L_1 Adaptive Output Feedback Controller with Operating Constraints for Solid Oxide Fuel Cells

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Abstract: Control on solid oxide fuel cells (SOFC) is challenging due to its nonlinearity, time-varying uncertainties, tight operating constraints and modeling difficulties. The L_1 adaptive output feedback controller for systems of unknown relative degree is introduced for the SOFC output voltage control in this paper. It allows for fast and robust adaptation, and provides improved transient performance. Its advantages of not enforcing a strictly positive real condition along with the low-pass filtered control signal bring it the potential to be applied in wide industrial processes. In the study of the SOFC control, a dynamic SOFC model is first built; then a L_1 adaptive output feedback controller is designed only using the nominal working conditions of the SOFC model. Through setting the operating constraints at proper locations, the closed-loop stability is maintained in the presence of hard constraints by the symmetric structure of the L_1 adaptive control loop. A simulation comparison is made in the SOFC constant voltage control process between the L_1 adaptive controller and a linear disturbance model predictive controller (DMPC) for their almost equal complexity in designs. The result shows the advantage of the L_1 adaptive controller in disturbance rejections for its faster transient response.

1 INTRODUCTION

Solid oxide fuel cell (SOFC) is a kind of high efficiency, environment friendly power generation assembly, which converts the chemical energy in fuel and oxidant directly to electricity. Because of the shortage of resources and increasing environment pollutions, governments and technicians all over the world pay more attentions on the research and development of SOFC today. SOFC in the application of massive distributed power sources has been considered to be a potential candidate to replace the traditional thermal cycle power generation. The SOFC system has severe output nonlinearity and tight operating constrains. It also has time-varying uncertainties and is hard to model. These features bring the major challenges on the control methods of SOFC systems. Because effective control on SOFC system can improve operation efficiency, extend the stack lifespan, and improve the quality of power, more and more research has been taken on designing high-performance controllers working with nonlinear and uncertain dynamic characteristics of SOFC plants in recent years.

Most of the research work is based on model predictive control (MPC) methods (Pukrushpan et al., 2002; Vahidi et al., 2004; Aguiar et al., 2005; Stiller et al., 2006). The conventional MPC is a receding-horizon linear quadratic control law, but it can be extended for nonlinear control by incorporating nonlinear prediction models. Fuzzy prediction models and data-driven prediction models are mainly used in nonlinear SOFC predictive controls. A fuzzy Hammerstein model is used as the predictive control model to achieve online control of an SOFC system (Huo et al., 2008). In order to control the stack temperature of a SOFC within a safe range, an online nonlinear MPC scheme based on an improved T-S fuzzy model is proposed (Yang et al., 2009). Its control sequence could be obtained by the branch-and-bound method. The nonlinear predictive controller based on an improved radial basis function neural network is applied (Wu et al., 2008). It controls the voltage and guarantees fuel utilization within a safe range and uses the genetic algorithm for parameter optimizations. In order to reduce the heavy computing load in nonlinear MPC

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control, which is mainly caused by the nonlinear optimization and on-line model identifications, the disturbance model predictive control (Muske and Badgwell, 2002) is introduced for SOFC (Pan and Shen, 2012). It has less computing load and can deal with some nonlinearity and uncertainty characteristics. But it is non-adaptive and cannot guarantee the closed-loop stability while achieving a fast disturbance-rejection.

In this paper, we try to introduce another advanced control approach, $\mathcal{L}_1$ adaptive control, for designing the SOFC control system. $\mathcal{L}_1$ adaptive control offers its own set of attractive features, including fast and robust adaptation. In addition to the conventional asymptotic performance characterization, $\mathcal{L}_1$ adaptive control permits transient analysis for both control signal and system response. Furthermore, this methodology has been extended to systems with unknown time-varying parameters (Cao and Hovakimyan, 2007), to systems with nonlinear uncertainties (Cao and Hovakimyan, 2008), to systems with un-modeled internal and actuator dynamics (Cao and Hovakimyan, 2008), to systems in the presence of non-zero trajectory initialization error (Cao and Hovakimyan, 2008), and to a certain output feedback framework (Cao and Hovakimyan, 2009). $\mathcal{L}_1$ adaptive control has been very successfully applied in unmanned flight controls which have nonlinearities, time-varying disturbances, unknown parameters and un-modeled dynamics.

