of a; and subsequent pricing given a. Note that in general $u_2 \leq a \leq V_1$. To obtain a feasible set of prices for a given a, we require that the regularity condition is satisfied and the prices are such that the firm's imposed condition that is $p_a < \theta_1$ and $p_{a+1} \geq \theta_1$ is satisfied.

Now supposing that we have identified a, then the problem of optimally pricing the product line can be broken down into managing three sets of items consisting of items from 1 to $u_2 - 1$ attracting customers from segment 1, items from u_2 to a attracting customers from both segments and items from $a+1$ to V attracting customers from segment 2. We refer to these item sets as "tiers".

The problem reduces to optimally solving three pricing problems with different boundary conditions for each tier. Let,
$l_1 = u_2 - 1$ be the number of items in tier 1
$l_2 = a - u_2 - 1$ be the number of items in tier 2
$l_3 = V - a$ be the number of items in tier 3.
Then the adjusted average costs for the tiers are

$$A_1 = \frac{\sum_{k=1}^{l_1} c_k + p_{u_2}}{l_1 + 1}, \quad A_2 = \frac{\sum_{k=1}^{l_2} c_k + p_a}{l_2 + 1}$$

and $A_3 = \frac{\sum_{k=1}^{l_3} c_k + \theta_2}{l_3 + 1}$

Let $\Pi_i = (p_i - c_i)D_i$ denote the profit generated by the i^{th} item in the line, where D_i, the demand for the i^{th} item is

$$D_i = \begin{cases} M_1\,(p_{i+1} - p_i) & 1 \leq i \leq u_2 - 1 \\ (M_1 + M_2)\,(p_{i+1} - p_i) & u_2 \leq i < a \\ M_1\,(\theta_1 - p_i) + M_2\,(p_{i+1} - p_i) & i = a \\ M_2\,(p_{i+1} - p_i) & a < i \leq V \end{cases}$$

where, $p_{V+1} = \theta_2$; $M_1 = N_1/(\theta_1 - c_{u_1})$; $M_2 = N_2/(\theta_2 - c_{u_2})$.

Solving for the optimal prices involves setting the partial derivative of Π with respect to $\Pi = \sum \Pi_i$ equal to zero for each of the three segments individually. The boundary conditions used are: $p_0 = c_0 = 0$, $p_{l_1+1} = p_{u_2}$, $p_{l_2+1} = p_a$, $p_{l_3+1} = p_{V+1} = \theta_2$. The optimal prices obtained are

$$p_i = \begin{cases} iA_1 - \sum_{k=1}^{i-1} c_k & 1 \leq i \leq l_1 \\ iA_2 - \sum_{k=1}^{i-1} c_k + \left(\frac{l_2 + 1 - i}{l_2 + 1}\right)(p_{u_2} - c_{u_2}) & 1 \leq i < l_2 \\ iA_3 - \sum_{k=1}^{i-1} c_k + \left(\frac{l_3 + 1 - i}{l_3 + 1}\right)(p_a - c_a) & 1 \leq i < l_3 \end{cases}$$

The optimal price of the u_2 item is given as,

$$p_{u_2} = \frac{\frac{M_1}{M_1+M_2}\left(\frac{-\sum_{k=1}^{l_1} c_k}{l_1+1}\right) + \frac{c_{u_2}}{l_2+1} + \frac{\sum_{k=1}^{l_2} c_k}{l_2+1} + \frac{p_a}{l_2+1}}{2 - \frac{M_1}{M_1+M_2}\left(\frac{l_1}{l_1+1}\right) - \frac{l_2}{l_2+1}} \quad (3)$$

For the special case of $u_2 = 1$, the lowest acceptable item for the second customer segment is the same as that of the first customer segment. In this case tier 2 merges with tier 1 and equation (3) need not be evaluated.

The optimal price of the a^{th} item at the reservation price boundary is given as,

$$p_a = \frac{\frac{M_1\theta_1}{M_1+M_2} + z_1 A_3 + c_a z_2 - \frac{\sum_{k=1}^{l_2} c_k}{l_2+1} + \frac{(p_{u_2} - c_{u_2})}{l_2+1}}{1 + z_2 - \frac{l_2}{l_2+1}} \quad (4)$$

$$\textit{where} \quad z_1 = \frac{M_2}{M_1 + M_2} \quad \textit{and} \quad z_2 = 1 - z_1\left(\frac{l_3}{l_3 + 1}\right)$$

Simultaneously solving equation (3) and equation (4) gives the value of p_a and p_{u_2}, which can then be used to solve the rest of the prices.

3.1 Optimal Product Line Partition

Thus far, we assumed that the item a, which segments the product line was known to us. It is clear though that an appropriate choice of a is required to maximize product line profits. First we formalize an intuitive observation, which says that items provide higher margins when they are limited to the higher customer segment than when they are made available to the lower customer segment.

Proposition 1: Assuming regularity conditions, the price for any item under $W(a)$ will be higher than under $W(a+1)$.

Proof: Available from authors.

Consider next any tier t within the product line with l items having costs $c_1^t, c_2^t, \ldots, c_l^t$.

