Modeling Neutron Dynamics in Nuclear Reactor using Fractional-order Point Reactor Kinetics Model with Adiabatic Temperature Feedback

Vishwesh A. Vyawahare[1] and P. S. V. Nataraj[2]
[1]*Department of Electronics Engineering, Ramrao Adik Institute of Technology, Nerul, Navi Mumbai, India*
[2]*IDP in Systems and Control Engineering, Indian Institute of Technology Bombay, Powai, Mumbai, India*

Keywords: Nuclear Reactor, Point Reactor Kinetics Model, Dynamic Model, Fractional-order Modeling.

Abstract: This paper deals with the development and analysis of fractional-order (FO) point reactor kinetics (FPRK) model with reactivity feedback for a nuclear reactor. Incorporation of adiabatic temperature feedback of reactivity makes this model nonlinear. It basically forms a system of coupled, nonlinear ordinary differential equations. The nonlinear subprompt critical FPRK model is developed and analyzed in detail. Fundamental motivation for this model is the fact that neutron transport inside the core of a reactor is truly a subdiffusion. The work presented here analyzes the effect of temperature feedback on the neutron concentration dynamics inside reactor core which is modeled using fractional differential equations. The system of nonlinear differential equations is solved numerically. The analysis clearly establishes the fact that the proposed model is 'stable' in the sense that it predicts self-limitting power excursions. The model presented in this paper constitutes an important step in the development of fractional-order model for a nuclear reactor, which can be used to achieve better control and operation.

1 INTRODUCTION

The heart of a nuclear power plant is the nuclear reactor. In this, the heat energy is generated by carrying out a controlled fission of nuclei of fissile radioactive materials with the help of neutrons. Fission reactions are a result of neutrons moving inside the reactor core and colliding with the nuclei of core material. Due to the use of radioactive materials and high probability of this fission chain reaction becoming uncontrollable, utmost care has to be taken to design, construct, maintain, and operate/control a nuclear reactor. In view of this, the mathematical modeling of nuclear reactor is a key step in designing an efficient and safe reactor. As given in (Duderstadt and Hamilton, 1976; Glasstone and Sesonske, 2002), this reactor model is fundamentally based on the model of neutron transport in reactor core. Thus, the validity and applicability of reactor model will depend on how perfectly one models the neutron transport in its core.

In the classical analysis, the diffusion approximation of neutron transport is used widely. The integer-order (IO) neutron diffusion equation (in one-dimension), based on the Fick's constitutive law, is

$$\frac{1}{v}\frac{\partial \phi(x,t)}{\partial t} + (\Sigma_a - \nu\Sigma_f)\phi(x,t) = D\frac{\partial^2 \phi(x,t)}{\partial x^2}, \quad (1)$$

where v is the neutron velocity, $\phi(x,t)$ is the neutron flux at location x at time instant t, D is the diffusion coefficient, ν is the average number of neutrons emitted per fission reaction, and Σ_a, Σ_f are the respective macroscopic cross-sections of absorption and fission reactions. But the concept of modeling neutron transport as diffusion has some problems, viz., the diffusion model is applicable mainly in the moderator of the core and should not be used to model the neutron movements near the regions with strong absorption, next, it predicts infinite speed of propagation of neutrons (Beckurts and Wirtz, 1964; Meghreblian and Holmes, 1960; Espinosa-Paredes et al., 2008). In an attempt to rectify these shortcomings and achieve a better representation of neutron movements, an FO neutron telegraph equation model was proposed in (Vyawahare and Nataraj, 2010; Vyawahare and Nataraj, 2013b) (see (2)). It was developed using the stochastic technique of continuous-time random walk (CTRW) as given in (Compte and Metzler, 1997) for a slab reactor.

$$\frac{\tau^\alpha}{v^\alpha}\frac{\partial^{2\alpha}\phi(x,t)}{\partial t^{2\alpha}} + M_1\frac{\partial^\alpha \phi(x,t)}{\partial t^\alpha} + M_2\phi(x,t) = D\frac{\partial^2 \phi(x,t)}{\partial x^2} \quad (2)$$

where $0 < \alpha < 1$, and $M_1 = \tau^\alpha(\Sigma_a - \nu\Sigma_f) + 1/v^\alpha$ and $M_2 = \Sigma_a - \nu\Sigma_f$. The terms like $\frac{\partial^\alpha}{\partial t^\alpha}$ denote the Caputo

Vyawahare V. and Nataraj P..
Modeling Neutron Dynamics in Nuclear Reactor using Fractional-order Point Reactor Kinetics Model with Adiabatic Temperature Feedback.
DOI: 10.5220/0005038103520360
In *Proceedings of the 4th International Conference on Simulation and Modeling Methodologies, Technologies and Applications* (SIMULTECH-2014), pages 352-360
ISBN: 978-989-758-038-3

fractional time-derivatives of order α (see (4)). The FO point reactor kinetics (FPRK) model was reported in (Vyawahare and Nataraj, 2012), which is a system on coupled nonlinear ordinary differential equations (ODEs):

$$\begin{aligned} \frac{d^\alpha}{dt^\alpha}P(t) &= \frac{\rho(t)-\beta}{\Lambda}P(t)+\lambda C(t), \\ \frac{d}{dt}C(t) &= \frac{\beta}{\Lambda}P(t)-\lambda C(t), \end{aligned} \quad (3)$$

with one delayed neutron group (1G). Here $P(t)$ is the reactor power, $\rho(t)$ is the reactivity, Λ is the mean generation time between the birth of neutron subsequence absorption inducing fission, $C(t)$ is the average concentration of the delayed neutrons, and λ is the average decay constant of the delayed neutrons. Various versions of the FPRK model were reported in (Vyawahare and Nataraj, 2013a). The FPRK model (or the PRK model in general) forms a set of coupled, nonlinear differential equations in reactor power $P(t)$ and delayed neutron concentration $C(t)$.

