Adaptive Strategies for Collaborative Work with Scale Quad-rotors

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Abstract: The purpose of this paper is to present strategies for the control of movement of rigid bodies via force actuators, possibly redundant. After a nonlinear feedback linealization of the considered dynamic models and the application of a suitable controller, an adaptive neural network based control component is incorporated in order to cope with modeling errors and disturbance rejection. An online sequential quadratic programing optimization with equality and inequality constraints assures an adequate configuration of actuator forces. Application to collaborative work in the transportation of a rigid body using a squadron of scale quad-rotors is studied.

1 INTRODUCTION

In collaborative work between different agents concerning dynamic processes of mechanical nature, such as the problem of air transport of rigid bodies, arises the need to allocate adequate efforts to maintain a path and pose previously established. In this paper this idea is applied to the collaborative work in carrying a rigid load along a given path (Sreenath et al., 2013) (Lee et al., 2013), using a squadron of scale quad-rotors.

The hierarchical process of collaborative assignments is shown. First, the problem of tracking a path and pose is solved for a rigid body, which is the load to be transported. Secondly, through an allocation procedure based on nonlinear programming, the efforts to apply at local mooring in the body transport are determined. Finally, the determination of the orbits of transport agents should follow, as well as the efforts to which they are subjected by the mooring links with the transported body. Scale quad-rotors are capable of aggressive maneouvering, as can be seen in (Huang et al., 2009), (Mellinger et al., 2012). In this last stage arises the need for adaptive augmentation, to overcome deficiencies encountered during the modeling process or due to changing environmental conditions.

The structure of this paper is as follows: Section 2 introduces the development of the equations of motion of the rigid body. In Section 3 follows the rigid body tracking control formulation. Section 4 focuses the allocation of forces attached to the mooring of the rigid body. The force allocation is formulated as a nonlinear programming problem. Section 5 presents the quad-rotor simplified modeling according to a lagrangian formulation. The quad-rotor path error tracking controller is developed along the Section 6 and the simulations carried out with a platoon of quad-rotors are brought in Section 7. Finally the conclusions are presented in Section 8. We have also included a small appendix on the method of feedback linearization as an introduction to the chosen control technique.

2 RIGID BODY DYNAMICS

For the rigid body subjected to external forces, the resulting movement equations can be described developing the Lagrangian:

$$\begin{aligned} T &= \frac{1}{2}m\dot{\xi}^\top\dot{\xi} + \frac{1}{2}\omega^\top J\omega \\ V &= mgz \\ \omega &= Q(\eta)\dot{\eta} \\ J_\eta &= Q(\eta)^\top J Q(\eta) \end{aligned} \tag{1}$$

Also

$$\tilde{\omega} = \begin{pmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{pmatrix} = R(\eta)\dot{R}^\top(\eta) \tag{2}$$

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$$Q(\eta) = \begin{pmatrix} c_\theta c_\psi & -s_\psi & 0 \\ c_\theta s_\psi & c_\psi & 0 \\ -s_\theta & 0 & 1 \end{pmatrix}$$
$$R(\eta) = \begin{pmatrix} c_\theta c_\psi & c_\theta s_\psi & -s_\theta \\ c_\psi s_\theta s_\phi - s_\psi c_\phi & s_\psi s_\theta s_\phi + c_\psi c_\phi & c_\theta s_\phi \\ c_\psi s_\theta c_\phi + s_\psi s_\phi & s_\psi s_\theta c_\phi - c_\psi s_\phi & c_\theta c_\phi \end{pmatrix} \tag{3}$$

Here m denotes the rigid body mass. ξ and ω are the position and angular velocity in global inertial coordinates and body coordinates, respectively. η are the Euler angles pitch ϕ, roll θ and yaw ψ. $R(\eta)$ is the transformation matrix representing the rigid body pose and c_θ, s_θ stand for $\cos(\theta), \sin(\theta)$, etc. The inertia matrix is

$$J = \begin{pmatrix} J_1 & J_{12} & J_{13} \\ J_{12} & J_2 & J_{23} \\ J_{13} & J_{23} & J_3 \end{pmatrix} \tag{4}$$

The resulting movement equations are

$$\begin{aligned} \ddot{\xi} &= m^{-1}(f_0 + f) \\ \ddot{\eta} &= J_\eta^{-1}\left(\frac{1}{2}\frac{\partial}{\partial\eta}(\dot{\eta}^\top J_\eta \dot{\eta}) - \dot{J}_\eta \dot{\eta} + \tau\right) \end{aligned} \tag{5}$$

with $f_0 = (0, 0, -mg)^\top$ and f, τ the external control forces and torques respectively.

