

# Equalisation of Measured Optical MIMO Channels

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**Abstract:** Within the last years multiple-input multiple-output (MIMO) transmission has reached a lot of attention in the optical fibre community. Theoretically, the concept of MIMO is well understood. However, practical implementations of optical components are in the focus of interest for further computer simulations. That's why in this contribution the specific impulse responses of the  $(2 \times 2)$  MIMO channel, including a 1.4 km multi-mode fibre and optical couplers at both ends, are measured for operating wavelengths of 1326 nm and 1576 nm. Since semiconductor diode lasers, capable of working at different wavelengths, are used for the characterization of the underlying optical MIMO channel, inverse filtering is needed for obtaining the respective impulse responses. However, the process of inverse filtering also known as signal deconvolution is critical in noisy environments. That's why different approaches such as Wiener and parametric filtering are studied with respect to different optimization criteria. Using these obtained impulse responses a baseband MIMO data transmission is modelled. In order to create orthogonal channels enabling a successful transmission, a MIMO zero forcing (ZF) equaliser is implemented and analysed. Our main results given as an open eye-diagram and calculated bit-error rates show the successful implementation of the MIMO transmission system.

## 1 INTRODUCTION

Aiming at further increasing the fibre capacity in optical transmission systems the concept of MIMO, well studied and wide-spread in radio transmission systems, has led to increased research activities in this area (Singer et al., 2008; Winzer, 2012; Richardson et al., 2013). Theoretical investigations have shown that similar capacity increases are possible compared to wireless systems (Kühn, 2006; Tse and Viswanath, 2005). The basis for this approach is the exploitation of the different optical mode groups. However, the practical implementation has to cope with many technological obstacles such as mode multiplexing and management. This includes mode combining, mode maintenance and mode splitting. In order to improve existing simulation tools practical measurements are needed. That's why in this contribution a whole optical transmission testbed is characterized by its respective impulse responses obtained by high-bandwidth measurements.

In order to describe the optical MIMO testbed at different operating wavelengths semiconductor laser diodes with a pulse width of 25 ps are used. Since the used picosecond laser generator doesn't guaran-

tee a fully flat frequency spectrum in the region of interest, inverse filtering has to be applied to obtain the MIMO impulse responses. However, the process of inverse filtering also known as signal deconvolution is critical in noisy environments. That's why different approaches such as Wiener and parametric filtering are studied with respect to different optimization criteria such as the mean square error (MSE) and the imaginary error parameter introduced by Gans (Gans, 1986). Using the measured impulse responses a MIMO baseband transmission system can be constructed. In order to exploit the full potential of the MIMO system, properly selected signal processing strategies have to be applied. The focus of this work is on the whole testbed functionality including the signal processing needed to separate the data streams. Based on computer simulations the end-to-end functionality of the whole testbed is demonstrated and appropriate quality criteria such as the eye-diagram and the bit-error rate (BER) are calculated.

The novelty of this paper is given by the proven testbed functionality, which includes the whole electro-optical path with the essential optical MIMO components of mode combining and splitting. The next logical step is the implementation of the MIMO

receiver modules such as automatic clock recovery, frame synchronisation, channel estimation and equalisation as demonstrated in (Köhnke et al., 2014).

The remaining part of the paper is structured as follows: In section 2 the optical MIMO testbed and its corresponding system model are introduced. The further processing of the measured impulse responses, which is carried out by inverse filtering, is described in section 3. The obtained results are given in Section 4. Finally, Section 5 shows our concluding remarks.

## 2 OPTICAL MIMO SYSTEM MODEL

An optical MIMO system can be formed by feeding different sources of light into the fibre, which activate different optical mode groups. This can be carried out by using centric and eccentric light launching conditions and subsequent combining of the activated different mode groups with a fusion coupler as show in Fig. 1 (Ahrens and Lochmann, 2013; Sandmann et al., 2014).