Note that the Caputo fractional time-derivative definition is considered in models (2) and (3) which is defined as follows (Samko et al., 1997). The Caputo fractional derivative (FD) of order $\alpha \in \mathbb{R}^+$ of a causal function $f(t)$ is given by

$${}_0D_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)}\int_0^t \frac{f^n(\tau)}{(t-\tau)^{\alpha-n+1}}d\tau, \quad (4)$$

with $n \in \mathbb{N}, n-1 < \alpha < n$, and $f^n(\tau)$ is the n^{th}-order derivative of the function $f(t)$. It is seen that this definition requires $f(t)$ to be n-times differentiable and furthermore this derivative has to be integrable. This condition makes definition (4) quite restrictive. Nevertheless, it is preferred by engineers and physicists because FO differential equations (FDEs) with Caputo derivatives have same initial conditions (ICs) (which have well-defined physical meanings) as that for the integer-order differential equations. Recently, fractional derivatives have been extensively used for modeling a variety of systems and processes (Das, 2011; Magin, 2006), and also in control (Monje et al., 2010). One of the major applications of FDEs is in modeling of anomalous diffusion occurring in complex systems (Compte and Metzler, 1997). These FDE models are found to be more realistic and compact than their counterparts, the classical integer-order models. For a detailed history and an exhaustive bibliography of fractional calculus and its applications, see (Machado et al., 2011).

The point reactor kinetics model establishes dependence of the neutron flux or power in reactor core on reactivity. The remarkable feature about reactor mechanism is that *the reactivity also depends on the power*. So there is an *inherent* feedback (negative, in fact) present in the reactor (Duderstadt and Hamilton, 1976; Hetrick, 1993). There is a kind of 'cyclic' mechanism related to neutron flux and reactivity: reactivity affecting power, which in turn affects the reactivity. The justifications explaining the dependence of reactivity on the power can be summarized simplistically as

1. Reactor power depends on the reactivity.
2. Core temperature depends on the reactor power.
3. Reactivity depends on the core temperature.

In this paper, we consider this feedback mechanism to develop FO point reactor kinetics (FPRK) model with temperature feedback of reactivity, which mimics the situation of a subprompt critical reactor subjected to a small positive reactivity ($\rho_0 < \beta$). Literature survey reveals that there have been only two attempts in which the analysis of FPRK model with reactivity feedback is carried out. The first of these references (Espinosa-Paredes et al., 2014) analyzes the FPRK model with Newtonian reactivity feedback and uses the FPRK model developed in (Espinosa-Paredes et al., 2011). The second contribution (Vyawahare and Nataraj, 2014) reports the analysis of FO Nordheim-Fuchs model with adiabatic temperature feedback of reactivity. To the best of our knowledge, this is for the first time that the development of a subprompt critical nonlinear fractional-order point reactor kinetics model using adiabatic temperature feedback of reactivity for a nuclear reactor is reported.

The proposed FPRK model is based on the fundamental assumption of considering the neutron transport as anomalous diffusion, particularly subdiffusion (Klages et al., 2008; Metzler and Klafter, 2000). The literature survey reveals that there have been attempts to develop other types of fractional-order models of the neutron transport and the nuclear reactor, (Espinosa-Paredes et al., 2008; Das and Biswas, 2007; Sardar et al., 2010), (Das et al., 2011; Espinosa-Paredes et al., 2011), (Kadem, 2009; Kadem and Baleanu, 2010). For a detailed and rigorous review on PRK models in general, see (Espinosa-Paredes et al., 2011).

The paper is organized as follows. Next section discusses in brief the inherent reactivity feedback mechanism present in nuclear reactor. In section 3, development of the proposed FPRK model is presented. Analysis of the proposed FO model is given in section 4. A comparison of the results with the IO point reactor kinetics (IPRK) model in terms of time evolution of power, reactivity and reactor core temperature is presented. Issues related to the use of various solution methods (both analytical and numer-

ical) for solving the stiff FPRK model are discussed in detail. Conclusion is given in section 5. The appendix at the end gives the numerical algorithm used for solving the fractional-order model.