An extension approach of the $\mathcal{L}_1$ adaptive output feedback control (Cao and Hovakimyan, 2009) to systems of unknown relative degree may deal well with the control problems of SOFC, e.g., time-varying uncertainties with unknown rate of variations caused by load disturbances. Compared to other $\mathcal{L}_1$ adaptive control methods, this approach adopts a new piece-wise continuous adaptive law along with the low-pass filtered control signal. It allows for achieving arbitrarily close tracking of the reference signals, and the transfer function of its reference system is not required to be strictly positive real (SPR). Stability of this system is guaranteed by its design via small-gain type argument. These features show that this $\mathcal{L}_1$ adaptive control approach may have great potential to be applied in wide industrial processes.

In this paper, we reproduce a SOFC simulation model as the plant; then take advantage of its nominal working conditions to design a $\mathcal{L}_1$ adaptive output feedback controller. Through the analysis on the controller framework, the operating constraints are set to the proper position in the loop. It holds the closed-loop stability in the presence of the hard constraints. Simulations comparing to a linear disturbance model predictive controller (DMPC) on the SOFC model show that $\mathcal{L}_1$ adaptive control has better disturbance-rejection performance and much faster temporary regulating-process on the SOFC constant voltage.

The paper is organized as follows. Section 2 describes the dynamic SOFC model. In Section 3, $\mathcal{L}_1$ adaptive output feedback controller with operating constraints is designed. For making an evaluation on the new controller, a linear DMPC controller is designed in Section 4. The simulation results along with some discussions on these two SOFC control methods are presented in Section 5. Section 6 concludes the paper.

2 DYNAMIC MODEL OF SOFC

Many nonlinear dynamic models of SOFC with detailed descriptions on cell internal processes are too complicated to support a controller designing process. The model reported in the paper (Padullés and Ault, 2000) describes the key characteristics of the SOFC dynamic process in the Laplace transform domain. It shows challenging control problems owing to SOFC's nonlinear dynamics, tight operating constraints and unexpected disturbances. It has been taken as a benchmark commonly studied in the SOFC control literature (Huo et al., 2008; Wu et al., 2008; Yang et al., 2009). Therefore, this model will be adopted as the SOFC plant for the $\mathcal{L}_1$adaptive controller design. Some preconditions (Padullés and Ault, 2000) are stated in the following: The gases are ideal; The stack is fed with hydrogen and air; The flow ratio of hydrogen to oxygen is kept at 1.145; Lump gas pressures are considered in the channels along the electrodes; The temperature is stable; The exhaust of each channel is via a single orifice, and the ratio of pressures between the interior and exterior of the channel is large enough to consider that the orifice is choked; The Nernst equation can be applied. This dynamic SOFC model consists of the fuel processor and the fuel cell stack, as shown in Fig. 1, where E denotes the stack output voltage (V), q_f the natural gas flow rate (mol/s), and I the external current load (A); p_{H2} , p_{O2} , and p_{H2O} denote the partial pressures (Pa) of hydrogen, oxygen, and water, respectively; $q_{H2,in}$ and $q_{O2,in}$ are the input flow rates of hydrogen and oxygen (mol/s), respectively. The model is described in the following and all the parameters are annotated in Table 1.

2.1 The Fuel Processor

The fuel processor converts fuels such as natural gas to hydrogen and byproduct gases. From the viewpoint of control analysis, a first-order transfer function with time constant τ_f can well describe the dynamic reform process from the natural gas input N_f to the hydrogen-rich fuel $q_{H_2,in}$. The fuel processor can be simply represented by

$$\frac{q_{\mathrm{H2,in}}}{N_f} = \frac{1}{1+\tau_f S} \tag{1}$$

Hydrogen reacts with oxygen in SOFC and generates water. The flow ratio between hydrogen and oxygen is represented by

$$\gamma_{H-O} = \frac{q_{H2,in}}{q_{O2,in}} \tag{2}$$

For having a complete reaction by the excessive oxygen, take the ratio $\gamma_{H\text{-}O}$ as 1.145 (Padullés and Ault, 2000).