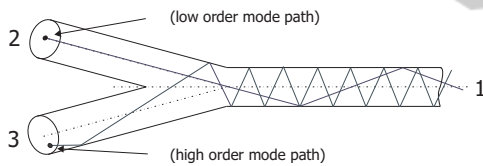


Figure 1: Transmitter side fusion coupler for launching different sources of light into the MMF.

The different sources of light lead to different power distribution patterns at the fibre end depending on the transmitter side light launch conditions. Fig. 2 highlights the measured mean power distribution pattern at the end of a 1.4 km multi-mode fibre. At the end of the MMF transmission line a similar fusion coupler is used for splitting the different mode groups.

The measurement setup depicted in Fig. 3 shows the testbed with the utilized devices for measuring the system properties of the optical MIMO channel in form of its specific impulse responses needed for modelling the MIMO data transmission.

A picosecond laser unit is chosen for generating the 25 ps input pulse. This input pulse is used to measure separately the different single-input single-output (SISO) channels within the MIMO system. Since the used picosecond laser unit doesn't guarantee a fully flat frequency spectrum in the region of interest, the captured signals have to be deconvolved. The obtained impulse responses are forming the base for modelling the MIMO transmission sys-

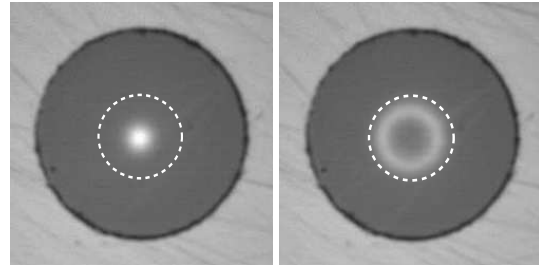


Figure 2: Measured mean power distribution pattern when using the fusion coupler at the transmitter side (left: centric mode excitation; right: eccentric mode excitation); the dotted line represents the  $50\mu\text{m}$  core size.

tem. Fig. 4 highlights the resulting electrical MIMO system model.

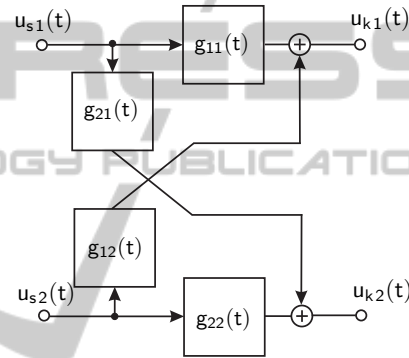


Figure 4: Electrical MIMO system model (example:  $n = 2$ )

## 3 MEASUREMENT CAMPAIGN AND SIGNAL DECONVOLUTION

Since the process of signal deconvolution is critical in noisy environments, different filtering processes such as Wiener and parametric filtering are studied in order to guarantee a high quality of the deconvolution process defined by the mean square error (MSE) and the imaginary error parameter introduced by Gans (Gans, 1986).

A linear time-invariant system is defined uniquely by its impulse response, or its Fourier transform as the corresponding transfer function. For the determination of the impulse response  $g_k(t)$  (see also Fig. 5) an appropriate formed input signal  $u_1(t)$  is needed. Unfortunately, an ideal Dirac delta pulse with a frequency independent transfer function is practically not viable. In real systems adequate impulses compared to the Dirac delta pulse must be used. For the determination of the impulse response in optical transmission systems impulses as specified in (Ahrens

