

# Optimization of Adaptive Up-and-Down Method for Equivalent Time Signal Conversion

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**Keywords:** Equivalent Time Conversion, UWB Radar, Adaptive Methods, Up-and-Down Method.

**Abstract:** This study relates to comparator type equivalent time signal conversion. The influence of parameters of combined adaptive up-and-down method on accuracy of signal conversion is studied. Optimal values of parameters are obtained, using which the minimal amount of the samples is ensured. As an input signal, a model of UWB radar signal (sine monocycle) is used. Method is intended for application in UWB radar receivers. The design guidelines for application of the method in UWB radar receivers are outlined.

## 1 INTRODUCTION

Two types of equivalent time converters are known, namely, gate type converters (Kahrs, M., 2003) and comparator type converters (Херманис, Э. Х., Карклиньш, В. Г., 1977), (Bergman, D. I., 2006.), (Askerzade, I. N., 2006). This article refers to the comparator type equivalent time converters. In such converters, the measurement of the instantaneous value of the signal is achieved through comparisons with a known threshold. The threshold is changed according to chosen procedure until the signal is measured with sufficient precision. One of the most efficient comparator type methods is the combined adaptive *up-and-down* method, named as  $\alpha$ - $k$ -*up-and-down* method (Krumin'sh, K. and Plotsin'sh, V., 2013). The aim of this study is to optimize the parameters  $r$ ,  $n_1/n$  and  $v$  of this method. The study uses an example of designing UWB radar receiver.

The simplest of comparator type signal conversion method is *up-and-down* method (Krūmiņš, K. and Kārklīņš, V., 2005). According to this method, the measurement of the signal proceeds as follows. In a specified phase  $t = t_i$  signal  $U_{1,i}$  is compared with the noisy threshold  $e$  of comparator. Therefore, we can assume that the signal  $U_{1,i}$  is masked with an additive noise  $X$  (variance  $DX = \sigma_1^2$  and mean value  $EX = 0$ ):

$$u_{1,i} = U_{1,i} + X. \quad (1)$$

The threshold is adjusted according to expression:

$$e_{i,j+1} = e_{i,j} + s \cdot \text{sign}(U_{1,i} - e_{i,j}), \quad (2)$$

where  $s$  is the step of the method.

After a sufficiently large number  $n$  of comparisons, the last value of threshold is used as a measurement result:

$$u_{2,i} = e_{i,n}. \quad (3)$$

Afterwards the phase is shifted to the next point, and the value of signal in that phase is measured. The initial value of threshold in next phase point remains the last threshold in previous phase point:

$$e_{i+1,1} = e_{i,n}. \quad (4)$$

According to the combined method the measurement of instantaneous value of signal proceeds as follows. Total number  $n$  of comparisons is divided in two parts  $n_1 + n_2 = n$ . In the first part the threshold is changed according to *up-and-down* procedure (2) with step  $s = s_0$ . After  $n_1$  comparisons the number  $n_1^+$  of threshold exceedances is counted. Afterward the value of step  $s = s_i$  is calculated according to expression

$$s_i = s_0 \cdot \left( k \left| \frac{n_1^+}{n_1} - 1 \right|^r + v \right), \quad (5)$$

where:

$k$  - coefficient of adaptation,  $r$  and  $v$  - parameters of method.

In the second part of measurement the *up-and-down* procedure proceeds according to (2) with the step (5). The result of measurement, similarly to the *up-and-down* method, is the last value of threshold  $u_{2,i} = e_{i,n}$ . However, the initial value of the threshold in the next point is set to

$$e_{i+1,1} = e_{i,n} + \alpha \cdot (u_{2,i} - u_{2,i-1}), \quad (6)$$

where  $\alpha$  - coefficient of adaptation.

The optimization of parameters of the combined method occurred at the specified noise suppression and accuracy of signal measurement. As an input signal, a model of UWB radar signal was used. There are known several models of UWB radar signals (Chen, X. and Kiaei, S., 2002). We used a sine monocycle

$$u_{1,i} = A_1 \cdot \sin\left(\frac{2\pi}{n_T} i\right), \quad (7)$$

where:  $A_1$  - amplitude of the signal;  $n_T$  - the number of phase points per monocycle period.