2 REACTIVITY FEEDBACK MECHANISM IN NUCLEAR REACTOR

The feedback mechanism in the reactor can be represented using the block diagram in Fig. 1 (see (Duderstadt and Hamilton, 1976), (Hetrick, 1993)). Let the

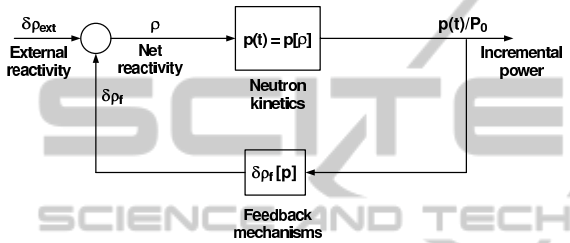

Figure 1: Closed-loop configuration with reactivity feedback.

reactor be represented by the PRK model. We start with the assumption that the reactor is operating at a steady-state equilibrium power level P_0. Now there will be two types of reactivities present in the reactor,

$$\begin{aligned} \rho_f[P_0] &\equiv \text{feedback reactivity due to } P_0, \\ \rho_0 &\equiv \text{external reactivity.} \end{aligned} \tag{5}$$

The feedback reactivity $\rho_f[P_0]$ mostly corresponds to a negative reactivity, trying to reduce the neutron flux, and ultimately the power. Hence the feedback shown in Fig. 1 is to be considered as a negative feedback, and is also known as the power defect in reactivity (Duderstadt and Hamilton, 1976). If we allow this process to continue, it will result into the gradual reduction in the number of fission reactions and so in the number of neutrons. The reactor will gradually become more and more subcritical and a time will come when the reactor will eventually shut down. In order to keep the reactor running and maintain its criticality, an external (positive) reactivity ρ_0 must be applied (like withdrawal of control rods) to balance the negative reactivity such that

$$\rho(t) = \rho_0 + \rho_f[P_0] = 0. \tag{6}$$

Now let the power change to a new value $P(t)$. The incremental power is defined as the deviation of the power from the equilibrium value,

$$p(t) \equiv P(t) - P_0. \tag{7}$$

The corresponding incremental changes in the reactivities can be expressed as

$$\begin{aligned} \delta\rho_{ext}(t) &= \rho_{ext}(t) - \rho_0, \\ \delta\rho_f[p] &= \rho_f[P] - \rho_f[P_0], \end{aligned} \tag{8}$$

where, $\rho_f[P] \equiv$ feedback reactivity due to $P(t)$, and $\rho_{ext}(t) \equiv$ external reactivity to counterbalance $\rho_f[P]$. As a result, the net reactivity input to the reactor is comprised of two components,

$$\rho(t) = \delta\rho_{ext}(t) + \delta\rho_f[p], \tag{9}$$

which is depicted in the block diagram of Fig. 1.

3 FPRK MODEL WITH REACTIVITY FEEDBACK

In this section, we develop the FPRK model with feedback of reactivity. Note that we consider the situation of one delayed group (1G) only, although extending this model to the six delayed group case is quite trivial and straightforward.

First, we consider the IPRK model with reactivity feedback (Hetrick, 1993), which is a system of three nonlinear IO ordinary differential equations (IDEs):

$$\begin{aligned} \frac{d}{dt}P(t) &= \frac{\rho(t)-\beta}{\Lambda}P(t) + \lambda C(t), \\ \frac{d}{dt}C(t) &= \frac{\beta}{\Lambda}P(t) - \lambda C(t), \\ \frac{d}{dt}\rho(t) &= \frac{d}{dt}\gamma(t) - K_C\alpha_T P(t), \end{aligned} \tag{10}$$

where, $\gamma(t)$ is the impressed reactivity, α_T is the temperature coefficient of reactivity and K_C is the reciprocal of the reactor heat capacity. The ICs are $P_0 = P(0)$, $C_0 = C(0)$, and $\rho_0 = \rho(0)$.

Now we derive the FPRK model with reactivity feedback. The 1G FPRK model (3) is rewritten:

$$\begin{aligned} \frac{d^\alpha}{dt^\alpha}P(t) &= \frac{\rho(t)-\beta}{\Lambda}P(t) + \lambda C(t), \\ \frac{d}{dt}C(t) &= \frac{\beta}{\Lambda}P(t) - \lambda C(t). \end{aligned} \tag{11}$$

We now append to this system the reactivity feedback equation. Temperature feedback for time-varying reactivity is given by (Hetrick, 1993)

$$\rho(t) = \gamma(t) - \alpha_T(T(t) - T_0), \tag{12}$$

where $\gamma(t)$ is the impressed time-varying reactivity and $T(t)$ is the reactor core temperature with T_0 being the initial temperature at $t = 0$. Next the adiabatic model is:

$$\frac{d}{dt}T(t) = K_C P(t). \tag{13}$$

Differentiating (12) with respect to t, and using (13), we get the ODE for reactivity, as

$$\frac{d}{dt}\rho(t) = \frac{d}{dt}\gamma(t) - \alpha_T K_C P(t). \tag{14}$$

Thus we have a system of three nonlinear differential equations: an FDE for $P(t)$ and two ODEs for $C(t)$ and $\rho(t)$ as

$$\begin{aligned} \frac{d^\alpha}{dt^\alpha}P(t) &= \frac{\rho(t)-\beta}{\Lambda}P(t) + \lambda C(t), \\ \frac{d}{dt}C(t) &= \frac{\beta}{\Lambda}P(t) - \lambda C(t), \\ \frac{d}{dt}\rho(t) &= \frac{d}{dt}\gamma(t) - K_C\alpha_T P(t). \end{aligned} \tag{15}$$

This is the FPRK model with reactivity feedback. It is also appended with three ICs P_0, C_0, and ρ_0.

In the next section, we analyze and compare the IO and FO models (10) and (15) for a step-type reactivity insertion. This will give us a clear picture of how the reactor power varies for a sudden change in reactivity input with the negative temperature feedback when subdiffusive neutron transport framework is used.