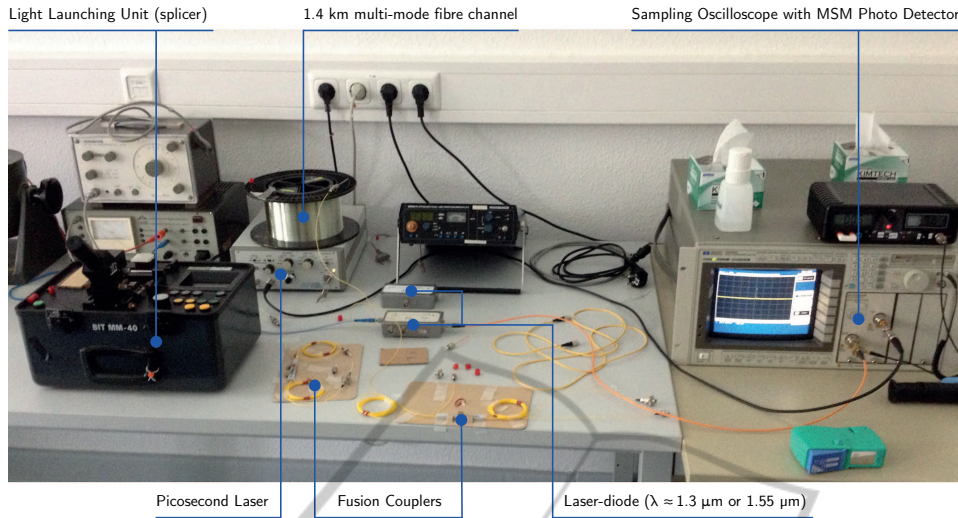


Figure 3: Measurement setup for determining the MIMO specific impulse responses.

et al., 2013) have proven to be useful. Additionally, when analysing the characteristics of any practical system, the measured impulse  $u_3(t)$  is affected by noise. The resulting transmission system model is depicted in Fig. 5. The measured impulse  $u_3(t)$  can

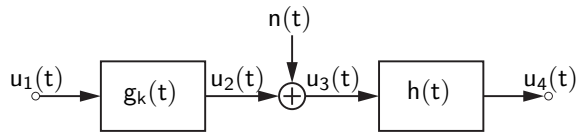


Figure 5: Transmission system model.

be decomposed into two parts, namely, the low-pass filtered output signal  $u_2(t)$  and the noise part  $n(t)$  resulting in

$$u_3(t) = u_1(t) * g_k(t) + n(t) . \quad (1)$$

In the absence of the noise term, i. e.  $n(t) = 0$ , the system characteristic  $g_k(t)$  can be easily obtained by inverse filtering and is given as

$$g_k(t) \circlearrowright G_k(f) = \frac{U_3(f)}{U_1(f)} . \quad (2)$$

Unfortunately, the measured impulse  $u_3(t)$  is affected by the noise term  $n(t)$ , which is assumed to be white and Gaussian distributed. Under these conditions inverse filtering is not working properly anymore. In order to improve the quality of the signal deconvolution different filter functions  $h(t)$  are applied and the filtered signal results in

$$u_4(t) = u_1(t) * g_k(t) * h(t) + n(t) * h(t) . \quad (3)$$

This filter operation affects both the low-pass filtered output signal  $u_2(t)$  and the noise term  $n(t)$ . With an

appropriate selected filter function the estimation of the impulse response  $g_k(t)$  yields to

$$\hat{g}_k(t) \circlearrowright \hat{G}_k(f) = \frac{U_4(f)}{U_1(f)} . \quad (4)$$

Hereinafter, two different filter functions are studied to estimate the impulse response  $g_k(t)$  based on the measured impulse  $u_3(t)$ . Commonly, the mean square error (MSE) between the impulse response  $g_k(t)$  and the estimated impulse response  $\hat{g}_k(t)$  is chosen as a quality indicator. It is expressed as

$$F_{\text{MSE}} = E\{[g_k(t) - \hat{g}_k(t)]^2\} \rightarrow \min. , \quad (5)$$

where  $E\{\cdot\}$  denotes the expectation functional.

Firstly, the Wiener filter  $h_w(t)$  is investigated. It is based on finding the optimal solution for minimizing the MSE when comparing the signal  $u_2(t)$  with the filter output signal  $u_4(t)$ . It is calculated as

$$E\{[u_2(t) - u_3(t) * h_w(t)]^2\} \rightarrow \min. , \quad (6)$$

Assuming the signal  $u_2(t)$  and the noise  $n(t)$  are uncorrelated, the Wiener filter transfer function results in (Vaseghi, 2000, pp. 191-194)

$$H_w(f) = \frac{S_{22}(f)}{S_{22}(f) + S_{nn}(f)} , \quad (7)$$

where  $S_{22}(f)$  is the power spectral density (PSD) of the signal  $u_2(t)$  and  $S_{nn}(f)$  is the noise PSD of the signal  $n(t)$ .