2.2 The Fuel Cell Stack

Applying Nernst's equation and taking into account ohmic, concentration, and activation losses (i.e., η_o, η_c and η_a), the stack output voltage is given by

$$E = N_0\left(E_0 + \frac{RT}{2F}\ln\frac{p_{H2}\sqrt{(p_{O2}/101325)}}{p_{H2O}}\right) - \eta_o - \eta_c - \eta_a \tag{3}$$

Where
$$p_{H2} = \frac{K_{H2}^{-1}}{1+\tau_{H2}S}(q_{H2} - 2K_r I) \tag{4}$$

$$p_{O2} = \frac{K_{O2}^{-1}}{1+\tau_{O2}S}(q_{O2} - K_r I) \tag{5}$$

$$p_{H2O} = 2K_r I \frac{K_{H2O}^{-1}}{1+\tau_{H2O}S} \tag{6}$$

$$\eta_a = \alpha + \beta \log I \tag{7}$$

$$\eta_c = -\frac{RT}{2F}\ln\left(1 - \frac{I}{I_L}\right) \tag{8}$$

$$\eta_o = Ir \tag{9}$$

Equation (4)-(6) represent the dynamic characteristics of the partial gas pressure of hydrogen, oxygen and water inside the anode channel associated with their molar flows through the anode valve respectively. Take hydrogen as an example to derive it. Consider the molar flow of Hydrogen is proportional to its partial pressure inside the channel and have

$$\frac{q_{H2}}{p_{H2}} = K_{H2} \tag{10}$$

Take time derivative on the perfect gas equation of hydrogen and obtain

$$\frac{dp_{H2}}{dt} = \frac{RT}{V}(q_{H2,in} - q_{H2,o} - q_{H2,r}) \tag{11}$$

Where $q_{H2,in}$ is the input hydrogen flow, $q_{H2,o}$ is the output hydrogen flow, and $q_{H2,r}$ is the hydrogen flow that reacts. According to the basic electrochemical relationships, $q_{H2,r}$ can be calculated by

$$q_{H2,r} = \frac{NI}{2F} = 2K_r I \tag{12}$$

Apply (10) and (12) in (11) and then take the Laplace transform, obtaining (4) and the equation $\tau_{H2} = V/(K_{H2}RT)$.

The transfer functions of oxygen and steam are derived as well.

The fuel utilization which is one important operating variable and may affect the performance of SOFC is defined as

$$u_f = \frac{q_{H2,in} - q_{H2,O}}{q_{H2,in}} = \frac{q_{H2,r}}{q_{H2,in}} = \frac{NI}{2Fq_{H2,in}} \tag{13}$$

The desired range of fuel utilization is from 0.7 to 0.9. The overused ($u_f > 0.9$) and underused ($u_f < 0.7$) fuel conditions should be prevented. An overused condition could lead to permanent damage to the cells due to fuel starvation and an underused-fuel situation results in unexpectedly high cell voltages (Vahidi et al., 2004).

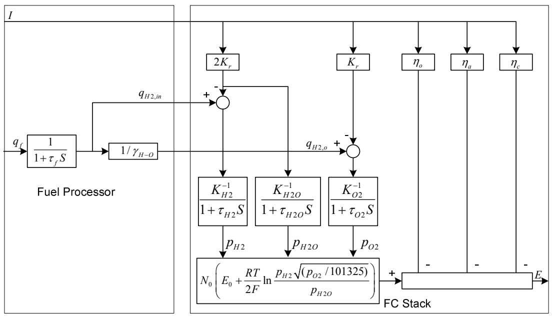

Figure 1: Dynamic model of SOFC.

Table 1: Parameters of the SOFC.

Para-meter	Value	Representation
T	1273 K	Absolute temperature
F	96,485 C /mol	Faraday's constant
R	8.314 J/(mol·K)	Universal gas constant
E_0	1.18 V	Ideal standard potential
N_0	384	Number of cells in series in the stack
Kr	0.996×10^{-3} mol/(s·A)	Constant, $Kr = N_0/4F$

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Table 1: Parameters of the SOFC. (Cont.)

Para-meter	Value	Representation
K_{H2}	8.32×10^{-6} mol/(s·Pa)	Valve molar constant for hydrogen
K_{H2O}	2.77×10^{-6} mol/(s·Pa)	Valve molar constant for water
K_{O2}	2.49×10^{-5} mol/(s·Pa)	Valve molar constant for oxygen
τ_{H2}	26.1 s	Response time of hydrogen flow
τ_{H2O}	78.3 s	Response time of water flow
τ_{O2}	2.91 s	Response time of oxygen flow
τ_{H-O}	1.145	Ratio of hydrogen to oxygen
r	0.126 Ω	Ohmic loss
τ_f	5 s	Time constant of the fuel processor
∂	0.05	Tafel constant
β	0.11	Tafel slope
I_L	800 A	Limiting current density

3 L_1 ADAPTIVE CONTROLLER WITH INPUT CONSTRAINTS ON SOFC

Several aspects should be considered in the design of a controller for the SOFC system.