Now the optimal price is given as,

$$p_i = (iA_t - \sum_{k=1}^{i-1} c_k^t) + \left(1 - \frac{i}{l_t + 1}\right)(p_l^{t-1} - c_l^{t-1}) \quad (5)$$

The regularity condition $p_{i+1} - p_i > 0$ gives,

$$A_t - c_i - \frac{(p_l^{t-1} - c_l^{t-1})}{l_t + 1} > 0 \quad (6)$$

Since c_l^t is the highest cost item, the regularity condition can be restated as,

$$c_l^t + \frac{(p_l^{t-1} - c_l^{t-1})}{l_t + 1} < A_t \quad (7)$$

Therefore, maintaining the regularity condition implicitly requires that no item cost exceeds the adjusted average cost. This puts a constraint on the item composition of the product line since inclusion and deletion of items affects the adjusted average cost.

Now, even if the regularity condition is satisfied and all the items have distinct prices, the firm's conditions $p_a < \theta_1$ and $\theta_1 \leq p_{a+1}$ can get violated due to a linking effect. The condition $p_a < \theta_1$ is satisfied if the regularity condition for segment 1 is satisfied given that ratio of M_1 to M_2 is not very large or very small. This linking effect and the regularity constraint imposed over the product line composition makes the problem hard to solve.

In Proposition 2, we establish that the violation of the regularity condition for an item composition in which the firm offers a total of a items to the first customer segment implies that the regularity condition would be violated if any more items are offered to first segment. This result serves as a stopping criterion for the search for a.

Proposition 2: If the condition $p_a < \theta_1$ is violated for $W(a)$ then corresponding condition $\bar{p}_{a+1} < \theta_1$ will be violated for $W(a+1)$.

Proof: Available from authors.

3.2 Optimally Solving the Two Segment Pricing Problem

We next describe the procedure to determine the optimal item composition.

Procedure:

Step 1: $a = u_2$. Solve the pricing problem and check if the linking condition $\theta_1 \leq p_{a+1}$ is satisfied. If not, then let $a = a+1$ and solve the new pricing problem until the linking condition is satisfied giving a feasible solution. Let the feasible solution is obtained at $a = h$ and $z = 1$ and the profit is $\Pi(z)$.

Step 2: Let $a = a+1$ and $z = z+1$. If regularity condition is satisfied then resolve the pricing problem resulting in profit $\Pi(z+1)$. Repeat step 2 until regularity condition is violated such that $p_a \geq \theta_1$ or $a = V_1$.

Step 3: Let $a = u_2 - 1$. This is disjoint segment scenario. Check if the regularity condition is satisfied. If not, let $a = a-1$ until the regularity condition is satisfied. Let $z = 0$ and $z' = a$. Solve the pricing problem for such an a giving profit $\Pi(0)$.

Step 4: Let $Z = argmax_z\{\Pi(z)\}$. If $Z = 0$, then $a = z'$ else $a = h+Z$. Prices of the items are chosen as per the prices for the item composition with a.

In the case that the two customer segments overlap the constraint $p_{u_2} > \theta_1$ may not to be satisfied. The profit function can be shown to be strictly concave and $p_{u_2} = \max\{\theta_1, p^*_{u_2}\}$.

4 COMPUTATIONAL RESULTS

To test the model we collected online retail prices of items for three different item categories from large national retail chains in United States. We refer to these different product line data sets as set 1, set 2 and set 3. The three categories we considered are,

Set 1. Kenmore single room air-conditioners at Sears.com with six variants,

Set 2. Apple ipods at Bestbuy.com with five variants, and

Set 3.Kenmore compact refrigerators at Sears.com with six variants.

First, we estimate the item costs, by randomly selecting a cost within $35\% - 45\%$ of the retail price. We consider two customer segments, assuming that the population of segment 1 is three times the population of segment 2. The values of different parameters we use for our pricing model for two customer segments are shown in Table 1. We assume that first segment of customers considers all the items for purchase if they are priced within their reservation price and therefore $u_1 = 1$ for all the sets. Segment 2 treats different sets of product line differently and therefore, u_2 varies from set to set.

Table 1: Base Problem Sets .

Sets	**1**	**2**	**3**
u_1	1	1	1
u_2	2	2	3
θ_1	230	155	195
θ_2	380	275	330

4.1 Pricing Model Performance on Base Data

To empirically test the performance of our pricing model in terms of its predictability, we solved each data set under the test parameters and compare the prices proposed by our model with respect to current retail prices. For computational purpose, we assume that the retailer provides the last two items in each product line data set exclusively to the second customer segment.

Tables 2, 3 and 4 shows the results of applying our pricing model to the three sets respectively. First column of the table represents the index of item number. Second column is the estimate of item costs and the third column shows the retail prices. Then, we determine the proposed price, shown in the fourth column of each table, by solving the pricing problem using our model.

The last column in each table shows the percent price difference between the proposed price and the retail price. These results indicate that the proposed prices from our pricing model closely resemble the current trend of retail prices.

Table 2: Comparison of Prices - Set 1 .