3 RIGID BODY TRACKING CONTROL

Given a path immersed in $\mathbb{R}^6$ and named as $\{\xi_r, \eta_r\}$, define the tracking error as:

$$\begin{aligned} e &= \{e_\xi, e_\eta\} \\ e_\xi &= \xi_r - \xi \\ e_\eta &= \eta_r - \eta \end{aligned} \tag{6}$$

Imposing now a stable dynamics for the error,

$$\begin{aligned} \ddot{e}_\xi + k_d^\xi \dot{e}_\xi + k_p^\xi e_\xi &= 0 \\ \ddot{e}_\eta + k_d^\eta \dot{e}_\eta + k_p^\eta e_\eta &= 0 \end{aligned} \tag{7}$$

calling $\{\nu_\xi, \nu_\eta\}$ pseudo-controls, (5) is reduced to a double integrator dynamics

$$\begin{aligned} \ddot{\xi} &= \nu_\xi \\ \ddot{\eta} &= \nu_\eta \end{aligned} \tag{8}$$

Solving now (7) regarding $\{\nu_\xi, \nu_\eta\}$,

$$\begin{aligned} \nu_\xi &= \ddot{\xi}_r + k_d^\xi \dot{e}_\xi + k_p^\xi e_\xi \\ &= m^{-1} f_0 + m^{-1} f \\ \nu_\eta &= \ddot{\eta}_r + k_d^\eta \dot{e}_\eta + k_p^\eta e_\eta \\ &= J_\eta^{-1}\left(\frac{1}{2}\frac{\partial}{\partial\eta}(\dot{\eta}^\top J_\eta \dot{\eta}) - \dot{J}_\eta \dot{\eta} + \tau\right) \end{aligned} \tag{9}$$

From (9) we obtain the controls that keep path and pose tracking. These controls are denoted as $\{f, \tau\}$.

4 FORCE DISTRIBUTION ON THE TIE POINTS

Consider now a set of points $p_i, i = 1, \cdots, n_p$ given in rigid body coordinates, just where moorings are anchored. Let be $F_i, i = 1, \cdots, n_p$ the mooring forces. The mooring forces F_i must be contained in a viable subspace of its space configuration. For example, in case that the clamping is performed by a platoon of quad-rotors, which is the chosen setup in this paper, these forces should be directed to the upper region of the geometric space in relation to the horizontal plane. The forces will point to the configuration platoon. The determination of each mooring force $F_i, i = 1, \cdots, n_p$ is focused as follows, where $a \wedge b$ is a vector product and $< a, b >$ is a scalar product:

1. A set of equality relations g^E concerning the efforts needed to control the rigid body.

$$g^E := \begin{cases} f = \sum_{i=1}^{n_p} F_i \\ \tau = \sum_{i=1}^{n_p} (R(\eta) p_i) \wedge F_i \end{cases} \tag{10}$$

2. The following relationship guarantees a conical opening concerning the mooring forces. It is established as a condition of no intersection between the sphere $|P - P_0| = r$ and the line

$$P = p_i + \lambda \frac{F_i}{|F_i|},\ 0 < \lambda < \infty \tag{11}$$

such that $P_0 = r_o e_3$

$$\begin{aligned} g_1^I &:= < R(\eta) p_i - r_o e_3, F_i >^2 \\ &\leq\ |F_i|^2 \left(|R(\eta) p_i - r_o e_3|^2 - r^2\right) \\ &,\ i = 1, \cdots, n_p \end{aligned} \tag{12}$$

where $e_3 = (0, 0, 1)^\top$. Also r_o, r are parameters governing the cone aperture (see figure 1).

3. The constraints

$$g_2^I := < F_i, e_3 > \geq 0,\ \{i = 1, \ldots, n_p\} \tag{13}$$

are needed for aperture regularization.

4. The objective function to be minimized is

$$F = \sum_{i=1}^{n_p} \|F_i\| \tag{14}$$

The nonlinear optimization problem can be formulated as:

$$\begin{array}{l}\text{Minimize } F(X) \text{ such that}\\ X \in \{g^E(X) \cap g_1^I(X) \cap g_2^I(X)\}\end{array} \tag{15}$$

The process of minimization is fast and efficient using a minimization method such as the Sequential Quadratic Programming Method (Nocedal and Wright, 1999). The initial point at each sample time is the solution previously found at the previous sample time.

Once determined the forces, the cabled ties are propagated in the corresponding direction to a desired length l_i, at which point the respective quad-rotor will absorb the effort. Thus, the position reference trajectory of the quad-rotors is obtained by:

$$\xi_r^i = R(\eta)p_i + \xi + l_i \frac{F_i}{|F_i|}, \; i = 1, \cdots, n_p \tag{16}$$

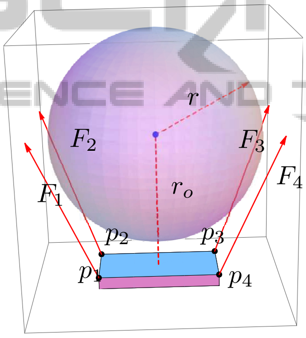

Figure 1: Auxiliary sphere in the definition of feasible lifting forces.