4 ANALYSIS OF THE PROPOSED MODEL

In this section, we carry out a thorough analysis of the proposed FPRK model (15). We make the reactor below prompt critical, that is a step reactivity variation of much smaller magnitude ($\rho_0 < \beta$) is impressed. It will be seen that the negative temperature feedback of reactivity limits the power rise, eventually bringing it back to zero (Duderstadt and Hamilton, 1976; Hetrick, 1993; Nahla, 2009). Since this nonlinear model is very difficult to solve analytically, we go for the numerical solution. The Adams-Bashforth-Moulton method, used widely in the field of fractional calculus and FO control was used to solve these models. The variations in power, precursor concentration, temperature, and reactivity are obtained for different values of fractional power α. Some issues related to the convergence of the numerical method are reported and discussed.

A certain amount ρ_0 of positive reactivity is suddenly inserted into the reactor. We need to keep the reactor below prompt critical (Hetrick, 1993) by choosing $\rho_0 < \beta$. Thus, $\gamma(t) = \rho_0 \Rightarrow \frac{d}{dt}\gamma(t) = 0$. So the ODE for reactivity in IPRK and FPRK models (10), (15) becomes

$$\frac{d}{dt}\rho(t) = -\alpha_T K_C P(t). \tag{16}$$

We use the data from (Duderstadt and Hamilton, 1976): $\beta = 0.0075$, $\lambda = 0.08$ sec^{-1}, $\Lambda = 10^{-3}$ sec. The ICs chosen are $P_0 = 1$ watts, $C_0 = 93.75$, and $\rho_0 = 0.0025$. We take $K_C = 0.05$ K/MW sec and $\alpha_T = 5 \times 10^{-5}$ K^{-1}. Variation in the reactor temperature $T(t)$ is also studied. It is obtained using (12) (with $\gamma(t) = \rho_0$) as

$$T(t) = \frac{\rho_0 - \rho(t)}{\alpha_T} + T_0. \tag{17}$$

As a convention it is assumed that $T_0 = 0°$C.

The IPRK system (10) was solved using the MATLAB ODE solver `ode15s` suitable for the stiff ODEs (Mathworks, 2005). This particular solver was chosen because the PRK models incorporating temperature feedback of reactivity, in general, form a stiff system of nonlinear ODEs (Espinosa-Paredes et al., 2011; Aboanber and Nahla, 2004). The step-size was $h = 1 \times 10^{-3}$ sec. Time-variation of power, delayed neutron precursor concentration, reactivity, and temperature are shown in Fig. 2.

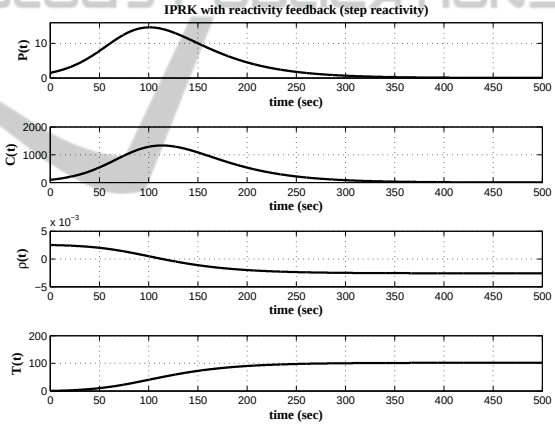

Figure 2: IPRK model with reactivity feedback: plots for $P(t)$, $C(t)$, $\rho(t)$, and $T(t)$ for a step reactivity input.

As we see, the power starts rising due to the insertion of positive reactivity. However, its rate of increase is much slower. This peculiar behaviour is due to the presence of delayed neutrons as they help in slowing down the dynamics of the reactor. This increase in power causes the reactor temperature to rise. The adiabatic negative temperature feedback shows its effect and reactivity starts decreasing. The power attains a peak value of $P_{max} = 14.63$ watts at the instant $t = 101.75$ sec. The reactivity and the precursor concentration at this instant are 0.0004523 and 1289.13 respectively. Finally, the power reduces to zero with the reactivity settling at -0.002592. The final core temperature is 101.83°C.

Next we solve and analyze the FPRK model (15). The same data and ICs are considered. As it is im-

possible to obtain a closed-form solution for the nonlinear FDE system, we opt for other techniques for its solution. To mention explicitly, the Adomian Decomposition Method (ADM) (Daftardar-Gejji and Jafari, 2005), and Variational Iteration Method (VIM) (Odibat and Momani, 2006) were tried. These methods as such don't come under the category of numerical methods, because they provide solutions in the form of a power series with easily computable terms. These methods are claimed to have many advantages over the classical numerical methods, viz., no discretization, high accuracy, minimal calculations, to name a few. We tried to implement these methods for our problem using Mathematica (Ruskeepaa, 2009). But due to inherent stiff nature of the FPRK model, these methods did not work and a convergent solution could not be achieved.

It was then decided to use an improved version of the Adams-Bashforth-Moulton (ABM) algorithm (Diethelm, 2010) which is based on the Predictor-Corrector scheme for the FDE system (Diethelm et al., 2002). This method worked perfectly for the given FDE system. It should be noted that the order of convergence for the ABM method is a non-decreasing function of the fractional order α. Only two values of fractional order α, 0.7, and 0.9 are considered as we could not make the algorithm converge for smaller values of α. Salient steps in the ABM algorithm are given in the Appendix.