A more simple filter choice when estimating the impulse response  $g_k(t)$  is represented by predefined parametric filter functions. A possible transfer function presented in (Nahman and Guillaume, 1981) and

studied more closely in (Sandmann et al., 2013) is given by

$$H_R(f) = \frac{|U_1(f)|^2}{|U_1(f)|^2 + \gamma \cdot |C(f)|^2}, \quad \gamma \in \mathbb{R}, \quad (8)$$

with:

$$|C(f)|^2 = 6 - 8 \cos(2\pi f T_a) + 2 \cos(4\pi f T_a), \quad (9)$$

where  $T_a$  is the sampling period. The regularisation function  $H_R(f)$  is a low-pass filter with the parameter  $\gamma$  influencing the sharpness of the filter and hence determining the cutoff frequency. In order to appropriately select this parameter the MSE criterion (5) can be applied for the optimisation. In practical measurements the knowledge of the original impulse response  $g_k(t)$  is not given. Therefore, another criterion is needed in order to properly select the  $\gamma$ -parameter for practical measurements. A promising criterion was introduced by Gans (Gans, 1986), where the root mean square of the deconvolved imaginary part of  $\hat{g}_k(t)$  is used for finding the parameter of the regularisation function. This optimisation criterion can be expressed as

$$F_{\text{Gans}} = E\{[\text{Im}\{\hat{g}_k(t, \gamma)\}]^2\} \rightarrow \min. \quad (10)$$

Using this criterion multiple local minima can occur and therefore another criterion described by Nahman and Guillaume in (Nahman and Guillaume, 1981, pp. 22) should be taken into consideration when choosing the  $\gamma$  value of the regularisation filter. This error criterion is defined as the MSE between the measured receive signal  $u_3(t)$  and the simulated receive signal  $u_1(t) * \hat{g}_k(t, \gamma)$ , where  $u_1(t)$  is the measured input impulse. It is described as follows

$$F_{\text{Error}}(\gamma) = E\{[u_3(t) - u_1(t) * \hat{g}_k(t, \gamma)]^2\}. \quad (11)$$

In order to compare the quality of the estimated impulse responses using the regularisation filter to the quality achieved by the Wiener filter, the following system is studied: The input impulse is a Dirac delta pulse with  $u_1(t) = U_s T_s \delta(t)$ , with  $U_s = 1$  V,  $T_s = 1$  ms and  $T_s/T_a = 20$ . The chosen impulse response is

$$g_k(t) = \frac{1}{T_s} \text{rect}\left(\frac{t}{T_s}\right). \quad (12)$$

In this case the filter output signal  $u_2(t)$  is an rectangular impulse with the amplitude  $U_s$ . The deconvolution quality results are depicted in Fig. 6 as a function of the signal-to-noise-ratio  $E_s/N_0$  with the parameter  $E_s$  defining the signal energy of  $u_2(t)$  and the noise power spectral density  $N_0$  of the signal  $n(t)$ .

When applying the regularisation filter  $H_R(f)$  the optimal  $\gamma$  values as well as the MSE are decreasing

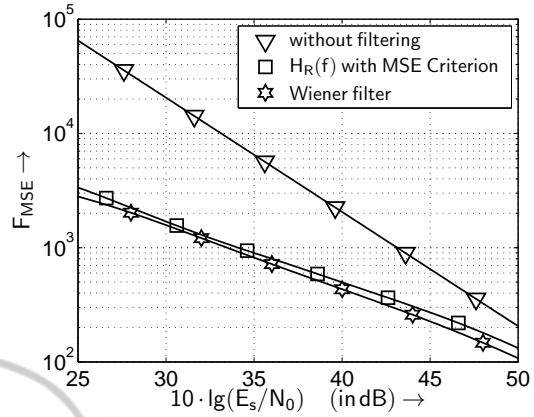


Figure 6: Quality  $F_{\text{MSE}}$  of the deconvolved impulse responses as a function of signal energy to noise power spectral density using different filter functions.

with increasing  $E_s/N_0$ . Therein the parameter  $E_s$  describes the average symbol energy and  $N_0$  the power spectral density of the white noise process  $n(t)$ . The achievable quality of the estimated impulse responses using the regularisation filter together with the MSE optimisation criterion comes close to the Wiener filter results. The benefit of using a filter function is clearly visible.