First, we know from the modeling work in Section 2 that the nonlinear SOFC model composed of (1) to (13) has the Wiener-type output nonlinearity. Suppose the operating temperature and pressure of the SOFC is kept constant, then the stack terminal voltage E is mainly influenced by the inlet hydrogen flow $q_{H2,in}$ and the current I. The operating stack voltage usually shows significant changes at low and high current loads, even shows a rapid deterioration caused by overloaded current. Thus the stack current is often taken as the main disturbance variable to the process.

Second, the feasible operation area of SOFC shows that it is impossible for SOFC to maintain a simultaneous constant fuel utilization u_f and constant output voltage E operating regime for a range of current I. The constant voltage control is much safer than the constant fuel utilization control for the fuel utilization performs (Wu et al., 2008).

In order to achieve a constant stack voltage control under drastic current load disturbances, a $\mathcal{L}_1$ adaptive output feedback controller is designed for keeping both the SOFC output voltage at set point and the fuel utilization within the safe range. Fig.2 shows the structure of the $\mathcal{L}_1$ adaptive output feedback control loop for a SOFC process.

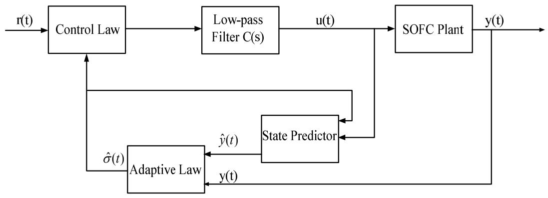


Figure 2: $\mathcal{L}_1$ Adaptive output feedback control system for SOFC.

3.1 Problem Formulation

Describe the controlled SOFC voltage dynamics as the following single-input single-output system:

$$y(s) = A(s)(u(s) + d(s)) \tag{14}$$

where y∈R is the SOFC terminal voltage, u∈R is the hydrogen flow rate, A(s) is a strictly proper unknown transfer function of unknown relative degree nr, for which only a known lower bound 1< dr <nr is available, d(s) is the Laplace transform of the time-varying uncertainties and disturbance d(t) = f(t, y(t)), where f is an unknown map, subject to the following assumption:

Assumption 1 There exist constants L>0 and L_0>0 such that for all t⩾0:

$$\left|f(t, y_1) - f(t, y_2)\right| \le L\left|y_1 - y_2\right| \tag{15}$$

$$\left|f(t, y)\right| \le L\left|y\right| + L_0 \tag{16}$$

Assumption 2 There exist constants L_1>0, L_2>0 and L_3>0 such that for all t⩾0:

$$\left|\dot{d}\right| \le L_1\left|\dot{y}(t)\right| + L_2\left|y(t)\right| + L_3 \tag{17}$$

where the numbers L, L_0, L_1, L_2, L_3 can be arbitrarily large.

Assumption 3 The DC gain of the nominal working point of SOFC is known.

Let r(t) be a given bounded continuous reference input signal. The control objective is to design an adaptive output feedback controller u(t) such that the system output y(t) tracks the reference input r(t) following a desired reference model

$$y(s) = M(s)r(s) \tag{18}$$

where M(s) is a minimum-phase stable transfer function of relative degree dr > 1. Thus we can rewrite the system in (14) as:

$$y(s) = M(s)(u(s) + \sigma(s)) \quad (19)$$

$$\sigma(s) = ((A(s) - M(s))u(s) + A(s)d(s)) / M(s) \quad (20)$$

Let ($A_m \in R^{N\times N}$, $b_m \in R^N$, $c_m \in R^N$) be the minimal realization of M(s). Thus the system in (19) can be rewritten as:

$$\begin{aligned} \dot{x}(t) &= A_m x(t) + b_m (u(t) + \sigma(t)) \\ y(t) &= c_m^T x(t), x(0) = x_0 \end{aligned} \quad (21)$$

3.2 L$_1$ Adaptive Output Feedback Controller

$\mathcal{L}_1$ Adaptive controller consists of the state predictor, the adaptation law and the control law.

