On the Stabilization of the Flexible Manipulator
Liapunov based Design. Robustness.

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Keywords: Hyperbolic Partial Differential equations, Hamilton Variational Principle, Energy Identity, Liapunov Energy Functional, Feedback Stabilization.

Abstract: This work deals with dynamics and control of the flexible manipulator viewed as a system with distributed parameters. It is in fact described by a mixed problem (with initial and boundary conditions) for a hyperbolic partial differential equation, the flexible manipulator being assimilated to a rod. As a consequence of the deduction of the model *via* the variational principle of Hamilton from Rational Mechanics, the boundary conditions result as "derivative" in the sense that they contain time derivatives of higher order (in comparison with the standard Neumann or Robin type ones). To the controlled model there is associated a control Liapunov functional by using the *energy identity* which is well known in the theory of partial differential equations. Using this functional the boundary stabilizing controller is synthesized; this controller ensures high precision positioning and additional boundary damping. All this synthesis may remain at the formal level, mathematically speaking. The rigorous results are obtained by using a one to one correspondence between the solutions of the boundary value problem and of an associated system of functional differential equations of neutral type. This association allows to prove in a rigorous way existence, uniqueness and well posedness. Moreover, in several cases there is obtained global asymptotic stability which is robust with respect to the class of nonlinear controllers - being in fact absolute stability. The paper ends with conclusions and by pointing out possible extensions of the results.

1 INTRODUCTION

A. In order to start a motivation for the present paper and the reported research, we shall reproduce from a survey paper in the field (Dwivedy and Eberhard, 2006): "Robotic manipulators are widely used to help in dangerous, monotonous, and tedious jobs. Most of the existing robotic manipulators are designed and build in a manner to maximize stiffness in an attempt to minimize the vibration of the end-effector to achieve a good position accuracy. This high stiffness is achieved by using heavy material and a bulky design. Hence, the existing heavy rigid manipulators are shown to be inefficient in terms of power consumption or speed with respect to the operating payload. Also, the operation of high precision robots is severely limited by their dynamic deflection, which persists for a period of time after a move is completed. The settling time required for this residual vibration delays subsequent operations, thus conflicting with the demand of increased productivity. These conflicting requirements between high speed and high accuracy have rendered the robotic assembly task a challenging research problem. Also, many industrial manipulators face the problem of arm vibrations during high speed motion.

In order to improve industrial productivity, it is required to reduce the weight of the arms and/or to increase their speed of operation. For these purposes it is very desirable to build flexible robotic manipulators. Compared to the conventional heavy and bulky robots, flexible link manipulators have the potential advantage of lower cost, larger work volume, higher operational speed, greater payload-to-manipulator-weight ratio, smaller actuators, lower energy consumption, better maneuverability, better transportability and safer operation due to reduced inertia. But the greatest disadvantage of these manipulators is the vibration problem due to low stiffness."

In terms of Dynamical Systems, vibration quenching is tightly connected with the basic problem of stability and stabilization; both stabilization and vibration quenching are achieved by feedback control if,especially increased stiffness is to be avoided. At its turn a good stabilizing structure may be achieved provided a sound mathematical model of the dynamics

Danciu D., Popescu D. and Rasvan V..
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DOI: 10.5220/0005051705080518
In *Proceedings of the 11th International Conference on Informatics in Control, Automation and Robotics* (ICINCO-2014), pages 508-518
ISBN: 978-989-758-039-0

is available. Or, the flexible manipulator belongs to the class of the *controlled objects with distributed parameters having one space dimension for parameter distribution*. More specific, reduced stiffness results in blocking the possibility of neglecting this parameter distribution along the length of the manipulator arm. Consequently, the modeling of the manipulator arm is made by assimilating it to a rod/beam. With respect to this we would like to point out another survey (Russell, 1986), where various "energy conservative and dissipative beam models" are described: Euler- Bernoulli model, Rayleigh model, Timoshenko model, models with Kelvin-Voight dissipation. If the models are deduced using the generalized variational principle of Hamilton, then considering one model or another depends on the expressions adopted for the potential energy and various forces involved in modeling.

B. The Hamilton approach in modeling beams for flexible robot manipulators as well as for other engineering devices (Răsvan, 2014) leads to somehow unusual mathematical structures i.e. initial boundary value problems for hyperbolic partial differential equations *with derivative boundary conditions*. From these models it is obvious that the natural damping of the systems consists of the quite weak distributed and boundary dampings; moreover, stability improvement which is in close connection with vibration quenching has to be achieved by feedback control. At its turn this feedback may be space distributed or boundary or combined.