In order to determine the quality of the estimated impulse responses, which are practically obtainable using the Gans' criterion (10), the following optical system configuration at a operating wavelength at 1576 nm is studied: The measured input impulse of the picosecond laser is depicted in Fig. 7 for different operating wavelengths with a pulse width of approximately 25 ps. The impulse response is now assumed

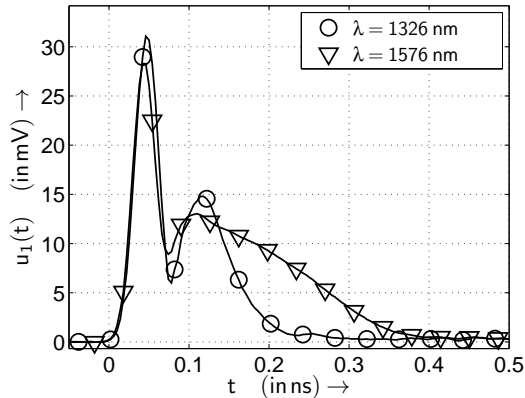


Figure 7: Measured input impulses at different operating wavelengths  $\lambda$ . to be Gaussian

$$g_k(t) = A e^{-\pi(t/T_s)^2}, \quad (13)$$

where  $T_s = 0.8$  ns and  $T_s/T_a = 200$ . The scaling factor  $A$  is chosen to maintain  $E_s/T_s = 1$  V<sup>2</sup> of the sig-

nal  $u_2(t)$  and to ensure the unit  $s^{-1}$  of the impulse response. Fig. 8 shows the quality of the obtained impulse responses using the two filter functions mentioned before. The regularisation filter is applied for both optimisation criteria resulting in  $\gamma$  values depicted in Fig. 9. The  $\gamma$  values are also decreasing with

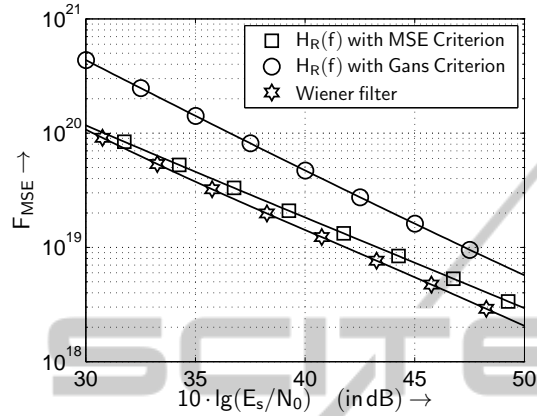


Figure 8: Quality  $F_{MSE}$  of the deconvolved impulse responses as a function of signal energy to noise power spectral density using different filter functions.

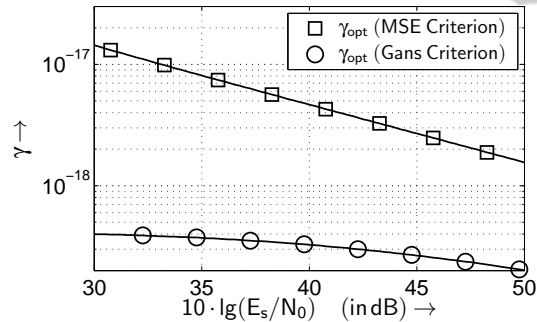


Figure 9: Choice of optimal  $\gamma$  when filtering with  $H_R(f)$  minimizing the MSE and using the Gans criterion.

