A Possibility of Fast Running of *Tyrannosaurus rex* by the Result of Evolutionary Computation

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Abstract: The author examined the effectiveness of the optimization strategy of evolutionary computation and the conventional simulated annealing method when studying the locomotor motion of bipedal animals. The simulated annealing method is known as a powerful tool for finding near-optimal solutions for combinatorial problems such as the NP-complete problem. However, the author found the evolutionary computational strategy more effective at finding near-optimal solutions of the running motion of bipedal animals. The author conducted extensive simulations of the running motion of the large, bipedal dinosaur *Tyrannosaurus rex* based on realistic, biological parameters. The author's simulations found that *T.rex* could run quickly, up to 14 m/s, which is faster than the beings.

1 INTRODUCTION

Simulated annealing is well known as a powerful simulation technique for obtaining near-optimal solutions for combinatorial problems such as the NP-complete problem, which is typically a traveling salesman problem (TSP) (Usami and Kano, 1995; Usami and Kitaoka, 1997). Furthermore, the statistical physics theory has proven that a slow annealing schedule leads to a near-optimal solution. Regarding TSP, the simulated annealing method is said to be equally appropriate with evolutionary computation methods such as the genetic algorithm (GA) (Holland, 1992; Goldberg, 1989). In addition, it is easier to solve TSP in program coding by using simulated annealing rather than GA.

On the contrary, the evolutionary computation strategy is known to be applicable to a variety of optimization problems. For example, the evolutionary computation strategy has been used in searching for a near-optimal solution of animal locomotion (Sellers and Manning, 2009; Usami et al., 1998). In this paper, the author has tested the conventional simulated annealing method and the evolutionary computation method to obtain the locomotion pattern of a bipedal animal. Consequently, the author found that evolutionary computation is more effective than the simulated annealing technique in this case. The reason may lie in the parameter dependence sensitivity of the total

system; a slight change in the parameters induces large changes in the locomotion pattern.

On the basis of this finding, the author conducted extensive simulations to determine the running motion of the bipedal carnivorous dinosaur *Tyrannosaurus rex* (*T.rex*). *T.rex* is the largest bipedal theropod that lived in the Cretaceous period (145–66 million years ago (Ma)). Its maximum estimated weight was up to 8 tons (Hutchinson et al., 2011). In 2002, Hutchinson and Garcia published a paper titled, "*Tyrannosaurus* was not a fast runner" (Hutchinson and Garcia, 2002). They assumed several patterns of midstance posture in running motion and calculated the required muscle mass to hold that posture. Using the Froude number ($Fr = v^2/hg$, where v, h, and g represent velocity, hip height, and the gravity constant, respectively), they stated that *T.rex* could not run at a speed of 20 m/s (Alexander, 1976, 1983, 1989, 2006). However, their discussion was based on static mechanics; no explicit speed estimation was involved in the framework of their study. In 2009, Sellers and Manning reported the first numerical simulation study for this problem. They published a result stating that a running speed of 9–10 m/s was possible for *T.rex*. However, a faster running speed would have been problematic (Fig.1).

In this paper, the author presents the numerical simulation results of *T.rex's* running motion. The simulation methodology is compared with the

simulated annealing and evolutionary computation methods, and the results of extensive numerical simulations are presented.
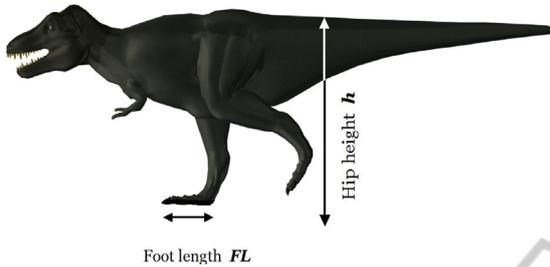
Figure 1: *Tyrannosaurus rex* and its hip height.

2 SIMULATION OF *T. REX*'S RUNNING MOTION

2.1 Mechanics

The motion of running is a periodic one; hence, expressing the time change of each joint angle using the Fourier expansion series is appropriate. The validity of this method was checked in advance using human locomotion. A human's running motion was captured by a combination of optical measurements and the use of a force plate on the ground. These data were analyzed by the reliable system VICON (Vicon Motion Systems). The time change of each joint angle was expressed using the Fourier expansion series. Convergence within 1% accuracy was checked, taking into account the 5th order of the Fourier expansion. Thus, expressing the 5th order of the Fourier expansion is a good method for describing the periodic motion of each joint.