5 QUAD-ROTOR: SIMPLIFIED MODELING AND LAGRANGIAN FORMULATION

The generalized coordinates for a quad-rotor are $q = (\xi_q, \eta_q)$ where $\xi_q = (x_q, y_q, z_q)$, denote the position of the center of mass concerning the inertial frame and $\eta_q = (\psi_q, \theta_q, \phi_q)$ are the three Euler angles (yaw, pitch and roll) representing the quad-rotor pose. The total quad-rotor kinetic energy is given by T_q and the potential energy is given by V_q with the corresponding lagrangian $L_q = T_q - V_q$ (Avila-Vilchis et al., 2003) (Koo et al., 2001), with

$$\begin{aligned} T_q &= \frac{1}{2} m_q \dot{\xi}_q^\top \dot{\xi}_q + \frac{1}{2} \omega_q^\top J_q \omega_q \\ V_q &= m_q g z_q \\ \omega_q &= Q(\eta_q)\dot{\eta}_q \end{aligned} \tag{17}$$

Here m_q denotes the mass of the quad-rotor and $R(\eta_q)$, $Q(\eta_q)$ are defined in (3). The quad-rotor inertia matrix is given by

$$\begin{aligned} J_q &= \begin{pmatrix} J_1^q & J_{12}^q & J_{13}^q \\ J_{12}^q & J_2^q & J_{23}^q \\ J_{13}^q & J_{23}^q & J_3^q \end{pmatrix} \\ J_{\eta_q} &= Q(\eta_q)^\top J_q Q(\eta_q) \end{aligned} \tag{18}$$

Here J_{η_q} is the inertia matrix regarding the inertial frame. The movement equations are:

$$\begin{aligned} \ddot{\xi}_q &= -m_q^{-1}(f_0 - F_i + R(\eta_q) f_b) \\ \ddot{\eta}_q &= J_{\eta_q}^{-1}(\eta_q) \left[\frac{1}{2} \frac{\partial}{\partial \eta_q} \left(\dot{\eta}_q^\top J_{\eta_q} \dot{\eta}_q \right) \right. \\ &\quad \left. - \dot{J}_{\eta_q} \dot{\eta}_q + R(\eta_q) \tau_{\eta_q} \right] \end{aligned} \tag{19}$$

with

$$f_b = \begin{pmatrix} 0 \\ 0 \\ u \end{pmatrix}, \; f_0 = \begin{pmatrix} 0 \\ 0 \\ -m_q g \end{pmatrix} \tag{20}$$

and $\tau_{\eta_q} = (\tau_{\psi_q}, \tau_{\theta_q}, \tau_{\phi_q})$ being the moments regarding the body frame. Those moments can be modeled in a first degree of approximation, and in the local frame without considering rotor dynamics, as:

$$\begin{aligned} u &= \sum_{j=1}^{4} f_j \\ f_j &= B_o \omega_j^2 \\ \tau_{\psi_q} &= (D_o/B_o)(f_2 + f_4 - f_1 - f_3) \\ \tau_{\theta_q} &= l(f_4 - f_2) \\ \tau_{\phi_q} &= l(f_3 - f_1) \end{aligned} \tag{21}$$

where f_j are the lifting forces in each rotor, ω_j the corresponding angular velocities, l the diagonal distance between axes of the respective rotors, and D_o, B_o are drag and thrust factors, respectively. The relationship between $\{f_b, \tau_{\eta_q}\}$ and rotations ω_j is:

$$\begin{aligned} \omega_1^2 &= \frac{1}{4B_o D_o l}(D_o l u - 2D_o \tau_{\phi_q} - B_o l \tau_{\psi_q}) \\ \omega_2^2 &= \frac{1}{4B_o D_o l}(D_o l u - 2D_o \tau_{\theta_q} + B_o l \tau_{\psi_q}) \\ \omega_3^2 &= \frac{1}{4B_o D_o l}(D_o l u + 2D_o \tau_{\phi_q} - B_o l \tau_{\psi_q}) \\ \omega_4^2 &= \frac{1}{4B_o D_o l}(D_o l u + 2D_o \tau_{\theta_q} + B_o l \tau_{\psi_q}) \end{aligned} \tag{22}$$

6 QUAD-ROTOR ERROR TRACKING CONTROLLER

Each quad-rotor will assume the controller role at each mooring link. Each quad-rotor must provide enough effort to counterbalance the mooring force F_i, while maintaining the required path defined through (16). Ideally the link to establish will read $\xi_r^i = \xi_q^i$ where ξ_r^i are the reference position of the center of gravity of the i-th quadrotor and are given by (16) (theoretical anchor point). Instead we will impose a stable error dynamics criteria such as

$$\ddot{e}^i + K_d \dot{e}^i + K_p e^i = 0 \ \{K_d \succ 0, K_p \succ 0\} \quad (23)$$

in which

$$e^i = (e_\xi^i, e_\eta^i) = P_r^i - P_q^i \quad (24)$$

with

$$\begin{aligned} P_r^i &= (\xi_r^i, \eta_r^i) \\ P_q^i &= (\xi_q^i, \eta_q^i) \end{aligned} \quad (25)$$