The FPRK model (15) is solved for two values of α and analysis is carried out in detail.

1. $\alpha = 0.7$

   The step-size used for the ABM method was $h = 0.05$ sec. The plots for $P(t)$, $C(t)$, $\rho(t)$, and $T(t)$ are shown in Fig. 3.

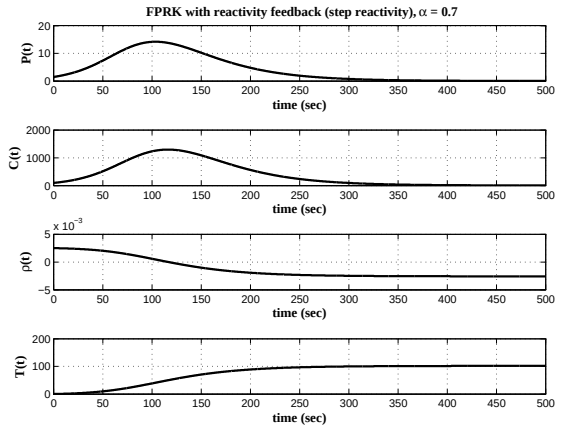

Figure 3: FPRK model with reactivity feedback ($\alpha = 0.7$): plots for $P(t)$, $C(t)$, $\rho(t)$, and $T(t)$ for a step reactivity input.

   We notice that the behaviour of this FPRK model in line with the reactor dynamics. The impressed step reactivity causes the power to shoot up, albeit at a slower rate. The power reaches to its peak value, $P_{max} = 14.2031$ watts (which is less than the P_{max} of IPRK model) at $t = 103.75$ sec. The reactivity at this peak value of power is 0.000454 and the corresponding precursor concentration is 1255.09. The rest of the dynamics is similar to the IPRK model. Negative reactivity required to bring power to zero is -0.002585. The temperature ultimately settles at 101.7043°C.

2. $\alpha = 0.9$

   Same step-size $h = 0.05$ sec. is used to solve the nonlinear FPRK model (15) with $\alpha = 0.9$. As said earlier, the behaviour of this FPRK model is very similar to that of the IPRK model. The effect of imposing a positive reactivity of $\rho_0 = 0.0025$ on power, precursor concentration, core temperature, and the reactivity itself is depicted in Fig. 4.

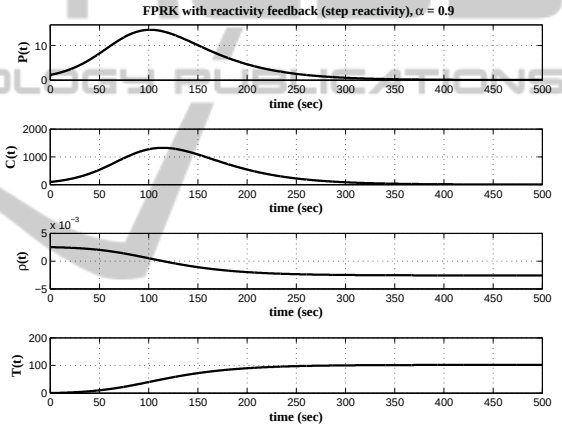

Figure 4: FPRK model with reactivity feedback ($\alpha = 0.9$): plots for $P(t)$, $C(t)$, $\rho(t)$, and $T(t)$ for a step reactivity input.

   The power rise as a result of achieving the subprompt criticality is slow. The peak power is 14.5434 watts. It occurs at $t = 101.75$ sec. The reactivity and precursor concentration at P_{max} are 0.0004513 and 1282.094 respectively. The power excursion is controlled by reduction in reactivity due to negative temperature feedback. Finally, a negative reactivity of -0.002591 brings the power to zero with reactor temperature settling at $T = 101.8262$°C.

To carry out a comparative study for the IO and FO models, these observations are compiled in tables 1 and 2. Survey of these two tables and the plots in Figs. 2, 3, and 4 bring following observations to our notice:

1. The power overshoot in all models is small. In case of FPRK models, for both $\alpha = 0.7$ and 0.9, the peak power attained was nearly equal to that

Table 1: IPRK and FPRK models with reactivity feedback: values of various quantities at P_{max} for step reactivity input.

α	P_{max} (watts)	P_{max} at (sec)	$\rho(t)$ at P_{max}	$C(t)$ at P_{max}
0.7	14.2	103.7	4.5e-4	1255
0.9	14.5	101.7	4.5e-4	1282
IPRK	14.6	101.1	4.5e-4	1289

Table 2: IPRK and FPRK models with reactivity feedback: values of various quantities at steady-state for step reactivity input.

α	$P(t)$ (watts)	$C(t)$	$\rho(t)$	T(t) °C
0.7	0.0258	2.9	-0.002585	101.7
0.9	0.0131	1.6	-0.002591	101.8
IPRK	0.0111	1.4	-0.002592	101.8

in the IPRK model. These maximum values in power occur almost at the same instant for all these models. Also, values of positive reactivity and precursor concentration at P_{max} are almost equal.