For the i-th joint angle, the expansion is expressed as follows:

$$\theta_i(t) = a_i^0 \sin(0t + \delta_i^0) + a_i^1 \sin(\omega t + \delta_i^1) + \cdots + a_i^5 \sin(5\omega t + \delta_i^5) \quad , \tag{1}$$

where a_i^j and δ_i^j are the amplitude and the phase of the *j*-th order of the expansion series for the *i*-th angle, respectively and ω is the angular velocity. The segmented structure of *T.rex* is the same as that of Hutchinson and Garcia's model, shown in Fig. 2 (a).

To study time-dependent dynamics, a solid object model is used to describe the motion of the *T. rex* limb. Namely, the *T.rex* model moves as one solid object for the external force according to the following equations:

$$m_{body} \frac{d^2 \vec{X}}{dt^2} = -m_{body} g \big|_y + \vec{F}(\vec{r}) \quad , \tag{2}$$

$$I \frac{d^2 \Phi}{dt^2} = \vec{r} \cdot \vec{F}(\vec{r}) \quad , \tag{3}$$

where $\vec{X}$ are Φ the position vector of the center of mass and the rotational angle of the object, respectively. The calculation is achieved in the sagittal plane, i.e., two-dimensional space *x* (horizontal) and *y* (vertical). *I*, g, and $\vec{F}(\vec{r})$ are *T. rex's* momentum of inertia, the gravitational constant, and the position vector to the point of the force, respectively. The second term in Eq. (2) expresses the fact that gravitational force acts in vertical direction *y*. The value of inertia *I* in this work is chosen to be $I = 19000$ kg·m². Note that Hutchinson et al.'s value is $I_{zz} = 19200$ kg·m² for a 6583 kg *T.rex*, where I_{zz} is the inertia around the axis perpendicular to the sagittal plane (Hutchinson et al. 2007). The value Bates et al. used is 18890.29 kg·m² for the "HAT" (Head-Arms-Torso) of a 6071.82 kg *T.rex* (Bates et al. 2009).
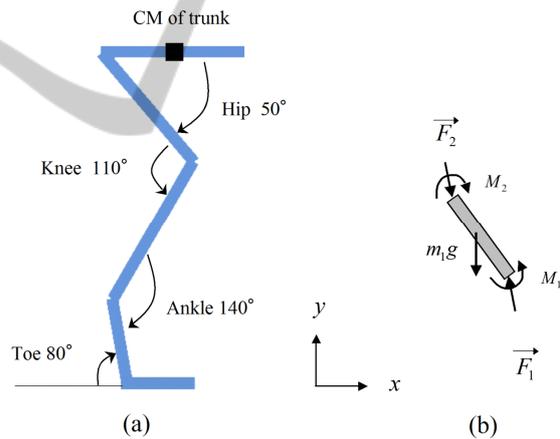
Figure 2: Segment model of a *T.rex* leg (a) and a free-body diagram (b). The angle denoted in (a) represents the best guess model in the reference (Hutchinson and Garcia, 2002).

Both studies used the same *T.rex* specimen, MOR 555; however, different reconstructions led to their slightly different estimations of inertia. The author's value is set close to these values. A solid object model is simple, yet is also known to express the dynamics of a moving object with many degrees of freedom (Usami et al., 1998).

To calculate joint torque, or the moment of force about the joint, a free-body diagram analysis is applied, as shown in Fig. 2 (b). For example, call the foot segment "segment 1", and define the mass and

moment of inertia as m_1 and I_1, respectively. The equations of motion for translation and rotation then become as follows in the (x, y) plane:

$$\vec{F}_1 - \vec{F}_2 - m_1 g\big|_y = m_1 \vec{a}_1, \tag{4}$$

$$\vec{x}_{1g} \times \vec{F}_1 - \vec{x}_{2g} \times \vec{F}_2 + M_1 - M_2 = I_1 \dot{\omega}_1, \tag{5}$$