the reference and actual trajectories of the i-th quadrotor. Establishing now

$$\begin{aligned} \ddot{e}_\xi^i &= \ddot{\xi}_r^i + m_q^{-1}(f_0 - F_i) + m_q^{-1} R(\eta_q^i) f_b^i \\ &= \nu_\xi^i \\ &= -k_{p\xi} e_\xi^i - k_{d\xi} \dot{e}_\xi^i \\ \ddot{e}_\eta^i &= \ddot{\eta}_r^i - J_{\eta_q^i}^{-1} \left[\frac{1}{2} \frac{\partial}{\partial \eta_q^i} \left(\dot{\eta}_q^{i\top} J_{\eta_q^i} \dot{\eta}_q^i \right) \right. \\ &\quad \left. - \dot{J}_{\eta_q^i} \dot{\eta}_q^i + R(\eta_q^i) \tau_q^i \right] \\ &= \nu_\eta^i \\ &= -k_{p\eta} e_\eta^i - k_{d\eta} \dot{e}_\eta^i \end{aligned} \quad (26)$$

with (ν_ξ^i, ν_η^i) the pseudo-control components and $k_{p\xi}$, $k_{p\eta}$, $k_{d\xi}$, $k_{d\eta}$ positive matrices. Here F_i is the mooring force attached to the i-th quad-rotor. The expression for $\ddot{e}^i$ is obtained substituting (19) in (23). From

$$\ddot{\xi}_r^i + m_q^{-1}(f_0 - F_i) + m_q^{-1} R(\eta_o^i) f_b^i = \nu_\xi^i \quad (27)$$

with $f_b^i = u^i e_3$ and solving for $\{u^i, \theta_o^i, \phi_o^i\}$ the system of equations

$$R(\eta_o^i) f_b^i = \Phi^i = m_q(\nu_\xi^i - \ddot{\xi}_r^i) + F_i - f_0 \quad (28)$$

we obtain

$$\begin{aligned} \eta_o^i &= \begin{cases} \theta_o^i = \arctan\left(\sqrt{{\Phi_y^i}^2 + {\Phi_z^i}^2},\ \Phi_x^i \right) \\ \phi_o^i = \arctan\left(-\Phi_z^i,\ -\Phi_y^i \right) \end{cases} \\ u^i &= \left\| \Phi^i \right\| \end{aligned} \quad (29)$$

where η_o^i is the needed attitude. Calling now as attitude correction

$$\Delta\eta^i := \begin{pmatrix} 0 \\ \theta_r^i - \theta_o^i \\ \phi_r^i - \phi_o^i \end{pmatrix} \quad (30)$$

and including the attitude correction in the attitude error dynamics, results in

$$\nu_\eta^i = -k_{p\eta}(e_\eta^i - \Delta\eta^i) - k_{d\eta} \dot{e}_\eta^i \quad (31)$$

which defines the control law:

$$\begin{aligned} \tau^i =& \dot{J}_{\eta_q^i} \dot{\eta}_q^i - \frac{1}{2} \frac{\partial}{\partial \eta_q^i} \left(\dot{\eta}_q^{i\top} J_{\eta_q^i} \dot{\eta}_q^i \right) \\ &+ J_{\eta_q^i} \left(\ddot{\eta}_r^i + k_{p\eta}(e_\eta^i - \Delta\eta^i) + k_{d\eta} \dot{e}_\eta^i \right) \end{aligned} \quad (32)$$

This control will stabilize the P_r^i trajectory tracking with a bounded error. In figure 2 the structure of the controller is shown, consisting of two proportional-derivative terms, namely PD_ξ, PD_η where S_ξ, S_η represent the operations described in equations (27) and (32) respectively. QR represents the plant (quadrotor) and C the generator of trajectory commands.

6.1 Adaptive Augmentation

In order to cancel the presence of unmodeled dynamics, two corrective components are added to the control loops presented in figure 2, which are generated by the single hidden layer neural network adaptive element defined by SHL-NN = (NN_ξ, NN_η). In what follows the control of each quadrotor is analyzed and the superindex i for the i-th quad-rotor is omitted for the sake of clarity. Also the q subindex is omited in ξ_q, η_q.

Let $\Delta = (\Delta_\xi, \Delta_\eta)$ be the vector of modeling errors. Equations (26) can be written as:

$$\begin{aligned} \ddot{e}_\xi &= \ddot{\xi}_r - (\ddot{\xi} + \Delta_\xi) \\ \ddot{e}_\eta &= \ddot{\eta}_r - (\ddot{\eta} + \Delta_\eta) \end{aligned} \quad (33)$$