increasing  $E_s/N_0$  for both criteria. It should be noted, that the  $\gamma$  values using the Gans' criterion are lower compared to the MSE criterion. This signifies that the measured signal  $u_3(t)$  is filtered less when applying the filter using the Gans' criterion in contrast to using the MSE criterion. As expected, the deconvolved impulse responses using the Wiener filter are showing the best quality of all applied filter functions closely followed by the estimated impulse responses filtered with the regularisation function using the MSE optimisation criterion. The quality of the estimated impulse responses using the Gans' criterion is still acceptable for a wide range of  $E_s/N_0$  values and provides a major improvement compared to the quality without filtering. The obtained results show further that the parametric regularisation filter function is a

good compromise compared to the Wiener filter with its high complexity.

Applying the described deconvolution processing to the  $(2 \times 2)$  MIMO testbed, the obtained impulse responses are depicted in Fig. 10-11. They are calculated by applying the regularisation filter in the deconvolution process with  $\gamma$  values respecting the Gans' and Error criterion. At an operating wavelength of

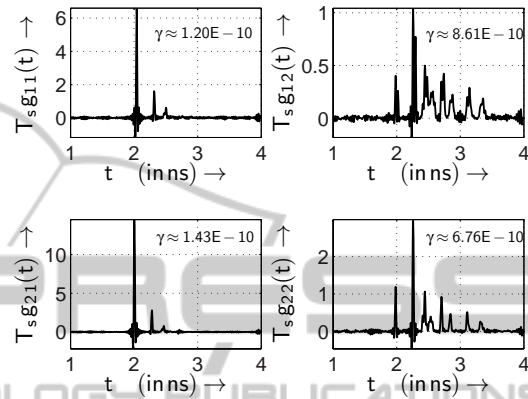


Figure 10: Deconvolved measured electrical MIMO impulse responses with respect to the pulse frequency  $f_T = 1/T_s = 620$  MHz at 1326 nm operating wavelength using the regularisation filter function with  $\gamma$  values according to the Gans' criterion.

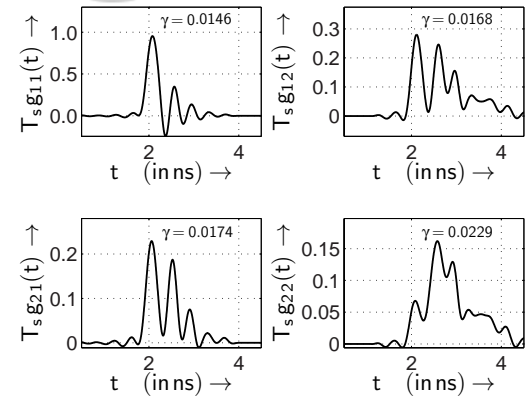


Figure 11: Deconvolved measured electrical MIMO impulse responses with respect to the pulse frequency  $f_T = 1/T_s = 620$  MHz at 1576 nm operating wavelength using the regularisation filter function with  $\gamma$  values according to the Gans' criterion.

1326 nm the modal structure can be identified. Considering the 1576 nm results the additional influence of the chromatic dispersion is clearly visible.

## 4 MIMO EQUALISATION AND SIMULATION RESULTS

In this section the MIMO baseband transmission system is constructed as illustrated in Fig. 12. It uses the deconvolved  $(2 \times 2)$  MIMO specific impulse responses  $g_{i,j}(t)$  (for  $i = 1, 2$  and for  $j = 1, 2$ ) depicted in Fig. 11 at 1576 nm operating wavelength. In this baseband system model the transmitter forms a rectangular pulse train and hence the transmit filter  $g_s(t)$  and the receive filter  $g_{ef}(t)$  are considered to be matched filters and are described in its non causal notation with

$$g_s(t) = g_{ef}(t) = \frac{1}{T_s} \text{rect}\left(\frac{t}{T_s}\right). \quad (14)$$