where $\vec{F}_1$, $\vec{F}_2$, and $\vec{a}_1$ are the force from the downside segment, the force from the upper segment, and acceleration, respectively. For rotational motion, $\vec{x}_{1g}$ and $\vec{x}_{2g}$ are the vectors from the center of mass of the 1-th segment to the points of acting force $\vec{F}_1$ and $\vec{F}_2$. M_1 and M_2 are the moments of force between the 0-th and 1-th and the 1-th and 2nd segments, respectively. For the case of the 1-st segment, $\vec{F}_1$ corresponds to the ground reaction force and $\dot{\omega}_1$ is the time derivative of the angular velocity. Inserting known terms $\vec{F}_1$, $\vec{a}_1$, $\dot{\omega}_1$, and M_1 into Eq. (5) yields unknown terms $\vec{F}_2$ and M_2. Thus, we obtain the moment of force on the upper segment.

Solid object approximation is used to find the motion of the whole body. In the expression, the total mass M, moment of inertia I, and gravitational constant g are 6071 kg, 9.80 m/s², and 19000 $\mathrm{kg \cdot m^2}$, respectively. The external force is the ground reaction force (GRF), which acts in vertical direction y according to the relation $F_y(y) = -ky - \gamma v_y$, where y and v_y are the depth from the horizontal level and the vertical velocity, respectively. This relation is composed using Hooke's law with spring constant k = 1.0×10^7 N/m, dumping term with coefficient γ = 2.0×10^5 Ns/m in our simulation. This model gives an appropriate solution of running motion with a wide range of parameters k and γ in the simulation.

2.2 Evolutionary Computation Method

The next task is searching the optimal Fourier coefficient for running motion; other parameters are fixed in the simulation. Using the computational method to obtain the optimal solution when there are many degrees of freedom is usually not an easy task; therefore, a variety of approximation methods have been proposed in many research areas. One of the most famous and well-studied methods is the genetic algorithm (GA) (Fraser and Burnell, 1970; Holland, 1975; Goldberg, 1989). Numerous studies have been published in many research areas concerning GA, which is based on the idea of gene evolution, as observed in actual life systems. In this method, a

digitized virtual gene is introduced and its evolution is simulated. The virtual gene falls into a stable state in which the value of evaluation function has a local minimum. However, the introduction of virtual genes is unnecessary for this study. Therefore, looking for another convenient approximation method is appropriate.

Another approximation method for obtaining a near-optimal solution is the evolutionary computation method (Sellers and Manning, 2009; Usami, 1998; Fogel, 1995). This method is as well known as the genetic algorithm method when searching for near-optimal solutions. The evolutionary computation method does not introduce a digitized virtual gene but changes system parameters directly. Parameters rapidly converge into the local minimum and the result is usually satisfactory. The evolutionary computation method is used on this problem.

First, several typical patterns of the running motion were created using the 3D software 3ds Max (from Autodesk). The typical patterns include various motions from flexed to upright. Next, we apply the dynamical simulation described above. At the first stage of evolutionary computation, the *T. rex* model usually falls on the ground in the simulation space. The parameters are then improved by the evolutionary computation method. The original set of parameters is slightly and randomly changed within a certain range. These sets of parameters are referred to as the children of the original parent set. Running motions with slightly changed parameters are calculated, and the best performing child is selected. The parent of the best performing parameter set again has children who have slightly different values from the parent; thus, the near-optimal solution for running motion is obtained as a result of the evolutionary computation method.

The simulation has many choices for the evaluation function to choose from when obtaining the appropriate solution and many function types were tried. Consequently, choosing between running motion as the product of ground reaction force or of forward velocity is suitable for this problem's evaluation function. This is mainly due to the fact that the legs of the segment model inevitably rotate around each joint, thus generating a driving force to move in any direction. Taking this condition then yields a smooth running motion for the segment model of *T. rex*.