By adding to the control effort the adaptive terms $\nu_{a\xi}, \nu_{a\eta}$ the following representation of the error dynamics is obtained:

$$\begin{aligned} \ddot{e}_\xi + k_{p\xi} e_\xi + k_{d\xi} \dot{e}_\xi + \nu_{a\xi} - \Delta_\xi = 0 \\ \ddot{e}_\eta + k_{p\eta} e_\eta + k_{d\eta} \dot{e}_\eta + \nu_{a\eta} - \Delta_\eta = 0 \end{aligned} \quad (34)$$

which can also be written as

$$\frac{d}{dt} \begin{pmatrix} e \\ \dot{e} \end{pmatrix} = \begin{pmatrix} O & I \\ -K_p & -K_d \end{pmatrix} \begin{pmatrix} e \\ \dot{e} \end{pmatrix} + B(\nu_a - \Delta) \quad (35)$$

with

$$\begin{aligned} K_p = \begin{pmatrix} k_{p\xi} & O \\ O & k_{p\eta} \end{pmatrix}, \ K_d = \begin{pmatrix} k_{d\xi} & O \\ O & k_{d\eta} \end{pmatrix} \\ B = \begin{pmatrix} O \\ I \end{pmatrix}, \ \nu_a = \begin{pmatrix} \nu_{a\xi} \\ \nu_{a\eta} \end{pmatrix}, \ \Delta = \begin{pmatrix} \Delta_\xi \\ \Delta_\eta \end{pmatrix} \end{aligned} \quad (36)$$

and with $e = (e_\xi, e_\eta)$. Here again, O, I are suitable null and identity matrices respectively. If the SHL-NN output signal ν_a perfectly cancels Δ, then we

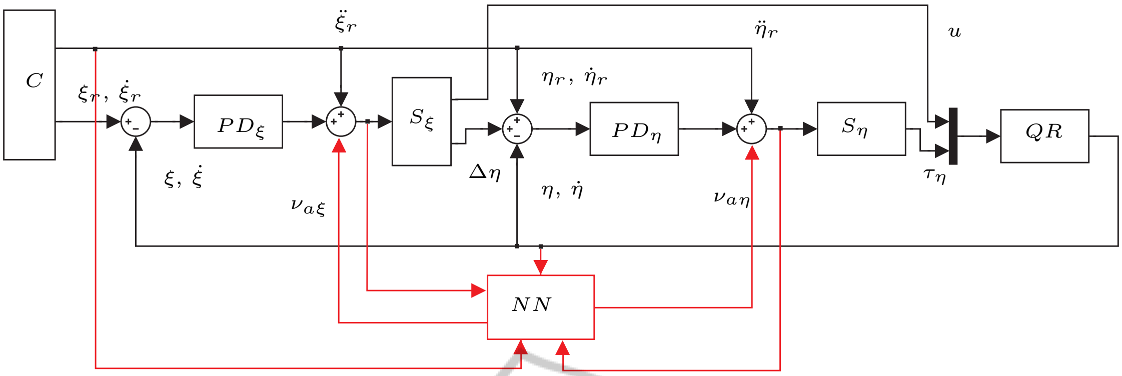

Figure 2: Augmented Linear Controller with an Adaptive SHL-NN.

have asymptotically stable error dynamics. ν_a has the structure

$$\nu_a = \left(W_\xi^\top \bar{\sigma}(V_\xi^\top \bar{\xi}), W_\eta^\top \bar{\sigma}(V_\eta^\top \bar{\eta})\right) \tag{37}$$

Weight propagation for $W_{\{\xi,\eta\}}, V_{\{\xi,\eta\}}$ is done according to the adaptation laws

$$\begin{aligned} \dot{W}_i &= -[(\bar{\sigma} - \bar{\sigma}' V_i^\top \bar{q}) r^\top + \kappa \|e\| W_i]\Gamma_{W_i} \\ \dot{V}_i &= -\Gamma_{V_i}[\bar{q}(r^\top W_i^\top \bar{\sigma}') + \kappa \|e\| V_i] \end{aligned} \tag{38}$$

with $r = (e^\top PB)^\top$, and $i = \{\xi, \eta\}$. The representation of $\bar{\sigma}(V_\xi^\top \bar{q})$ as $\bar{\sigma}$, as well as that of $\bar{\sigma}'$, is done for the sake of clarity. $\Gamma_{V_i} \succ 0, \Gamma_{W_i} \succ 0$ are definite positive matrices and $\kappa > 0$ is a real constant, being $\bar{q}$ the extended input vector, that is, $\bar{q} = (1, q)$ where q is the input vector.

6.2 Obtaining the Adaptation Laws

Let us consider the Lyapunov function

$$\begin{aligned} \mathbf{V}(e, \tilde{V}, \tilde{W}) = \frac{1}{2}\Big(e^\top Pe \\ + \mathrm{tr}\left(\tilde{W}^\top \Gamma_W^{-1} \tilde{W}\right) + \mathrm{tr}\left(\tilde{V}^\top \Gamma_V^{-1} \tilde{V}\right)\Big) \end{aligned} \tag{39}$$

where P solves the equation

$$A^\top P + PA + Q = 0,\ A = \begin{pmatrix} O & I \\ -K_p & -K_d \end{pmatrix} \tag{40}$$

with $-Q$ and P definite positive. In order to obtain the adaptation equations (38) we must follow the steps required to proof that, on the error orbits, the condition $\dot{\mathbf{V}} \leq 0$ is satisfied as explained in (Kannan and Johnson, 2002). The following steps are given in order to show the parameters regarding an adequate tuning of the controller. The details of the proof of convergence follow the above mentioned reference. Let us consider

$$\varepsilon = \nu_a^* - \Delta = W^{*\top} \bar{\sigma}(V^{*\top} \bar{q}) - \Delta \tag{41}$$

where W^*, V^* are the optimum values that best approximate Δ. The error dynamics is

$$\dot{e} = Ae + B\left(W^{*\top} \bar{\sigma}(V^\top \bar{q}) - W^\top \bar{\sigma}(V^{*\top} \bar{q}) + \varepsilon\right) \tag{42}$$