The state predictor is given by:

$$\begin{aligned} \dot{\hat{x}}(t) &= A_m \hat{x}(t) + b_m u(t) + \hat{\sigma}(t) \\ \hat{y}(t) &= c_m^T \hat{x}(t), \hat{x}(0) = x_0 \end{aligned} \quad (22)$$

Where $\hat{x}(t) \in R^N$ and $\hat{y}(t) \in R$ are the state and output of the predictor respectively; $\hat{\sigma}(t) \in R^N$ compensates the system disturbances and model mismatch. It is on-line estimated by the following adaptation law:

$$\begin{aligned} \hat{\sigma}(t) &= \hat{\sigma}(iT) \quad t \in [iT, (i+1)T], \\ \hat{\sigma}(iT) &= -\Phi^{-1}(T)\mu(iT), \quad i = 0,1,2,\ldots, \end{aligned} \quad (23)$$

Where T>0 is the sampling time of the adaptation law; and

$$\begin{aligned} \Phi(T) &\triangleq \int_0^T e^{\Lambda A_m \Lambda^{-1}(T-\tau)} \Lambda d\tau \\ \mu(iT) &= e^{\Lambda A_m \Lambda^{-1} T} 1_1 \tilde{y}(iT), \quad i = 0,1,2,\ldots. \end{aligned} \quad (24)$$

Where $\mathbf{1}_1 \in R^N$ be the basis vector with first element **1** and all other elements zero;

$$\tilde{y}(t) \triangleq \hat{y}(t) - y(t); \qquad \Lambda \triangleq \begin{bmatrix} c_m^T \\ D\sqrt{P} \end{bmatrix}, \Lambda \in R^{N\times N},$$

where $P=P^T>0$ satisfies the algebraic Lyapunov equation $A_m^T P + PA_m = -Q$, $Q>0$; and let $D \in R^{(N-1)\times N}$ satisfies $D(c_m^T(\sqrt{P})^{-1})^T = 0$.

The control law is defined via the output of the low-pass filter C(s):

$$u(s) = C(s)r(s) - \frac{C(s)}{M(s)} c_m^T (SI - A_m)^{-1} \hat{\sigma}(s) \quad (25)$$

The selection of C(s) and M(s) must ensure that

$$H(s) = A(s)M(s) / (C(s)A(s) + (1 - C(s))M(s)) \quad (26)$$

is stable and that the $\mathcal{L}_1$-gain of the system is bounded as follows:

$$\left\| H(s)(1 - C(s)) \right\|_{L_1} L < 1 \quad (27)$$

(Cao and Hovakimyan, 2009).

The above piece-wise continuous adaptive law with the low-pass filtered control signal allows for achieving arbitrarily close tracking of the input and the output signals of the reference system. The performance bounds between the closed-loop reference system and the closed-loop $\mathcal{L}_1$ adaptive system can be rendered arbitrarily small by reducing the step size of integration. It can be represented by the following equations.

$$\lim_{T\to 0} y(t) = y_{ref}(t), \qquad \lim_{T\to 0} u(t) = u_{ref}(t)$$

where T is the integration step of the $\mathcal{L}_1$ adaptive controller.

3.3 Operating Constraints for SOFC

Besides the design above, we need to put the input constraints in the $\mathcal{L}_1$ adaptive controller for the SOFC voltage control, i.e., letting $u_{\min} \le u(t) \le u_{\max}$ hold for all t⩾0, where u_{min}=0, u_{max}=1.7023mol/s given in the paper (Padullés and Ault, 2000). Considering the subtle symmetric structure of $\mathcal{L}_1$ adaptive control, we cannot constrain u(t) directly. $\hat{\sigma}(t)$ is sent into both the plant and the state predictor for cancellation. Its constraints can influence the value of u(t) but cannot change the stability of the closed loop. Thus, we have

$$\hat{\sigma}(t) = \begin{cases} \hat{\sigma}(iT), if\, \hat{\sigma}_{\min} < \hat{\sigma}(iT) < \hat{\sigma}_{\max} \\ \hat{\sigma}_{\min}, if\, \hat{\sigma}(iT) < \hat{\sigma}_{\min} \\ \hat{\sigma}_{\max}, if\, \hat{\sigma}(iT) > \hat{\sigma}_{\min} \end{cases} \quad (28)$$

$$t \in [iT, (i+1)T], \quad i = 0,1,2,\ldots.$$

Another point is the possible different DC gains between the plant and the state estimator. Because the nominal parameters of SOFC are available, we have Assumption 3. Dividing the output voltage reference by the nominal DC gains of the SOFC system, we get r(t) in control law (25).