A result of simulated annealing method is compared to the one of evolutionary computation method, which is shown in Fig.3. The simulation is

meant to find a running motion faster than 14 m/s. The optimization parameter is the vertical acceleration, which is related to the required muscle mass fraction mi (Eq. (6)). The smaller the value of mi, the larger the probability that T. rex is able to run fast.
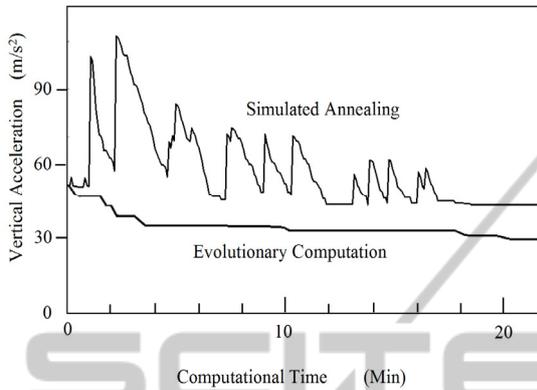
Figure 3: Comparison of the optimization profile in simulated annealing and evolutionary computation.

As is observed from Fig.3, evolutionary computation method shows better convergence to lower vertical acceleration rather than simulated annealing method. The reason may lay in strong sensitivity of the motion to the parameters set appeared in Eq.(1). Gradual improvement of solution by evolutionary computation method is appropriate for this case to obtain near optimal solution of the system.

Concerning to the dependence of selection and reproduction rule in the evolutionary computation, we have tested three well known methods. Those are proportionate selection, roulette wheel and roulette wheel plus elitist selection. Figure 4 displays the results for the case of 1000 indivituals, which are the same condition of Fig.3 as *T.rex* running motion.
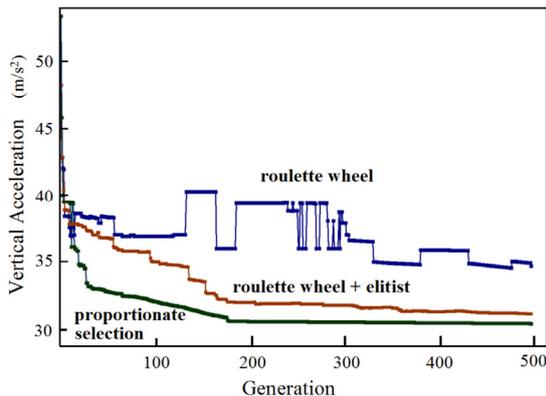
Figure 4: Comparison of the optimization profile in three different optimization methods in evolutionary computation.

As is observed from Fig.4, roulette wheel selection shows bad convergence to near optimal solution. However, the introduction of elitism to the roulette wheel selection makes remarkable improvement to the result. The elitism is a way to keep best individual into the calculation of next generation. In our calculation, proportionate selection gives the best result. It shows fast convergence, and gives the lowest value of vertical acceleration among all simulations.

2.3 Maximum Running Speed of *T.rex*

The main question of this research is, "What is the maximum running speed of *T.rex*?" This paper presents the author's simulation results. However, it should be noted that the results depend on simulation conditions. For example, our model of *T. rex* has segments with lengths of 1.13 m, 1.26 m, 0.699 m, and 0.584 m for the thigh, shank, metatarsus, and foot, respectively; these lengths are identical to those used with Hutchinson et al.'s works (Hutchinson and Garcia, 2002; Hutchinson, 2004; Gatesy et al., 2009). If these values change even slightly, the results may be different from those we describe in this section. However, concerning to biological parameters appeared in this theoretical formulation, we did extensive study to cover known biological parameters. The result is appeared in the seperate work (Usami, 2014).

We have calculated huge numbers of running simulation trials. Figure 5 shows some of those trials that resulted in fast running speeds. In general, the vertical acceleration increases with running speed; however, this tendency is not uniform across many different running patterns. Figure 5 shows the increased minimum vertical acceleration that occurred with increased running speed.
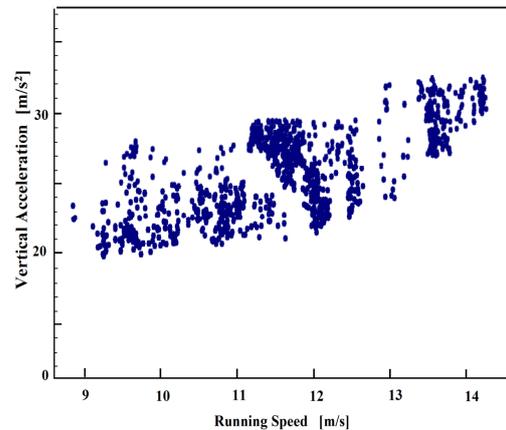
Figure 5: Data of running speed and vertical acceleration that appeared in our simulation.