Defining now $\tilde{W} = W - W^*, \tilde{V} = V - V^*$ and using the Taylor series expansion of σ with respect to V in the neighborhood of V^*, which is the optimum value, we obtain

$$\dot{e} = Ae + B\left(\tilde{W}^\top (\sigma - \sigma' V^\top \bar{q}) + W^\top \sigma' \tilde{V}^\top \bar{q} + w\right) \tag{43}$$

with

$$w = \varepsilon - W^{*\top}\left(\sigma^* - \sigma + \sigma' \tilde{V}^\top \bar{q}\right) + \tilde{W}^\top \sigma' V^{*\top} \bar{q} \tag{44}$$

Substituting now (38) and (43) in the expression of $\dot{\mathbf{V}}$ we have

$$\dot{\mathbf{V}} = -\frac{1}{2} e^\top Qe + e^\top PBw - \kappa \|e\| \,\mathrm{tr}\left(\tilde{Z}^\top Z\right) \tag{45}$$

where

$$Z = \begin{pmatrix} V & 0 \\ 0 & W \end{pmatrix},\ \tilde{Z} = Z - Z^* \tag{46}$$

Using $\mathrm{tr}(\tilde{Z}^\top Z) \leq \|\tilde{Z}\| \|Z^*\| - \|\tilde{Z}\|^2$ and following (Kannan and Johnson, 2002) there exist $a_0, a_1, c_3, \kappa > \|PB\| c_3$ such that

$$\begin{aligned} \dot{\mathbf{V}} = -\frac{1}{2}\lambda_{min}(Q)\|e\|^2 - (\kappa - \|PB\| c_3)\|e\| \|\tilde{Z}\|^2 \\ + a_0 \|e\| + a_1 \|e\| \|\tilde{Z}\| \end{aligned} \tag{47}$$

and, with $Z_m = \frac{a_1 + \sqrt{a_1^2 + 4a_0(\kappa - \|PB\| c_3)}}{\kappa - \|PB\| c_3}$,

$$\|e\| \geq \frac{a_0 + a_1 Z_m}{\frac{1}{2}\lambda_{min}(Q)} \Rightarrow \dot{\mathbf{V}} \leq 0 \tag{48}$$

Thus for convenient initial conditions, the tracking error e is ultimately uniformly bounded.

7 SIMULATION RESULTS

In this section we perform the simulation of a ty-pical maneuver, in which a rigid body is transported in an upward helical path, using four identical quad-rotors on the task. During transport the body and the quad-rotors suffer the action of an intense wind gust along the x,y,z axes modeled by

$$\begin{aligned} f_x &= \rho_x \exp\left(\left(\frac{t-t_x}{\sigma_x}\right)^2\right) \\ f_y &= \rho_y \exp\left(\left(\frac{t-t_y}{\sigma_y}\right)^2\right) \\ f_z &= \rho_z \exp\left(\left(\frac{t-t_z}{\sigma_z}\right)^2\right) \end{aligned} \tag{49}$$

where $\rho_x = \rho_y = \rho_z = 2, t_x = 25, t_y = 35, t_z = 40, \sigma_x = \sigma_y = \sigma_z = 2$. The main parametric values in this simulation are: $m = 3, J_1 = 1, J_2 = 2, J_3 = 4$, the mooring points in body coordinates $p_1 = (2,1,0.2)^\top$, $p_2 = (2,-1,0.2)^\top$, $p_3 = (-2,-1,0.2)^\top$, $p_4 = (-2,1,0.2)^\top$ and the mooring ropes with $l_1 = l_2 = l_3 = l_4 = 20$. For the restriction sphere $r = 3.5, r_o = 4$. The path and rigid body pose of the load g.c. along the trip are given by $P_\xi = (\rho\sin(\Omega t), \rho\cos(\Omega t), V_c t + h_o)^\top$ and $P_\eta = (0,0,\Omega t)^\top$ with $\Omega = 0.1, V_c = 0.1$. For each quad-rotor the main data is: $m_q = 2, J_1^q = 0.5, J_2^q = 0.5, J_3^q = 0.2, k_{p_\xi} = 16, k_{d_\xi} = 4, k_{p_\eta} = 120, k_{d_\eta} = 12, \gamma_{W_\xi} = \gamma_{W_\eta} = \gamma_{V_\xi} = \gamma_{V_\eta} = 2, \kappa_\xi = \kappa_\eta = 4$.