Let's introduce following notations:

$A^*$  - the maximum amplitude of the undistorted signal transformation;

$\sigma_1 = 1$  - the normalized standard deviation of the masking noise at the converter's input;

$\sigma_2$  - the standard deviation of the noise at the converter's output;

$\sigma_2^*$  - the maximum allowed standard deviation of the noise at the converter's output;

$\sigma_f$  - the coefficient showing the distortion of the signal's shape:

$$\sigma_f = \frac{std(u_2 - u_1)}{A_1}; \quad (8)$$

$c$  - the maximum allowed value of the distortion coefficient of the signal's shape for  $n_T$  points of the signal;

$N = 2n_T$  - the sweep length (the sine monocycle is located at the centre of the sweep) used for the determination of  $\sigma_f$ ;

$c_1 = c/\sqrt{2}$  - the maximum allowed value of the distortion coefficient of the signal's shape within the sweep range of  $2n_T$ ;

$\sigma_{f1} \leq c_1$  - criterion of signal conversion quality for amplitude  $A_1=1$  within the sweep range of  $2n_T$ ; and

$\sigma_{f2} \leq c_1$  - criterion of signal conversion quality for amplitude  $A_1 = A^*$  within the sweep range of  $2n_T$ .

The distortion coefficients of the signal's shape  $\sigma_{f1}$  and  $\sigma_{f2}$  are calculated within the sweep range of  $2n_T$ , because signal distortions may spread beyond the sine-wave period when adaptive methods are used.

The optimization of parameters was based on example of converter outlined in (Krumin'sh, K. and Plotsin'sh, V., 2013):  $\sigma_1 = 271 \mu V$ ;  $\sigma_2^* = 15 \mu V$ ;  $A^* = 25 mV$ ;  $\alpha = 0.7$ ;  $c = 0.05$ ;  $c_1 = 0.0353$ ;  $n_T = 50$ . By reducing the values of  $\sigma_1$ ,  $\sigma_2^*$  and  $A^*$  to relative values, we obtain  $\sigma_1 = 1$ ,  $\sigma_2^* = 0.055$  and  $A^* = 93$ .

Optimization of the parameters was carried out at  $n = 210$  because several tests showed that this value might be close to the minimum of required number of comparisons. As initial values of  $r$ ,  $n_1/n$  and  $v$  values  $r = 6$ ,  $n_1/n = 0.5$  and  $v = 1$  were used. At each selected pair of initial values of the parameters, the influence of the third parameter was studied. Research was conducted by statistical modeling.

## 2 THE RESEARCH OF INFLUENCE OF PARAMETER $r$ ON THE ACCURACY OF SIGNAL CONVERSION

The following values of parameter  $r$  were used:  $r = 1.0; 1.5; \dots; 9.5; 10.0; 100.0$ . The values of step  $s_0(k)$ , which fulfill the condition  $\sigma_2 \in (0.985\sigma_2^*; 0.995\sigma_2^*)$ , were found at each selected  $r$ . The obtained results are shown at Figure 1.

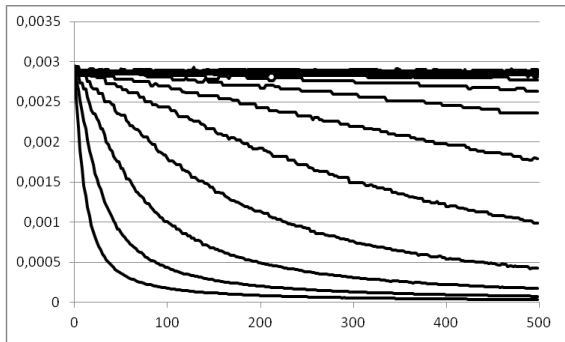


Figure 1: The values of step  $s_0(k)$  for fulfilling the condition  $\sigma_2 \in (0.985\sigma_2^*; 0.995\sigma_2^*)$  at different values of  $r$ . The corresponding dependencies are shown from  $r = 1.0$  (bottom) to  $r = 100$  (top).

As seen in Figure 1, the value of step decreases by increasing  $k$ , however, beginning with  $r \geq 5$  we can assume that the value of step does not depend on parameters  $k$  and  $r$ . This property is beneficial to use in practical implementation of  $\alpha$ - $k$ -up-and-down method.