For the value of vertical acceleration, two or three times larger value than gravity (9.8 m/s^2) is plausible for animal running motion. Thus, the maximum running speed that appeared in the simulation was around 14 m/s, which can be observed in Fig. 5.
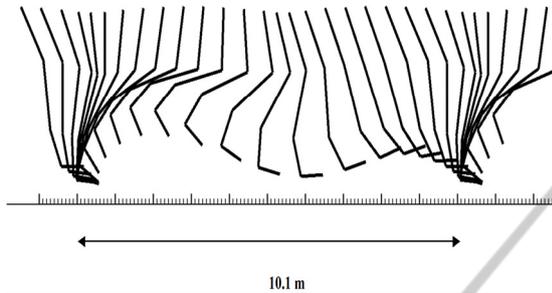
10.1 m

Figure 6: Stick diagram of the segment in running motion at a speed of 14.1 m/s, as it appeared in the simulation. The stride length is 10.1 m and the cyclic period is 0.716 s. This is the fastest running speed that appeared in the simulation with a moderate vertical acceleration of 28.3 m/s^2.

Figure 6 shows a sample of the calculation from this high-speed region with a velocity of 14.1 m/s and a moderate vertical acceleration of 28.3 m/s^2. The running motion with a 14.1 m/s velocity is shown as a stick diagram whose stride length is 10.1 m and whose cyclic period is 0.716 s.

Faster running speeds show considerably larger values for vertical acceleration; for example, a velocity of 20 m/s leads to a vertical acceleration of 60 m/s^2.

The required muscle mass m_i of a 14.1 m/s running speed during the stance phase is shown in Fig. 6. m_i represents the muscle mass fraction for the i-th joint to the total body mass:

$$m_i(\%) = \frac{100 M_i L d}{\sigma c r m_{\mathbf{body}} \cos\theta} \quad (6)$$

In Eq. (6), muscle density $d = 1.06 \times 10^3$ kgm^{-3} and the fraction of active muscle volume $c = 1$ are relatively reliable parameters. The total body mass m_{body} is not an intrinsic parameter because joint moment M_i includes the $m_{\mathrm{body}} \times$ (gravitational acceleration) term, which leads to cancelation of the factors. Although the expression does not contain a total mass factor, the theory is intended to apply in the case of *T. rex* with a total mass of 6 tons. The estimation results in $m_{\mathrm{body}} = 6071.82$ kg for the MOR 555 sample (Hutchinson et al., 2007).

L, r, and θ represent the muscle fiber length, moment arm, and pennation angle of the muscle fiber, respectively. The employed values for these parameters are the same as those of previous works

(Hutchinson and Garcia, 2002; Hutchinson, 2004; Gatesy et al., 2009).

σ represents the maximum muscle stress and setting this value is a controversial problem. The author's future publication describes this parameter in detail. For now, it is sufficient to say that the reported value of σ appears to span as wide a range as 11–220 N/cm^2. This is probably because of species' adaptation of muscle ability. The reported value of σ is in the range of 11–90.3 N/cm^2 for the human knee and ankle muscle groups. However, it is noted that much of the data for σ is located in the range of 20–40 N/cm^2. Thus, in this work, σ is set as 30 N/cm^2, which is the same as that of previous works (Hutchinson and Garcia, 2002; Hutchinson, 2004; Gatesy et al., 2009).