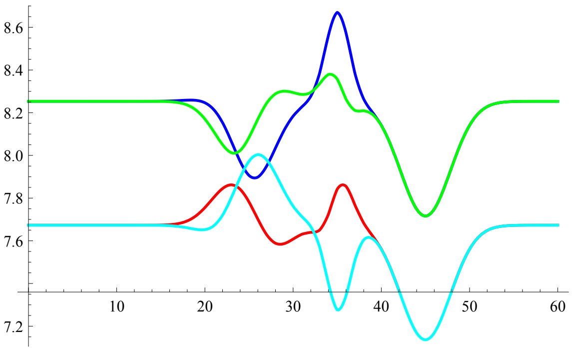

Figure 3: Module forces $|F_i|$ in the four ropes during the maneuver, including simultaneous wind gusts along the $\{x,y,z\}$ axes.

8 CONCLUSIONS

The method presented provides guidelines for the transport of rigid loads through the collaborative effort between agents. Within this approach can also addressed the problem of failures with consequent redistribution of loads among agents. Substituting stable dynamic error cancelation instead of equality holonomic links, can be an effective simplification procedure, according to the simulation results.

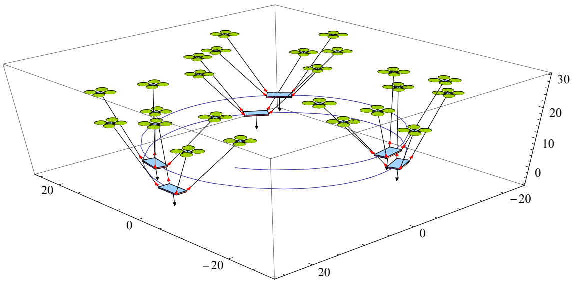

Figure 4: Evolution of a transport maneuver.

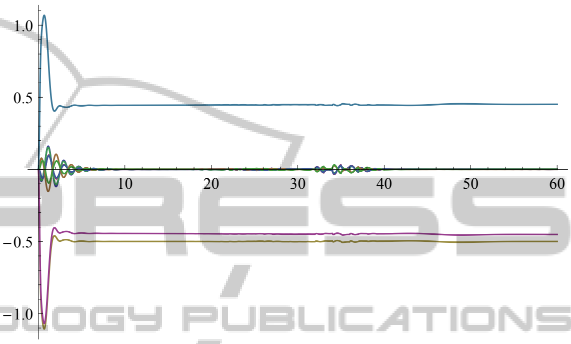

Figure 5: Evolution of W_ξ weights in a typical maneuver.

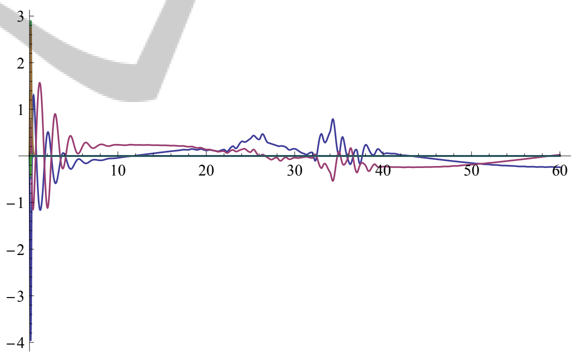

Figure 6: Evolution of W_η weights in a typical maneuver.

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APPENDIX

Approximate System Linearization

One common method for controlling nonlinear dynamical systems is based on approximate feedback linearization (Isidori, 1995), which depends on the relative degree of each controlled variable. For newtonian systems like the quad-rotor in a simplified approach, the regulated variables of interest, here represented as the vector q, have relative degree two

$$\ddot{q} = f(q, \dot{q}, u) \tag{50}$$

The control variables are represented by the vector u. A pseudo control ν is defined such that the dynamic relation between it and the system state is linear $\ddot{q} = \nu$ where $\nu = f(q, \dot{q}, u)$. Since the function $f(q, \dot{q}, u)$ is not exactly known, an approximation $\nu = \hat{f}(q, \dot{q}, u)$ is used which is invertible regarding u, resulting in

$$\ddot{q} = \nu + \Delta(q, \dot{q}, u) \tag{51}$$

where the modeling error is represented by

$$\Delta(q, \dot{q}, u) = f(q, \dot{q}, u) - \hat{f}(q, \dot{q}, u) \tag{52}$$

So the effective actuator can be calculated as

$$\hat{u} = \hat{f}^{-1}(q, \dot{q}, \nu) \tag{53}$$

Supposing in (51) that $\Delta(q, \dot{q}, u) = 0$ we can proceed in the stabilization problem, choosing a linear controller, a PD for instance, that will locally solve the regulation problem. A single hidden layer (SHL) neural network with conveniently adapted weights will be responsible for modeling error cancelation. Including a command path generator C, the former linear controller can be augmented through the architecture depicted in figure 7.