2. The inherent negative feedback of temperature helps in limiting the power overshoot and eventually brings it down to a negligible value. This dynamics is observed in both IO and FO models. The delayed neutron dynamics also works for this cause.
3. After the power excursion dies out, it settles to a very low value (almost equal for IPRK and FPRK models). A very little negative reactivity is required to bring this power to zero. The reactor temperature settles at around 101°C for all the models.
4. However a different scenario is observed in the dynamics of precursor concentration. The values of $C(t)$ when power reaches to its peak value are different for IPRK and FPRK models. Furthermore, the values also depend on the fractional derivative power α. It is seen that as the derivative order increases, precursor concentration at P_{max} increases (lowest for FPRK model with $\alpha = 0.7$ and highest for IPRK model). However, exactly opposite situation is observed during the steady-state. Now, the final value at which $C(t)$ settles after the power excursion has died out is largest for FPRK model with derivative order 0.7. It gradually decreases and is lowest for the IPRK model.

One may deduce that for the reactivity feedback case, the behaviours of IPRK and FPRK models (with $\alpha = 0.7, 0.9$) nearly coincide. However, it should be noted that a nuclear reactor is very critical and complex system and its safe operation is a very crucial issue. A slight variation in any parameter may have severe effect on performance of the system. In view of this, the results predicted by the proposed FPRK model may yield a better understanding of the nuclear reactor dynamics.

Furthermore, we think that the effect of considering subdiffusive neutron transport would be more saliently visible for lower values of α. But as mentioned earlier, we found it almost impossible to get the ABM method converged for smaller values of fractional differentiation order. Nevertheless, the results presented here confirm the validity of the developed FPRK model. It faithfully captures the behaviour of a reactor subjected to a subprompt step in reactivity under the influence of adiabatic temperature feedback.

Thus, in this section we analyzed the FPRK model with reactivity feedback. As the aim was to study the reactor with below prompt critical situation, the dynamics of precursor concentration was also considered in the model. The derived model was subjected to a detailed analysis. As a part of the study, we examined the behaviour of FPRK model when subjected to the step reactivity. The reactivity insertion was chosen in such a way so as to keep the reactor subprompt critical. This exercise confirms that the developed fractional-order nonlinear model with reactivity feedback faithfully represents the reactor dynamics. A comparison of this FO model with the classical IO model is also carried out and various results are presented.

5 CONCLUSIONS

To get a more reliable and realistic model of the nuclear reactor, it is necessary to consider processes by which the reactivity is affected by the neutron flux or power. Using this fact, in this paper, we proposed a new version of the fractional-order point reactor kinetics model considering the effect of temperature feedback on the reactivity. The adiabatic model of temperature dependence on power is used.

The proposed FPRK model is used to analyze small perturbations (below prompt critical) in the power. This model is a system of three coupled, nonlinear ODEs (one FDE + two IDEs). Various standard methods like VIM, ADM, were tried. But it was found that the stiff nature of the PRK model in general proved as a big hurdle in achieving the convergence for these methods. Finally, the ABM method was used to numerically solve the FO model. ‘It becomes more and more difficult to make a numerical

method applied to an FDE converge for smaller values of α': this frequently observed phenomenon was experienced for the model under consideration.

The behaviour predicted by the FO model was found to be in line with reactor physics. Each time, the power excursion was found to be self-limiting and therefore stable. Thus this paper presents a major step in the development and analysis of fractional-order model for a nuclear reactor under the consideration of reactivity feedback. The developed FO model very faithfully mimics the actual behaviour of the reactor in these situations. Also the developed FO model has broader applicability, and is easy to derive and solve. The classical integer-order model forms a special case of the proposed FO model.

The analysis carried out in this paper can be made more exhaustive by studying the FPRK models with other types of reactivity feedback mechanisms and carrying out a comparative study of these models. Further, it has been proved in literature that the heat transfer mechanism is better represented using fractional dynamics. Hence a more detailed study of the reactivity feedback in nuclear reactor can be achieved by additionally considering a fractional-order model for the temperature dependence of reactivity.

REFERENCES

Aboanber, A. E. and Nahla, A. A. (2004). On pade' approximations to the exponential function and application to the point kinetics equations. *Progress in Nuclear Energy*, 44(4):347–368.

Beckurts, K. H. and Wirtz, K. (1964). *Neutron Physics*. Springer-Verlag, Germany.

Compte, A. and Metzler, R. (1997). The generalized Cattaneo equation for the description of anomalous transport processes. *Journal of Physics A: Mathematical and General*, 30:7277–7289.

Connolly, J. A. (2004). *The numerical solution of fractional and distributed order differential equations*. PhD thesis, University of Liverpool, UK.

Daftardar-Gejji, V. and Jafari, H. (2005). Adomian decomposition: a tool for solving a system of fractional differential equations. *Journal of Mathematical Analysis and Applications*, 301:508–518.

Das, S. (2011). *Functional Fractional Calculus for System Identification and Controls*. Springer, Germany.

Das, S. and Biswas, B. B. (2007). Fractional divergence for neutron flux profile in nuclear reactor. *International Journal of Nuclear Energy Science and Technology*, 3(2):139–159.

Das, S., Das, S., and Gupta, A. (2011). Fractional order modeling of a PHWR under step-back condition and control of its global power with a robust $PI^{\lambda}D^{\mu}$ controller. *IEEE Transactions on Nuclear Science*, 58(5):2431–2441.

Diethelm, K. (2010). *The Analysis of Fractional Differential Equations: An Application-Oriented Exposition Using Differential Operators of Caputo Type*. Springer, Germany.