Item	Estimated Cost	Retail Price	Proposed Price	% Price Difference
1	41	99.99	95.11	-4.88
2	61	**139.99**	**149.23**	6.60
3	72	**189.99**	**193.74**	1.97
4	88	**229.99**	**227.26**	-1.19
5	107	299.99	306.84	2.28
6	155	379.99	367.42	-3.31

Table 3: Comparison of Prices - Set 2 .

Item	Estimated Cost	Retail Price	Proposed Price	% Price Difference
1	25	69.99	66.36	-5.18
2	38	**99.99**	**107.72**	7.74
3	53	**149.99**	**143.88**	-4.08
4	79	199.99	208.25	4.13
5	89	249.99	246.63	-1.35

Table 4: Comparison of Prices - Set 3 .

Item	Estimated Cost	Retail Price	Proposed Price	% Price Difference
1	31	74.98	75.77	1.05
2	47	119.99	120.54	-0.46
3	57	**154.99**	**149.30**	-3.67
4	64	**179.99**	**182.40**	1.34
5	107	269.99	264.27	-2.12
6	119	299.99	303.13	1.05

Based on our mathematical model, an analysis of profit by using prices generated from our pricing model as compared to the store retail prices shows an increase in profit by 0.5-3 %. The exact increases are 1.196 %, 2.575 % and 0.837 % for data sets 1, 2 and 3 respectively in favor of the prices generated by our model.

4.2 Comparison with the 1-Segment Model

We demonstrate the importance of considering two segments over the single segment model by considering the prices set 3. We estimate the reservation price (θ), for a single segment model by satisfying the regularity condition described in equation (2). The resulting value is \$408.00, which is very high as compared to θ_2 (\$330.00), rendering items 4, 5 and 6 beyond the buying capacity of even the second segment customers.

Figure 2 shows this large variation of proposed prices by the 1-segment model from the retail prices. Also notice in the figure that for the two segment model, theproposed prices are very close to the retail prices and within the reservation price of customers in segment 2.

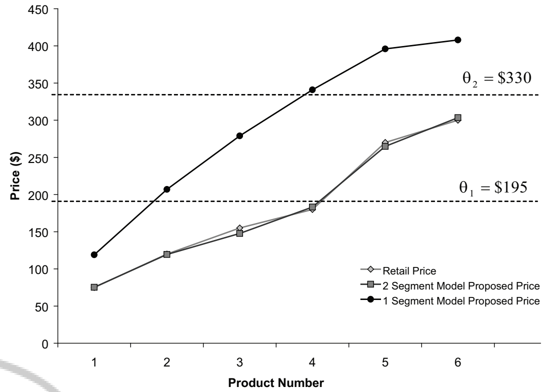

Figure 2: Two Segment Model Proposed Prices vs One segment Model Proposed Prices .

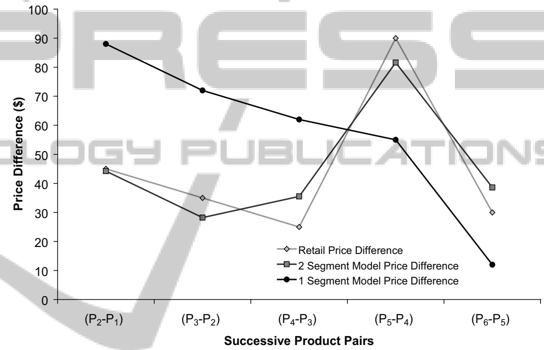

Figure 3: Two Segment Model Price gap vs One segment Model Price gap.

The price difference between successive items is shown in figure 3. In the 1-segment model, the price difference between successive items falls steeply. In the 2-segment model, the price difference does not fall steeply but has peaks at times when there is a change in the customer population as the higher segment can pay more and so this is an intuitively appealing and more appropriate representation of the market.

5 GENERAL S-CUSTOMER SEGMENT CASE

In general, the customer population can be divided into more than two customer segments, say S-customer segments. The segmentation of customers is based on the same two measures, namely the lowest acceptable item and the maximum reservation price. The total number of tiers formed shall be in the range of 1 to $2S-1$. Since there are more than two customer segments there is possibility of overlap of many different customer segments. If the lowest acceptable item or reservation price of any two customer segments are the same, then the total number of tiers accordingly. Nonetheless, as the number of tiers increases, the complexity of solving the problem also increases.

The S-segment model can be solved in a manner similar to the 2-segment model by partitioning the product line into tiers. The mathematical analysis as done for 2-customer segments is directly extendable to the S-customer segments with some generalization.Details about this procedure are omitted because of space. It is important to note that the procedure is computationally more demanding. However, the maximum number of computations that may be needed is $\prod_{j=1,S-1} (V+1-u_j)$, giving a worst-case complexity of $O(V^{S-1})$. However, because of the regularity condition and the requirement that $a_j \leq a_{j+1}$, many of these computations will not be needed and therefore the actual computational burden will be a lot less. Moreover, the number of customer segments is unlikely to be very large and so enumerating the whole problem is computationally not very costly.

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