Figure 7 shows the required muscle mass fraction m_i as it corresponds to the stance phase shown in Fig. 6. It is observed that no value of m_i exceeds 7%. Hutchinson states that if m_i surpasses 7%, the bipedal animal is less likely to run quickly (Hutchinson, 2004). Thus, the results of our numerical simulations suggest the possibility that *T. rex* could run quickly. The sum of m_i, $m_{\mathrm{total}} = \sum m_i$, is also shown in Fig. 7.
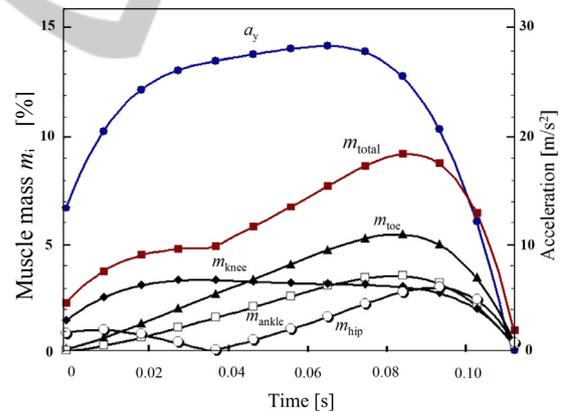
Figure 7: Required muscle mass fraction m_i for the i-th segment of the leg in the stance phase shown in Fig. 6. Black triangle, white square, black rhombus, and white circle represent m_i values for the toe, ankle, knee, and hip, respectively. The vertical acceleration of the center of mass is also shown as a black circle. In this case, the maximum vertical acceleration is 28.3 m/s^2, which yields a maximum required muscle mass $m_{\mathrm{total}} = 9.2\%$. Note that σ is set as 30 N/cm^2 in this graph.

In Hutchinson's work, m_i of the toe joint is omitted (Hutchinson, 2004) because the ankle extensors could have produced most of the required toe joint moments.

It is observed that the maximum value of m_{total} is

9.2%, yet computer-aided mass distribution analysis reveals an m_{total} of 14.2%–16.0% in other studies (Bates et al., 2009; Hutchinson et al., 2007). Thus, the result of even this criterion in the running simulation suggests the possibility that *T.rex* could run quickly.

If it is 36cm cranial that is a value obtained in the work (Hutchinson et al., 2007), the value of m_{total} is corrected as approximately twice larger one. A detailed discussion on the problem of the center of mass is given in forthcoming work (Usami, 2014).

The maximum value of vertical acceleration, as shown in Fig. 7, is 28.3 m/s². A vertical acceleration of 18.3 m/s² (~1.87 × 9.8) is allowed in Gatesy et al.'s estimation of static postures (Gatesy et al., 2009); however, they also state, "In light of the large number of options available to most theropods at running GRFs of 2-4 BW (Body Weight), further optimization analysis and consideration of the entire stride cycle may reveal why specific poses are chosen over so many alternatives." This study's result of 29.3 m/s² (~3.0 BW) is within this range. In addition, the entire stride cycle was obtained by dynamical calculation with well-described parameters. Thus, this work is a possible answer to their unsolved question.

2.4 Comparison with Other Works

This section provides a detailed discussion on the use of the Froude number to simulate a large, bipedal dinosaur running. In our simulation, the Froude number is calculated as Fr = 2.7–6.5 for a running velocity v = 8.9–14.1 m/s. The Froude number Fr is defined as

$$Fr = \frac{v^2}{hg} \ , \qquad (7)$$

where v, h, and g are the velocity, hip height, and gravity constant, respectively. Our simulations are shown as the white triangle in Fig. 8. The black triangle shows Sellers et al.'s data as reported in their Fig. 4 (Sellers and Manning, 2009). The black square shows the data of Hutchinson and Garcia for the case of h = 2.5 m (Hutchinson and Garcia, 2002).

From a fossilized footprint found in Texas, U.S.A., Farlow reported a dinosaur trackway of L_{st} = 6.59 m and a stride/hip height ratio of 4.3 (Farlow, 1981). From Farlow's data, the hip height and Froude number are calculated as h = 1.50 m and Fr = 8.39, respectively.
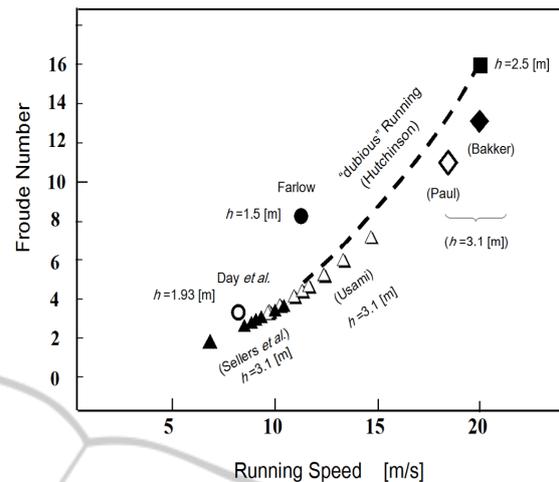
Figure 8: Froude number vs. dinosaur running speed from the literature and this study.