The dependencies of coefficients  $\sigma_{f1}(k)$  and  $\sigma_{f2}(k)$  of signal distortions were investigated within the same range of  $r$ . For calculating  $\sigma_{f1}(k)$  and  $\sigma_{f2}(k)$  the previously obtained values  $s_0$  were used. The acquired  $\sigma_{f1}(k)$  and  $\sigma_{f2}(k)$  are depicted correspondingly in Figure 2 and Figure 3.

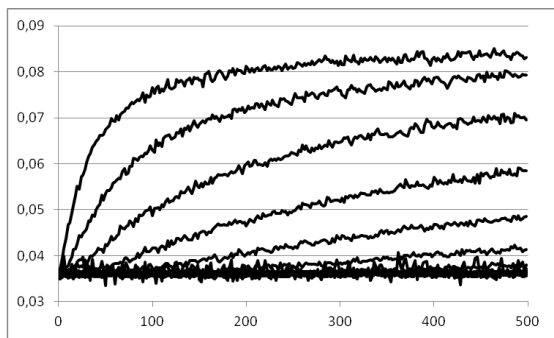


Figure 2: The dependence  $\sigma_{f1}(k)$  at different values of  $r$ . The corresponding dependencies are shown from  $r = 1.0$  (top) to  $r = 100$  (bottom).

As seen in Figure 2, the coefficient  $\sigma_{f1}(k)$  of distortion of small signal increases by increasing  $k$ , however, beginning with  $r \geq 4$  we can assume that the value of  $\sigma_{f1}(k)$  does not depend on parameters

$k$  and  $r$ . This is another beneficial property to use in practical implementation of  $\alpha$ - $k$ -up-and-down method.

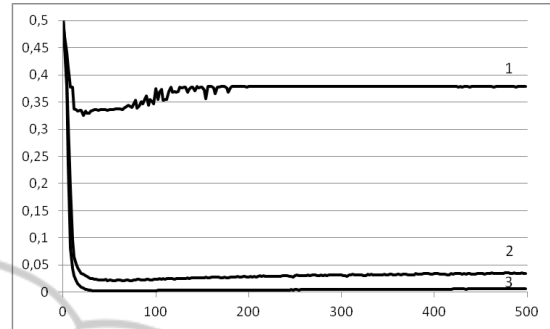


Figure 3: The dependence of coefficient  $\sigma_{f1}(k)$  of signal distortions at several values of  $r$ :  $r = 1.0$  – curve 1;  $r = 1.5$  – curve 2 and  $r = 2.0 \div 100$  – curve 3.

It was obtained that, beginning with  $r \geq 2$ , the dependencies  $\sigma_{f2}(k)$  practically do not differ and therefore in Figure 3 are depicted as one curve. Besides, the fulfilling of conditions  $r > 1.5$  and  $k > 30$  makes the coefficient  $\sigma_{f2}$  independent of  $k$ , and, moreover, less than 0.008. Therefore, the condition  $\sigma_{f2} < 0.0353$  is satisfied by a large margin.

From the above it follows that in the practical implementation of the method one should use the values  $r \geq 5$  of the parameter  $r$ . In addition, this study showed that the requirements  $\sigma_{f1} < 0.0353$  and  $\sigma_2 \in (0.985\sigma_2^*; 0.995\sigma_2^*)$  by adjusting the coefficient  $r$  cannot be satisfied at  $n = 210$ .

### 3 THE RESEARCH OF INFLUENCE OF THE RATIO $n_1/n$ ON THE ACCURACY OF SIGNAL CONVERSION

The following values of the ratio  $n_1/n$  were used:  $n_1/n = 0.05; 0.1; 0.2; \dots; 0.8; 0.9; 0.95$ . The values of step  $s_0(k)$ , which fulfill the condition  $\sigma_2 \in (0.985\sigma_2^*; 0.995\sigma_2^*)$ , and corresponding  $\sigma_{f1}(k)$  and  $\sigma_{f2}(k)$  were found at each selected  $n_1/n$ . Obtained results are shown accordingly in Figure 4, Figure 5 and Figure 6.