4 DMPC CONTROLLER DESIGN FOR A COMPARISON

In order to evaluate the performance of the $\mathcal{L}_1$ Adaptive controller for SOFC, we try to introduce the linear disturbance model predictive controller (Muske and Badgwell, 2002; Pannocchia and Rawlings, 2003) to be an evaluating reference. It is a kind of target-adaptive offset-free MPC with the advantages in disturbance rejection and offset-free tracking. The DMPC has been successfully applied in CSTR (Pannocchia and Rawlings, 2003) and become a fundamental approach where a variety of MPC approaches have derived. The $\mathcal{L}_1$ Adaptive controller and the DMPC controller have almost equal complexity in designs and computation load online, therefore we will compare their performance in the simulations. For clarity, a brief design of the DMPC controller for SOFC is presented. First, the augmented disturbance prediction model of SOFC is built in term of the conditions for detectability, and then the problem of estimating the augmented disturbance states is solved. As a result, an augmented observer is used to estimate the system states and the lumped mismatch. Last, the augmented disturbance model is adopted in the predictive control algorithm to realize the control of SOFC.

4.1 Disturbance Model and Estimator

We need to describe the SOFC plant approximately by a linear model with augmented disturbance states before the design of the DMPC controller. The following linearized discrete state-space model describes the controlled voltage system

$$\begin{aligned} x_{k+1} &= Gx_k + Hu_k \\ y_k &= Cx_k \end{aligned} \tag{29}$$

where $y\in R$ is the SOFC terminal voltage, $u\in R$ is the hydrogen flow rate, $x \in R^2$ is the process state and its rank represents the inertial of the process, $G\in R^{2\times2}$, $H\in R^{2\times1}$, $C\in R^{1\times2}$, (G, H) is stabilizable and (C,G) is detectable in the SOFC model. There must be some model-plant mismatch in using the linear model of Eq.29. We lump the mismatch along with load disturbances into an augmented state to make a disturbance model of SOFC

$$\begin{aligned} \begin{bmatrix} x_{k+1} \\ d_{k+1} \end{bmatrix} &= \begin{bmatrix} G & G_d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_k \\ d_k \end{bmatrix} + \begin{bmatrix} H \\ 0 \end{bmatrix} u_k \\ y_k &= \begin{bmatrix} C & C_d \end{bmatrix} \begin{bmatrix} x_k \\ d_k \end{bmatrix} \end{aligned} \tag{30}$$

where $d\in R$, $G_d\in R^{2\times1}$, $C_d\in R^{1\times1}$. Because the lumped disturbance is unmeasured, an estimator is needed for state-observing

$$\begin{aligned} \begin{bmatrix} \hat{x}_{k+1|k} \\ \hat{d}_{k+1|k} \end{bmatrix} &= \begin{bmatrix} G & G_d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{x}_{k|k-1} \\ \hat{d}_{k|k-1} \end{bmatrix} + \begin{bmatrix} H \\ 0 \end{bmatrix} u_k \\ &+ \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} (y_k - C\hat{x}_{k|k-1} - C_d\hat{d}_{k|k-1}) \end{aligned} \tag{31}$$

where $L_1\in R^{2\times1}$, $L_2\in R^{1\times1}$ are the predictor gain matrices for the state and the disturbance. Since the additional modes introduced by the disturbance are unstable, detectability of the augmented system is a necessary and sufficient condition for a stable estimator to exist.

4.2 Detectability of the Augmented State Space

The condition which ensures the observability of the augmented disturbance system is given in the following Lemma (Muske and Badgwell, 2002).

Lemma The augmented system presented in Eq. 30 is detectable if and only if (C,G) is detectable and

$$rank \begin{bmatrix} (I-G) & -G_d \\ C & C_d \end{bmatrix} = n + n_d \tag{32}$$

Where n is the number of the nonaugmented states, n_d is the number of the disturbances. In the SOFC system, n is 2 and n_d is 1. This Lemma implies that the maximum dimension of the disturbance d in Eq.30 such that the augmented system is detectable is equal to the number of measurements y. That gives us the guideline to design the augmented system. Because (C, G) is detectable, the disturbance model is defined by choosing the matrices G_d and C_d to hold Eq.32.