The total transmit power is normalised to  $P_s = 1 \text{ V}^2$  and a symbol pulse frequency of  $f_T = 1/T_s = 620 \text{ MHz}$  per data channel is used resulting in a total bit rate of 1.24 Gb/s for both channels. Both transmit signals  $u_{s,j}(t)$  are launched onto the  $(2 \times 2)$  MIMO channel. The filtered receive signals  $u_{e,i}(t)$  are sampled with  $kT_s$ , where  $k \in \mathbb{Z}$ . The system can be simplified by introducing the cumulative channel impulse response  $h_{i,j}(t)$  and the filtered noise  $w_i(t)$  expressed as follows

$$\begin{aligned} h_{i,j}(t) &= g_s(t) * g_{i,j}(t) * g_{ef}(t), & h_{i,j}(k) &= h_{i,j}(t) \Big|_{kT_s} \\ w_i(t) &= n_i(t) * g_{ef}(t), & w_i(k) &= w_i(t) \Big|_{kT_s} \end{aligned} \quad (15)$$

By utilising a data block transmission model (Raleigh and Cioffi, 1998; Raleigh and Jones, 1999) a vectorial notation can be applied as follows

$$\begin{aligned} \mathbf{c} &= (c[1] \ c[2] \ \dots \ c[K])^T \\ \mathbf{h}_{i,j} &= (h_{i,j}[1] \ h_{i,j}[2] \ \dots \ h_{i,j}[L])^T. \end{aligned} \quad (16)$$

Using the convolution matrices  $\mathbf{H}_{i,j}$  the transmission model can be described as

$$\begin{aligned} \mathbf{u}_1 &= \mathbf{H}_{11} \cdot \mathbf{c}_1 + \mathbf{H}_{12} \cdot \mathbf{c}_2 + \mathbf{w}_1 \\ \mathbf{u}_2 &= \mathbf{H}_{21} \cdot \mathbf{c}_1 + \mathbf{H}_{22} \cdot \mathbf{c}_2 + \mathbf{w}_2. \end{aligned} \quad (17)$$

Written in matrix notation

$$\begin{pmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} \\ \mathbf{H}_{21} & \mathbf{H}_{22} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{c}_1 \\ \mathbf{c}_2 \end{pmatrix} + \begin{pmatrix} \mathbf{w}_1 \\ \mathbf{w}_2 \end{pmatrix}. \quad (18)$$

Simplifying this equation results in

$$\mathbf{u} = \mathbf{H} \cdot \mathbf{c} + \mathbf{w}, \quad (19)$$

where the channel matrix  $\mathbf{H}$  contains the ISI as well as the crosstalk information. For obtaining the transmitted symbols unaffected from the channel

$$\mathbf{F} \cdot \mathbf{H} = \mathbf{I} \quad (20)$$

has to be fulfilled, where  $\mathbf{I}$  is a identity matrix and thus the equaliser matrix  $\mathbf{F}$  can be obtained as follows

$$\mathbf{F} = (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H. \quad (21)$$

Hereinafter, the equaliser matrix  $\mathbf{F}$  is applied to the received data vector  $\mathbf{u}$

$$\begin{aligned} \mathbf{y} &= \mathbf{F} \cdot \mathbf{u} \\ \mathbf{y} &= \mathbf{c} + \mathbf{F} \cdot \mathbf{w}. \end{aligned} \quad (22)$$

The benefit of applying this zero forcing (ZF) equaliser is the orthogonalisation of the transmission channels. Thus, the resulting equalised MIMO system can be described by two independent single input single output (SISO) channels. The disadvantage of using the ZF equaliser is the weighting of the noise term.

Eye diagrams of both received signals in the MIMO system after equalisation are shown in Fig. 13. Using the ZF equaliser both eyes are fully opened

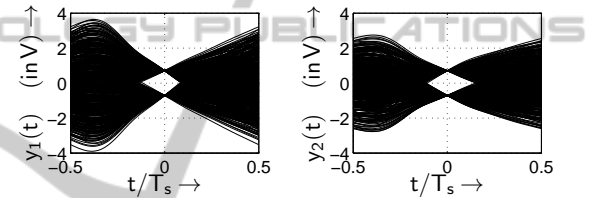


Figure 13: Eye diagram patterns of both received signals when applying zero forcing equalisation.

confirming its functionality. The MIMO bit-error rate (BER) simulation results are depicted in Fig. 14 and underline the functionality of the equaliser.