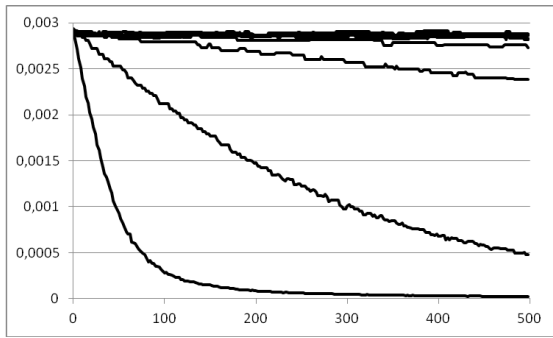


Figure 4: The values of step  $s_0(k)$  for fulfilling the condition  $\sigma_2 \in (0.985\sigma_2^*; 0.995\sigma_2^*)$  at different values of  $n_1/n$ . The corresponding dependencies are shown from  $n_1/n = 0.05$  (bottom) to  $n_1/n = 0.95$  (top).

As seen in Figure 4, condition  $n_1/n \geq 0.3$  implicates the independence of step  $s_0$  from parameters  $k$  and  $n_1/n$ .

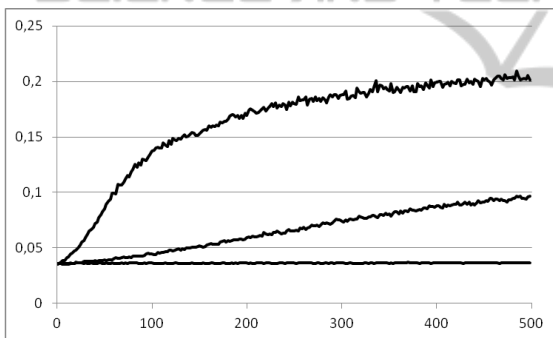


Figure 5: The dependence of coefficient  $\sigma_{f1}(k)$  of signal distortions at several values of  $n_1/n$ :  $n_1/n = 0.05$  (top);  $n_1/n = 0.1$  (middle) and  $0.2 \leq n_1/n \leq 0.95$  (bottom).

It was obtained that condition  $0.2 \leq n_1/n \leq 0.95$  makes the dependencies  $\sigma_{f1}(k)$  practically undistinguishable, therefore those dependencies in Figure 5 are depicted as one curve. Besides, the condition  $0.2 \leq n_1/n \leq 0.95$  implicates that  $\sigma_{f1}$  is independent from parameter  $k$ . Usage of  $n_1/n < 0.2$  increases the coefficient  $\sigma_{f1}$  of distortions and the accuracy of signal conversion worsens.

As seen in Figure 6, it's not beneficial to use the values  $n_1/n < 0.1$  because in this case the range of

allowable values of parameter  $k$  narrows. Also it is inadvisable to use the values  $n_1/n > 0.7$  because in this case the required value of  $\sigma_{f1}$  is ensured at large values of parameter  $k$ . The studies have shown that increase of parameter  $k$  makes the output noise spikier. Summarizing above-mentioned (see Figure 4, Figure 5 and Figure 6), one can use the values  $0.3 \leq n_1/n \leq 0.7$ .

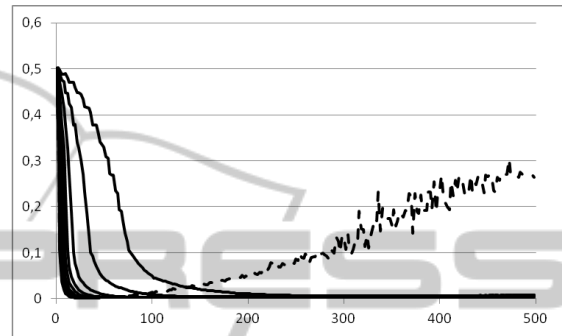


Figure 6: The dependence of coefficient  $\sigma_{f2}(k)$  of signal distortions at several values of  $n_1/n$ :  $n_1/n = 0.95$ ;  $n_1/n = 0.8$  ...;  $n_1/n = 0.1$  (respectively, from right to left) and  $n_1/n = 0.05$  (dashed curve).

In addition, this study showed that the requirements  $\sigma_{f1} < 0.0353$  and  $\sigma_2 \in (0.985\sigma_2^*; 0.995\sigma_2^*)$  by adjusting the coefficient  $n_1/n$  cannot be satisfied at  $n = 210$ .