Diethelm, K., Ford, N. J., and Freed, A. D. (2002). A Predictor-Corrector approach for the numerical solution of fractional differential equations. *Nonlinear Dynamics*, 29:3–22.

Diethelm, K., Ford, N. J., Freed, A. D., and Luchko, Y. (2005). Algorithms for the fractional calculus: A selection of numerical methods. *Computer Methods in Applied Mechanics and Engineering*, 194:743–773.

Duderstadt, J. J. and Hamilton, L. J. (1976). *Nuclear Reactor Analysis*. John Wiley & Sons, USA.

Espinosa-Paredes, G., del Valle-Gallegos, E., Núñez-Carrera, A., Polo-Labarrios, M.-A., Espinosa-Martínez, E.-G., and Vázquez-Rodríguez, R. (2014). Fractional neutron point kinetics equation with newtonian temperature feedback effects. *Progress in Nuclear Energy*, 73:96–101.

Espinosa-Paredes, G., Morales-Sandoval, J. B., Vázquez-Rodríguez, R., and Espinosa-Martínez, E.-G. (2008). Constitutive laws for the neutron transport current. *Annals of Nuclear Energy*, 35:1963–1967.

Espinosa-Paredes, G., Polo-Labarrios, M.-A., Espinosa-Martínez, E.-G., and del Valle-Gallegos, E. (2011). Fractional neutron point kinetics equations for nuclear reactor dynamics. *Annals of Nuclear Energy*, 38:307–330.

Glasstone, S. and Sesonske, A. (2002). *Nuclear Reactor Engineering: Vol. 1*. CBS Publishers & Distributors, India.

Hetrick, D. L. (1993). *Dynamics of Nuclear Reactors*. American Nuclear Society, USA.

Kadem, A. (2009). The fractional transport equation: an analytical solution and a spectral approximation by Chebyshev polynomials. *Applied Sciences*, 11:78–90.

Kadem, A. and Baleanu, D. (2010). Analytical method based on Walsh function combined with orthogonal polynomial for fractional transport equation. *Communications in Nonlinear Science and Numerical Simulation*, 15(3):491–501.

Klages, R., Radons, G., and Sokolov, I. M., editors (2008). *Anomalous Transport*. WILEY-VCH Verlag GmbH & Co.

Li, C. and Peng, G. (2004). Chaos in Chen's system with a fractional order. *Chaos, Solitons & Fractals*, 22(2):443–450.

Machado, J. T., Kiryakova, V., and Mainardi, F. (2011). Recent history of fractional calculus. *Communications in Nonlinear Science and Numerical Simulation*, 16(3):1140–1153.

Magin, R. L. (2006). *Fractional Calculus in Bioengineering*. Begell House Publishers, USA.

Mathworks (2005). *MATLAB Manual*. The Mathworks Inc., MATLAB version 7.1 (R14), USA.

Meghreblian, R. V. and Holmes, D. K. (1960). *Reactor Analysis*. McGraw-Hill Book Company, USA.

Metzler, R. and Klafter, J. (2000). The random walk's guide to anomalous diffusion: A fractional dynamics approach. *Physics Reports*, 339:1–77.

Monje, C. A., Chen, Y. Q., Vinagre, B. M., Xue, D., and Feliu, V. (2010). *Fractional-order Systems and Control: Fundamentals and Applications*. Springer-Verlag London Limited, UK.

Nahla, A. A. (2009). An analytical solution for the point reactor kinetics equations with one group of delayed neutrons and the adiabatic feedback model. *Progress in Nuclear Energy*, 51:124–128.

Odibat, Z. and Momani, S. (2006). Application of Variational Iteration Method to nonlinear differential equations of fractional order. *International Journal of Nonlinear Sciences and Numerical Simulation*, 1(7):15–27.

Ruskeepaa, H. (2009). *Mathematica Navigator: Mathematics, Statistics and Graphics*. Academic Press, USA.

Samko, S. G., Kilbas, A. A., and Marichev, O. I. (1997). *Fractional Integrals and Derivatives*. Gordon and Breach Science Publishers, Netherlands.

Sardar, T., Ray, S. S., Bera, R., Biswas, B., and Das, S. (2010). The solution of coupled fractional neutron diffusion equations with delayed neutrons. *International Journal of Nuclear Energy Science and Technology*, 5(2):105–113.

Tavazoei, M. S. and Haeri, M. (2007). A necessary condition for double scroll attractor existence in fractional-order systems. *Physics Letters A*, 367:102–113.

Vyawahare, V. A. and Nataraj, P. S. V. (2010). Modeling neutron transport in a nuclear reactor as subdiffusion: The neutron fractional-order telegraph equation. In *The 4th IFAC Workshop on Fractional Differentiation and its Applications*, Badajoz, Spain.

Vyawahare, V. A. and Nataraj, P. S. V. (2012). Development and analysis of the fractional point reactor kinetics model for a nuclear reactor with slab geometry. In *The 5th IFAC Workshop on Fractional Differentiation and its Applications*, Nanjing, China.

Vyawahare, V. A. and Nataraj, P. S. V. (2013a). Development and analysis of some versions of the fractional-order point reactor kinetics model for a nuclear reactor with slab geometry. *Communications in Nonlinear Science and Numerical Simulation*, 18:1840–1856.

Vyawahare, V. A. and Nataraj, P. S. V. (2013b). Fractional-order Modeling of Neutron Transport in a Nuclear Reactor. *Applied Mathematical Modelling*, 37:9747–9767.