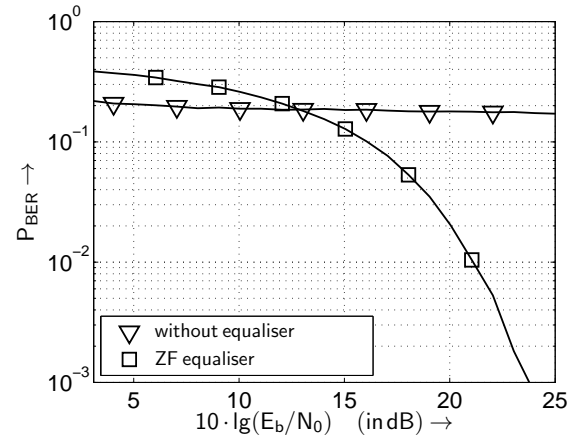
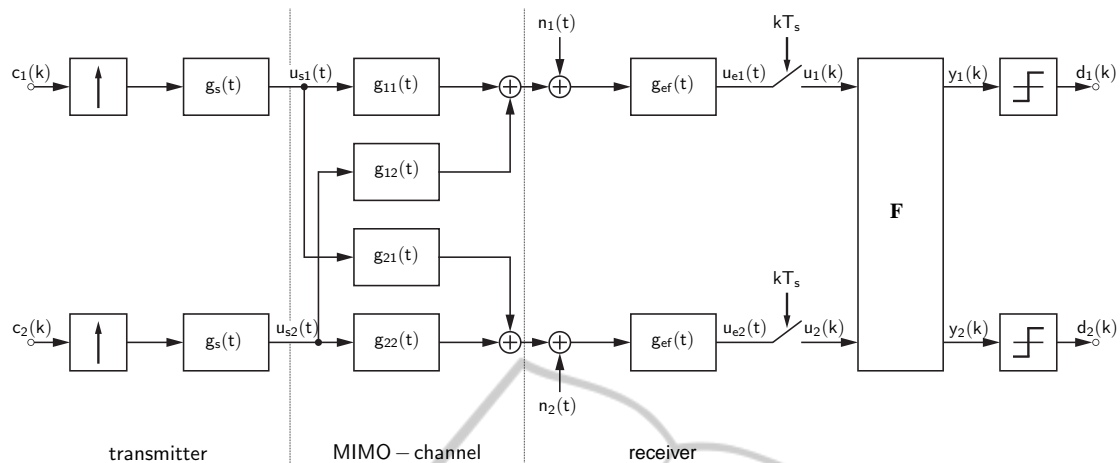


Figure 14:  $(2 \times 2)$  MIMO BER probability results with and without applying the zero forcing equalising method using the deconvolved MIMO impulse responses at 1576 nm operating wavelength and transmitting with a bit rate of 1.24 Gb/s.


 Figure 12:  $(2 \times 2)$  MIMO baseband transmission system model with discrete zero forcing equaliser.

## 5 CONCLUSIONS

In this contribution a  $(2 \times 2)$  optical MIMO communication system, consisting of a 1.4 km multi-mode fibre and optical couplers attached to both ends, has been analysed. The estimations of the MIMO specific impulse responses have been obtained for operating wavelengths of 1576 nm and 1326 nm using optimized signal deconvolution by applying the parametric regularisation filter. It has been shown that the quality of the estimated impulse responses significantly improves and is comparable to Wiener filtering. These estimated impulse responses have been used for modelling a baseband MIMO data transmission system. In order to receive the transmitted data unaffected from the data sent on the neighbouring channel zero forcing equalisation has been investigated. The successful implementation has been shown by the bit-error curves as well as by the open eye-diagram.

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