### 3 THE RESEARCH OF INFLUENCE OF THE PARAMETER $\nu$ ON THE ACCURACY OF SIGNAL CONVERSION

The following values of the parameter  $\nu$  were used:  $\nu = 0.1; 0.2; 0.3; \dots; 1.8; 1.9$ . The values of step  $s_0(k)$ , which fulfill the condition  $\sigma_2 \in (0.985\sigma_2^*; 0.995\sigma_2^*)$ , and corresponding  $\sigma_{f1}(k)$  and  $\sigma_{f2}(k)$  were found at each selected  $\nu$ . Obtained results are shown accordingly in Figure 7, Figure 9 and Figure 11.

As seen in Figure 7, the values of step  $s_0$  do not depend on  $k$ . Using this property, the values of

dependence  $s_0(v)$  were obtained by averaging  $s_0(k)$  at each  $v$  (see Figure 8).

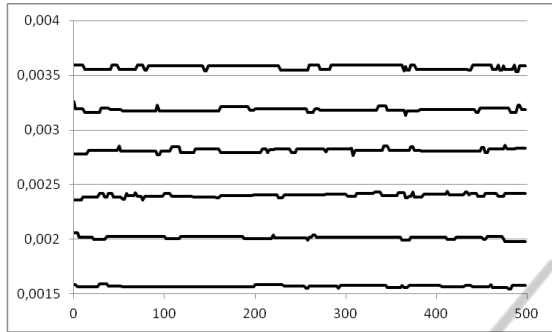


Figure 7: The values of step  $s_0(k)$  for fulfilling the condition  $\sigma_2 \in (0.985\sigma_2^*; 0.995\sigma_2^*)$  at different values of  $v$ :  $v = 0.5; 0.3; 0.2; 0.1; 1.5$  and  $1.9$  (corresponding dependencies are shown from top to bottom).

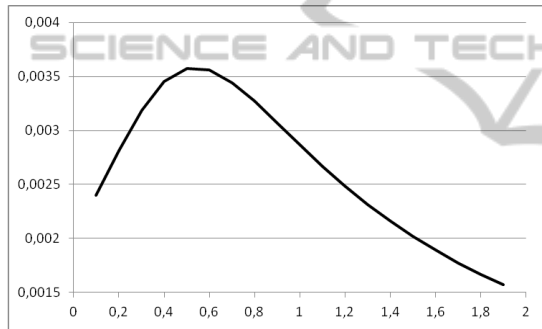


Figure 8: The dependence of step  $s_0(v)$  for fulfilling the condition  $\sigma_2 \in (0.985\sigma_2^*; 0.995\sigma_2^*)$ .

Using the obtained  $s_0(v)$  and corresponding  $v$ , the dependence  $\sigma_{f1}(k)$  was studied. As seen in Figure 9, the condition  $v \geq 0.3$  implicates independence of  $\sigma_{f1}$  from parameter  $k$ . Using this property, the dependence  $\sigma_{f1}(v)$  was studied at  $v \geq 0.3$  (see Figure 10).

As seen in Figure 10, the dependence  $\sigma_{f1}(v)$  has the optimum  $v_{opt} = 0.8$ , at which  $\sigma_{f1}(v_{opt}) = 0.035$ . Consequently, the requirement  $\sigma_{f1} \leq c_1$  is fulfilled. However, since the value of  $\sigma_{f1}(v_{opt})$  differs only slightly from the criterion  $c_1 = 0.0353$ , we can assume that  $n = 210$  is the minimal number of

comparisons for fulfilling criteria  $\sigma_{f1} \leq 0.0353$  and  $\sigma_2 \in (0.985\sigma_2^*; 0.995\sigma_2^*)$ .

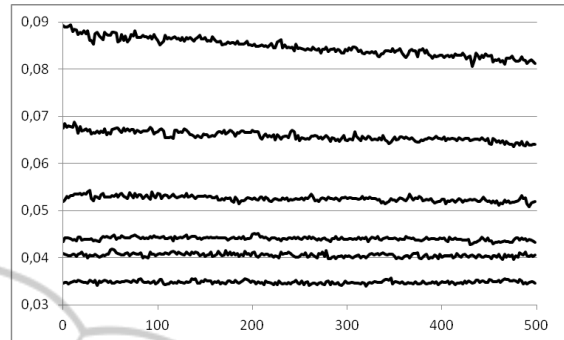


Figure 9: The dependence  $\sigma_{f1}(k)$  at different values of  $v$ :  $v = 0.1; 0.2; 0.3; 0.4; 1.4$  and  $0.8$  (corresponding dependencies are shown from top to bottom).