Vyawahare, V. A. and Nataraj, P. S. V. (2014). Development and analysis of fractional-order Nordheim-Fuchs model for nuclear reactor. In Daftardar-Gejji, V., editor, *Fractional Calculus: Theory and Applications*. Narosa Publishing House, India.

APPENDIX: FRACTIONAL SECOND-ORDER ADAMS-BASHFORTH-MOULTON METHOD

Here the fractional second- order Adams-Bashforth-Moulton (ABM) method which is used in section 4 is explained in brief. The main computational steps involved in the algorithm are presented here for the equispaced grid points. For details, refer to (Diethelm, 2010; Diethelm et al., 2005; Connolly, 2004). It is an extension of the classical ABM method used to numerically solve the first-order ODEs. It comes in the category of the so-called PECE (Predict, Evaluate, Correct, Evaluate) type since it involves calculation of the predictor value which is in turn used to compute the corrector value. This method and its variants are very popular in the field of fractional calculus and applied areas (Li and Peng, 2004; Tavazoei and Haeri, 2007). The algorithm explained below is for a single fractional differential equation. However, it can be easily modified to handle a system of FDEs.

Consider the single term FDE with Caputo FD

$$ {}_0D_t^{\alpha} y(t) = f(t, y(t)), \tag{18} $$

where $\alpha \in \mathbb{R}^+$ and with the appropriate initial conditions:

$$ D_t^k y(0) = y_0^{(k)}, \quad k = 0, 1, \ldots, m-1, \tag{19} $$

where, $m = \lceil \alpha \rceil$ is the ceil function. The equivalent Volterra integral equation is

$$ y(t) = \sum_{k=0}^{m-1} \frac{t^k}{k!} D_t^k y(0) + \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} f(\tau, y(\tau)) d\tau \tag{20} $$

The integration limits from 0 to t imply the nonlocal structure of the fractional derivatives.

The next step is to use the product trapezoidal quadrature formula to replace the integral in (20). We approximate the following integral

$$ \int_0^{t_{k+1}} (t_{k+1} - \tau)^{\alpha-1} g(\tau) d\tau, \tag{21} $$

as

$$ \int_0^{t_{k+1}} (t_{k+1} - \tau)^{\alpha-1} g_{k+1}(\tau) d\tau, \tag{22} $$

where $\tilde{g}_{k+1} \equiv$ piecewise linear interpolation for $g(t)$ with grid points at $t_j, j = 0, 1, 2, \ldots, k+1$. Thus we can write the integral (22) as

$$ \int_0^{t_{k+1}} (t_{k+1} - \tau)^{\alpha-1} g_{k+1}(\tau) d\tau = \sum_{j=0}^{k+1} a_{j,k+1} g(t_j), \tag{23} $$

for the equispaced nodes ($t_j = jh$ with some fixed step-size h). The values of $a_{j,k+1}$ are given for $j = 0$ as

$$\frac{h^\alpha}{\alpha(\alpha+1)}\left(k^{\alpha+1} - (k-\alpha)(k+1)^\alpha\right),$$

for $1 \le j \le k$ as

$$\left(\frac{h^\alpha}{\alpha(\alpha+1)}\right)(d),$$

where

$$d = (k-j+2)^{\alpha+1} + (k-j)^{\alpha+1} - 2(k-j+1)^{\alpha+1},$$

and for $j = k+1$ as

$$\frac{h^\alpha}{\alpha(\alpha+1)}.$$

So the corrector formula is

$$\begin{aligned} y_{k+1} &= \sum_{j=0}^{m-1} \frac{t_{k+1}^j}{j!} y_0^{(j)} \\ &+ \frac{1}{\Gamma(\alpha)} \sum_{j=0}^{k} a_{j,k+1} f(t_j, y_j) \\ &+ \frac{1}{\Gamma(\alpha)} \left(a_{k+1,k+1} f(t_{k+1}, y_{k+1}^P)\right), \end{aligned} \tag{24}$$

where now the predictor y_{k+1}^P is evaluated as

$$y_{k+1}^P = \sum_{j=0}^{m-1} \frac{t_{k+1}^j}{j!} y_0^{(j)} + \frac{1}{\Gamma(\alpha)} \sum_{j=0}^{k} b_{j,k+1} f(t_j, y_j), \tag{25}$$

with

$$b_{j,k+1} = \frac{h^\alpha}{\alpha}\left((k+1-j)^\alpha - (k-j)^\alpha\right). \tag{26}$$

For $0 < \alpha < 1$, the predictor and corrector expressions get modified as

$$y_{k+1}^P = y_0 + \frac{1}{\Gamma(\alpha)} \sum_{j=0}^{k} b_{j,k+1} f(t_j, y_j), \tag{27}$$

and

$$\begin{aligned} y_{k+1} = y_0 &+ \frac{1}{\Gamma(\alpha)} \sum_{j=0}^{k} a_{j,k+1} f(t_j, y_j) \\ &+ \frac{1}{\Gamma(\alpha)} \left(a_{k+1,k+1} f(t_{k+1}, y_{k+1}^P)\right) \end{aligned} \tag{28}$$

As already mentioned in section 4, the convergence of this algorithm deteriorates as $\alpha \to 0$. This algorithm was coded in MATLAB.