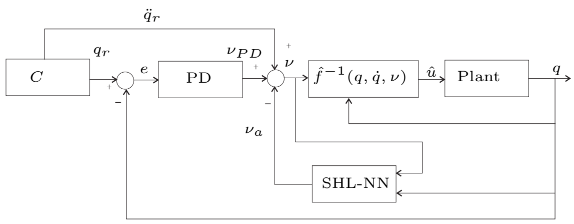

Figure 7: NN augmented adaptive control architecture.

The pseudo control signal in (51) is the sum of three components

$$\nu = \ddot{q}_r + \nu_{PD} - \nu_a \tag{54}$$

where $\ddot{q}_r$ is generated by C, ν_{PD} is generated by the PD controller and ν_a is generated by the adaptive element introduced to compensate for the model inversion error. The tracking error is computed as $e = [q_r - q, \dot{q}_r - \dot{q}]^T$ and the PD controller can be represented by

$$\nu_{PD} = \begin{bmatrix} K_p & K_d \end{bmatrix} e \tag{55}$$

so the tracking error dynamics is given by

$$\dot{e} = Ae + B(\nu_a - \Delta) \tag{56}$$

with

$$A = \begin{bmatrix} O & I \\ -K_p & -K_d \end{bmatrix},\ B = \begin{bmatrix} O \\ I \end{bmatrix} \tag{57}$$

where I and O are a suitable identity and null matrices respectively.

Adaptive Element

The adaptive element is implemented by a SHL-NN with conveniently tuned weights V,W such that

$$\nu_a = W^\top \bar{\sigma}(V^\top \bar{q}) \tag{58}$$

with $\bar{q} = [\nu, q]$. Given a sufficient number of hidden layer neurons and appropriate inputs, it should be possible to train a SHL-NN (Hornik et al., 1989) on line to cancel the effect of Δ. The weight matrices are

$$\begin{aligned} V &= \begin{pmatrix} v_{0,1} & v_{0,2} & \cdots & v_{0,n_2} \\ v_{1,1} & v_{1,2} & \cdots & v_{1,n_2} \\ \vdots & \vdots & \ddots & \vdots \\ v_{n_1,1} & v_{n_1,2} & \cdots & v_{n_1,n_2} \end{pmatrix} \\ W &= \begin{pmatrix} w_{0,1} & w_{0,2} & \cdots & w_{0,n_3} \\ w_{1,1} & w_{1,2} & \cdots & w_{1,n_3} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n_2,1} & w_{n_2,2} & \cdots & w_{n_2,n_3} \end{pmatrix} \end{aligned} \tag{59}$$

Here n_1, n_2, n_3 are the number of inputs, hidden layer nodes and outputs. Also $\bar{\sigma}(\xi) = (1, \sigma(\xi_1), \cdots, \sigma(\xi_{n_1}))^\top$. The scalar function σ is the sigmoidal activation function $\sigma(\xi) = 1/(1+e^{-\alpha\xi})$.

Contractibility

The transformation $\nu_a = W^\top \bar{\sigma}(V^\top \bar{q})$ must be contractive regarding ν_a. Note that Δ depends on ν_a through ν, whereas ν_a has to be designed to cancel Δ. Hence the existence and uniqueness of a fixed point solution for $\nu_a = \Delta(q,\dot{q},\nu_a)$ must be assumed. A sufficient condition is to ascertain that the map $\nu_a \to \Delta(q,\dot{q},\nu_a)$ is a contraction over the entire input domain of interest, or $\|\partial\Delta/\partial\nu_a\| < 1$. This condition is equivalent to (Kim, 2003) (Johnson, 2000) (Kannan and Johnson, 2002)

$$0 < \frac{1}{2}\left|\frac{\partial f}{\partial u}\right| < \left|\frac{\partial \hat{f}}{\partial u}\right| < \infty \tag{60}$$

Consider the system (50), the inverse law $\hat{u} = \hat{f}^{-1}(q,\dot{q},\nu)$ and the contractibility property, as well as the adaptation laws

$$\begin{aligned} \dot{W} &= -\Big((\bar{\sigma} - \bar{\sigma}' V^\top \bar{q}) r^\top + \kappa\|e\| W\Big)\Gamma_W \\ \dot{V} &= -\Gamma_V\Big(\bar{q}(r^\top W^\top \bar{\sigma}') + \kappa\|e\| V\Big) \end{aligned} \tag{61}$$

where $\bar{\sigma}'(\hat{z}) = \frac{\partial\bar{\sigma}(z)}{\partial z}\Big|_{z=\hat{z}}$ is the Jacobian matrix and $r = e^\top PB$. Also $P \succ 0$ is the unique positive definite solution for the Lyapunov equation $A^\top P + PA + Q = 0$ for any convenient $Q \succ 0$. A and B are defined in (57). Given (61) with $\Gamma_W \succ 0, \Gamma_V \succ 0$ and $\kappa > 0$, according to (Nardi, 2000), (Shin, 2005) the tracking error e uniform boundedness is assured.