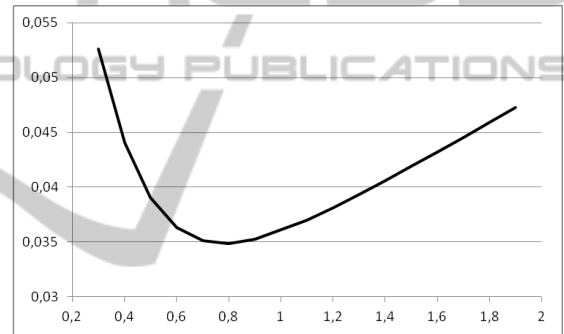


Figure 10: The dependence of coefficient  $\sigma_{f1}(v)$  of signal distortion.

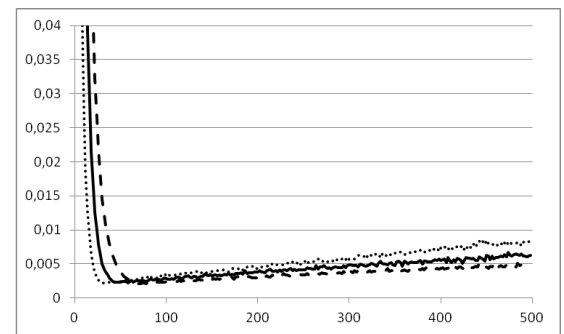


Figure 11: The dependence of signal distortion  $\sigma_{f2}(k)$  at different values of  $v$ :  $v = 0.1$  (continuous line);  $v = 0.5$  (dotted line) and  $v = 1.9$  (dashed line).

The study of dependence  $\sigma_{f2}(k)$  was carried out in the same range of parameter  $v$ :  $v = 0.1; 0.2; 0.3; \dots; 1.8; 1.9$ . It was obtained that in

the beginning of the range of parameter  $k$  the values of dependence  $\sigma_{f2}(k)$  decreases very rapidly from  $\sigma_{f2}(0) \approx 0.5$  to  $\sigma_{f2} < 0.004$ . Thereafter dependence  $\sigma_{f2}(k)$  slowly increases (see Figure 11). Besides it was obtained that the condition  $\sigma_{f2} \leq 0.0353$  is fulfilled for all values of  $k > 30$  regardless of  $v$ . However, in order to minimize  $\sigma_{f2}$ , it is recommended to use  $k \geq 70$ . Still, as was mentioned before, one should not choose too big values of parameter  $k$ .

Summarizing all these studies of optimization of parameters  $r$ ,  $n_1/n$  and  $v$  yields that the minimal number of comparisons for fulfilling the requirements  $\sigma_2 \in (0.985\sigma_2^*; 0.995\sigma_2^*)$ ,  $\sigma_{f1} \leq c_1$  and  $\sigma_{f2} \leq c_1$  is  $n = 210$ . The parameters at which this number is obtained, are:  $r = 6$ ,  $n_1/n = 0.5$ ,  $v = 0.8$ ,  $k = 70$ ,  $\alpha = 0.7$  and  $s_0 = 0.00325\sigma_1$ .

It should be mentioned that the usage of *up-and-down* method in our example requires the minimal number of comparisons  $n = 2076$ . That means that the application of this adaptive method provides almost 10-fold increase of speed of signal conversion. In order to show the efficiency of the combined method versus *up-and-down* method, the results of converting of sine monocycle with amplitude of  $A^* = 93$  at the same number of comparisons  $n = 210$  are seen in Figure 12.