4.3 Target-adaptive MPC Algorithm

The goal of tracking the steady-state target is to remove the effects of the estimated constant disturbance states in the MPC control. It is a kind of target-adaptive control. Given the current estimate of the disturbance $\hat{d}_{k|k}$, the state and input target are computed by solving the following quadratic program

$$\begin{aligned} &\min_{x_t, u_t} (u_t - u_s)^T R' (u_t - u_s) \\ &s.t. \\ &\quad \begin{bmatrix} I-G & -H \\ C & 0 \end{bmatrix} \begin{bmatrix} x_t \\ u_t \end{bmatrix} = \begin{bmatrix} G_d \hat{d}_{k|k} \\ -C_d \hat{d}_{k|k} + y_s \end{bmatrix} \\ &\quad u_{\min} \le u_t \le u_{\max} \\ &\quad y_{\min} \le Cx_t + C_d \hat{d}_{k|k} \le y_{\max} \end{aligned} \tag{33}$$

where $x_t \in R^2$, $u_t \in R$, $R' \in R$ and $R' > 0$, $y_s \in R$ and $u_s \in R$ are the setpoints of the controlled and manipulated variables respectively.

By tracking the steady-state target of the manipulated variable, the MPC controller solves the following optimization problem to obtain the input sequence

$$\begin{aligned} \min_{u_0, u_1, \ldots} J = \sum_{k=0}^{\infty} (y_k - y_s)^T Q (y_k - y_s) \\ + (u_k - u_t)^T R (u_k - u_t) \\ s.t. \quad Eq.30 \\ u_{\min} \le u_k \le u_{\max} \\ y_{\min} \le y_k \le y_{\max} \end{aligned} \tag{34}$$

where $Q \in R^+$ and $R \in R^+$.

With an augmented disturbance prediction model of SOFC and an online target-optimization algorithm, the DMPC controller can deal with the plant-model mismatch, unmodeled plant disturbances and achieve the zero offset output tracking. The design complexities of DMPC and $\mathcal{L}_1$ adaptive controller are at the same level.

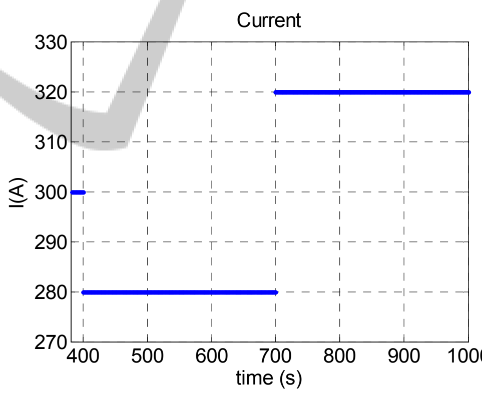
5 SIMULATIONS AND DISCUSSION

Design the $\mathcal{L}_1$ adaptive output feedback controller based on (22)-(25) for the SOFC model shown in Fig.1. The design information is shown as follows.

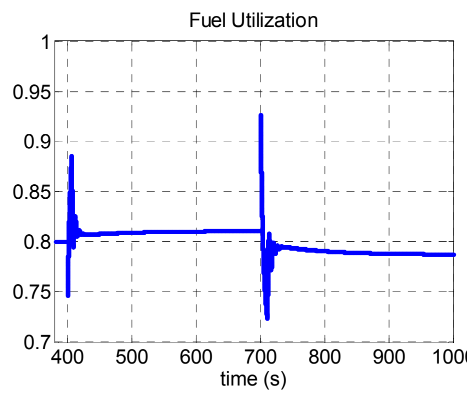
The reference model $M(s) = \frac{1}{s^2 + 1.4s + 1}$; the filter $C(s) = \frac{9}{s^2 + 25s + 9}$; the sampling interval T=0.01s; the offset range $\hat{\sigma}_{\min}$ = -0.4, $\hat{\sigma}_{\max}$ =0.4; the variables at the nominal working point, I=300A, q_f=0.746mol/s, E=341.7V.Only the nominal DC gain of the SOFC process is used for designing the $\mathcal{L}_1$ adaptive controller. Modeling is not needed for this control algorithm.

Because of its successful and wide applications, the model predictive control approach can act as a reference to evaluate the $\mathcal{L}_1$ adaptive output feedback controller. Considering there are many kinds of MPC approaches, we choose two ways to make these comparisons.