Day et al. reported a dinosaur trackway of L_{st} = 5.65 m found in 163-million-year-old strata in the U.K (Day et al., 2002) and reported an estimated hip height of h = 1.93 m from the foot and a running speed that might be v = 8.11 m/s. The corresponding Froude number in this case is calculated as Fr = 3.5. These observational data are shown as black and white circles in Fig. 8.

In Fig. 8, black and white triangles show the simulation results from Sellers and Manning (Sellers and Manning, 2009) and this paper, respectively. Note that Sellers and Manning presented data requiring a muscle mass of 22.5% for Fr = 3.8 and a running speed of 10.7 m/s. The black circle represents Farlow's data from the fossilized footprint with a hip height of h = 1.5 m (Farlow, 1981). The white circle represents Day et al.'s data from the footprint with a hip height of h = 1.93 m (Day et al., 2002). The black and white rhombuses are Bakker and Paul's estimations for running speeds of 20 m/s and 17.9 m/s, respectively, with an assumption of h = 3.1 m; their running speeds are based on the morphological consideration of muscle and limb structure (Bakker, 1986; Paul, 1988). Despite their different methodologies, all these works state that *T.rex* would run at a speed of 7–20 m/s.

On the contrary, only works using the static method state that *T.rex* would not be able to reach speeds of 20 m/s (Hutchinson and Garcia, 2002). In 2004, Hutchinson stated, "speeds >11 m/s remain dubious" (Hutchinson, 2004). The thick dashed curve in Fig. 6 shows Hutchinson's "dubious" range (Hutchinson, 2004), whereas the black square represents Fr = 16 with a running speed of 20 m/s

and a hip height of $h = 2.5$ m, data from which Hutchinson and Garcia claim no running could occur. (Hutchinson and Garcia, 2002).

From the data, it seems that Hutchinson et al.'s assumption of $Fr = 16$ would be a large value for *T. rex's* ability to run. However, note that speed estimation using Froude number is qualitative and has uncertainty in quantitative evaluation.

3 CONCLUSIONS

In 2011, Hutchinson *et al.* conducted 3D scanning of four adult and one juvenile specimens of well-preserved *T.rex* skeletons and analyzed their mass distributions (Hutchinson et al., 2011). In particular, remarkable from their report is an evaluation of the amount of extensor muscle for a leg. Because muscles are composed of extensor and flexor muscles, the evaluation of the extensor muscle is a monumental contribution to this field.
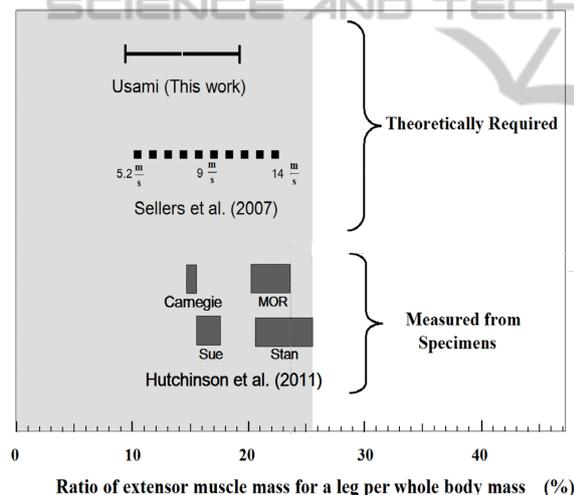
Figure 9: Ratio of extensor muscle mass for a leg per whole body mass. The bottom four specimens are CM 9380 (Carnegie), FMNH PR 2081 (Sue), MOR 555 (MOR), and BHI 3033 (Stan) (Hutchinson et al., 2011).

The ratio of the extensor muscle mass relative to the whole body mass is shown at the bottom of Fig. 9. The upper three data in Fig. 9 show theoretically required values of the ratio. Note that the most probable body mass estimation of these four specimens is in a range of 6000–9500 kg, which is heavier than the one assumed in this paper based on earlier studies. As seen from this graph, the theoretically required and measured data overlap. Thus, it cannot be said that *T.rex* could not run fast.

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