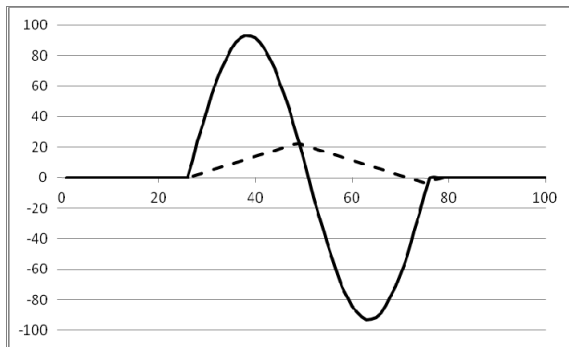


Figure 12: The result of sine monocycle conversion with combined method (continuous line) and *up-and-down* method (dashed line) at  $n = 210$ .

The result of signal conversion with *up-and-down* method (dashed line) is distorted beyond recognition while the result of conversion with combined method coincides with the input signal.

In designing of a similar converter it is advantageous to use the above-mentioned values of

parameters:  $0.3 \leq n_1/n \leq 0.7$ ,  $r \geq 5$ ,  $v = 0.8$ ,  $\alpha = 0.7$  and  $k \geq 30$ . The search of parameters  $s_0$ ,  $n$  and  $k$  should be carried out through computer simulation. Let's assume that the reduced noise suppression of the designed converter is  $\sigma_2^{**}$ . The modeling of signal conversion should begin with a choice of  $n$ . As an initial choice of  $n$ , one can use a following empirical formula:

$$n \approx 0.6525 \left( \sigma_1 / \sigma_2^{**} \right)^2 \tag{9}$$

Using chosen  $n$ , one should find the value of step  $s_0$  fulfilling the required sensitivity  $\sigma_2^{**}$ . As an approximate initial value of  $s_0$  one can use the following expression (Kruminsh, K., Plotsinsh, V., 2012):

$$s_0 = \sqrt{1.5625\sigma_1^2 + 4(\sigma_2^{**})^2} - 1.25\sigma_1 \tag{10}$$

The sensitivity requirement  $\sigma_2 \in (0.985\sigma_2^{**}; 0.995\sigma_2^{**})$  is tested at the chosen values of  $s_0$  and  $n$ . If  $\sigma_2 > 0.995\sigma_2^{**}$ , then the value of step should be decreased. Conversely, the value of step should be increased if  $\sigma_2 < 0.985\sigma_2^{**}$ . By applying such procedure, the value of step  $s_0$ , which fulfills the requirement for necessary sensitivity, is obtained. Afterwards the criterion  $\sigma_{f1} = 0.0353$  is tested. If  $\sigma_{f1} > 0.0353$ , then the number  $n$  should be increased and the procedure of finding the value of step  $s_0$ , as well as the testing of criterion  $\sigma_{f1} = 0.0353$  should be repeated. If  $\sigma_{f1} < 0.0353$ , then the number  $n$  should be decreased and the procedure of finding the value of step  $s_0$ , as well as the testing of criterion  $\sigma_{f1} = 0.0353$  should be repeated. In such way the values of  $s_0$  and  $n$  are obtained. In order to find the value of parameter  $k$ , the criterion  $\sigma_{f2} \leq 0,01$  will be used. The testing of this criterion should begin with value of  $k = 30$ . The value of  $k$  is increased until the criterion  $\sigma_{f2} \leq 0,01$  is fulfilled. Obtained in such way values of  $s_0$ ,  $n$  and  $k$  are optimal for fulfilling the required sensitivity and quality of signal conversion.

Remark: A more strict condition  $\sigma_{f2} \leq 0.01$  is used instead of  $\sigma_{f2} \leq 0.0353$  because dependence

$\sigma_{f_2}(k)$  is rather steep, and condition  $\sigma_{f_2} \leq 0.0353$  is fulfilled easily.

## 4 CONCLUSIONS

1. In implementation of the combined method one should use the following values of the parameters:  $0.3 \leq n_1/n \leq 0.7$ ,  $r \geq 5$  and  $v = 0.8$ .
2. The conducted research showed that the value of parameter  $k$  can be changed within wide range beginning with approximately  $k \geq 30$ .
3. With equal accuracy and sensitivity requirements for signal conversion the combined adaptive method provides a nearly 10-fold increase in performance than the *up-and-down* method.
4. Significant efficiency of adaptive method, illustrated by computer simulations, indicates advisability of analytical studies of this method.

## ACKNOWLEDGEMENTS

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