First, the linear offset-free disturbance model predictive controller (DMPC) presented in Section 4 is adopted for the comparison with the $\mathcal{L}_1$ adaptive output feedback controller. We apply $\mathcal{L}_1$ adaptive controller and DMPC controller respectively on the SOFC model. We put two step disturbances into the simulation experiments. Assuming at t = 400 s, a load disturbance causes the stack current to have a step change (from 300 to 280 A), and at t = 700 s, a load disturbance causes the stack current to have another step change (from 280 to 320 A). The step disturbances on the stack current are shown in Fig. 3(a). Fig. 3(b) shows the fuel utilization by $\mathcal{L}_1$ adaptive control. It is kept within the safe range. Fig. 3(c) and Fig. 3(d) compare the curves of the constant voltage by $\mathcal{L}_1$ and DMPC control. It shows that the $\mathcal{L}_1$ adaptive control has a shorter temporary regulating-process in the constant voltage control. For improving the robustness, the regulating of DMPC is much slower than that of $\mathcal{L}_1$ adaptive control, which can be seen from the control signal shown in Fig.3(e) and Fig.3(f). If we quicken the DMPC regulating, the DMPC control system may not be stable. Thus, $\mathcal{L}_1$ adaptive control has obvious advantages over DMPC control in the fast tracking and disturbance rejection in this case. Something to note, we cannot say the control performance of DMPC shown in Fig.3 is its best one, but it is its best in all our simulations.



(a) Disturbance of stack current.



(b) From $\mathcal{L}_1$ adaptive control.

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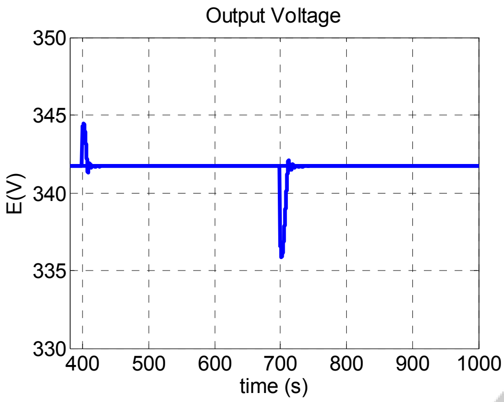
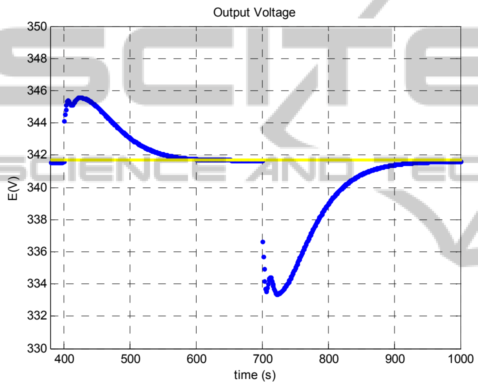
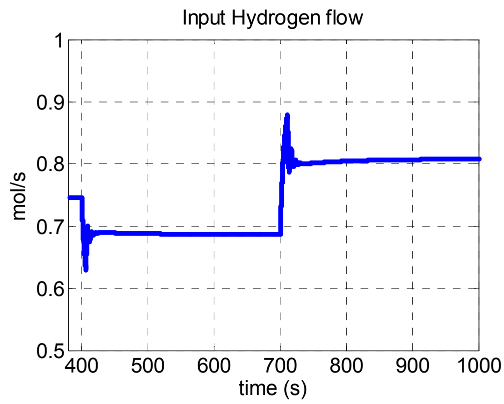


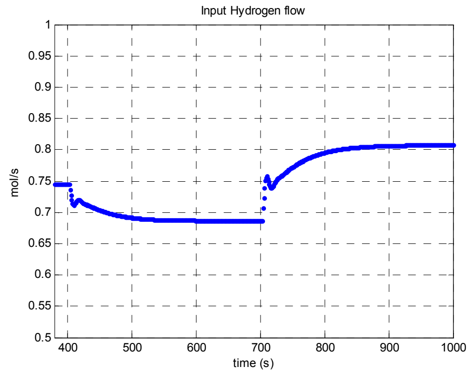
(c) From $\mathcal{L}_1$ adaptive control.



(d) From DMPC control.



(e) From $\mathcal{L}_1$ adaptive control.



(f) From DMPC control.

Figure 3: The simulation results of the $\mathcal{L}_1$ adaptive control and DMPC control on SOFC process.

Second, some comparisons are made with other published results of a nonlinear MPC (Li et al., 2011), we find that the $\mathcal{L}_1$ adaptive controller has a better rapidness under the guaranteed stability in the nonlinear SOFC process control and has much less online computation load.

6 CONCLUSIONS

This paper illustrates the fast-adaptation $\mathcal{L}_1$ adaptive controller design for the nonlinear SOFC process control. An output feedback controller is designed for SOFC system with unknown dynamics. Unlike model-based control, it only needs a few system parameters to design. The simulation results show that it has good capability of disturbance rejection and fast reference-tracking.

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