A rather widespread method for control synthesis is now the *control Liapunov function approach*. For the aims of this paper a good reference survey might be (de Queiroz et al., 2000). As it is known, finding a suitable Liapunov function(al) implies "guessing". With respect to this the *energy identity* represents a very useful hint in finding a good Liapunov function which allows synthesis of a stabilizing controller. Since this synthesis may be performed at the formal level (from the mathematical point of view), the closed loop structure should be considered as a mathematical problem *an und für sich* (in itself) and treated as such (existence, uniqueness, well posedness, stability).

Following some references e.g. (Icart et al., 1992; Cherkaoui and Conrad, 1992; de Queiroz et al., 2000) we shall consider here the rod model for the flexible manipulator which coincides basically with the vibrating string equation. For this model we shall perform the synthesis of the stabilizing controllers, obtain the closed loop model and discuss the theory it. As a consequence what is left of this paper is organized as follows. First the corresponding expressions for the kinetic and potential energies will be considered allowing to obtain exactly the rod model *via* the Hamilton principle. The involved external or virtual (accounting for constraints) forces are listed in order to obtain the controlled mathematical model. To it we associate the energy identity which will turn useful for controller synthesis. Once the closed loop model will be obtained, a section of the paper will be allocated to the basic theory. The stability discussion will be followed by the robust control issues and robustness properties. Finally a section of conclusions and hints for future research will end the exposition.

2 THE MATHEMATICAL MODEL AND THE ENERGY IDENTITY

A. We shall consider the flexible manipulator carrying a payload and being rotated through a hub, as suggested by (de Queiroz et al., 2000) and illustrated in Figure 1

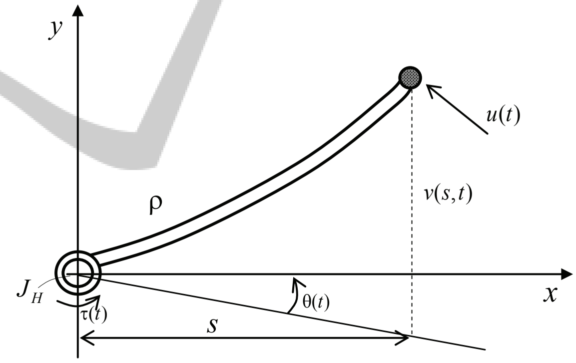

Figure 1: Flexible manipulator dynamics: s - current coordinate on the flexible arm; $\nu(s,t)$ - current local deflection of the flexible arm element including with respect to the inertia axis of the rotating hub; J_H - mass of the torque controlled rotating hub; m - payload mass; $u(t)$ - the control force at the payload; $\tau(t)$ - the control torque at the hub.

In the following we reproduce in brief the model deduction of (Răsvan, 2014). In order to make use of the variational principle of Hamilton we write down

1^{o} The kinetic energy of the controlled hub with the moment of inertia J_H, the flexible arm and the payload mass m, given by

$$E_k(t) = \frac{1}{2}\left[J_H\left(\dot{\theta}(t)\right)^2 + m\left(\frac{\mathrm{d}}{\mathrm{d}t}y(L,t)\right)^2 + \right. \\ \left. + \int_0^L \rho(s)y_t^2(s,t)\mathrm{d}s\right] \tag{1}$$

where $y(s,t)$ is the current position of the moving flexible cable at the local coordinate s (including the elastic deflection) and at the moment of time t, $\rho(s)$ being the local mass density at the local coordinate s. We shall have also

$$y(s,t) = s\theta(t) + \nu(s,t) \tag{2}$$

where θ is angular position of the rotating hub and $\nu(s,t)$ is position with respect to the rotating system of coordinates

2º The potential energy due to the strain energy of the flexible cable given by

$$E_p(t) = \frac{1}{2}\int_0^L T(s)\nu_s^2(s,t)\mathrm{d}s \tag{3}$$

where $T(s)$ is the tension of the flexible link of the arm.Remark that (2) implies $\nu_s(s,t) \equiv y_s(s,t) - \theta(t)$.

3º The external "forces" acting on this mechanical system are the following: the control torque $\tau(t)$, the thrust regulating force applied at the payload boundary, possible local and distributed perturbations. Even if they might be negligible, we shall include also the friction forces, in order to keep the model structure as close as possible to the most general case (Răsvan, 2014). The work of these forces is given by

$$\begin{aligned} W_m(t) &= (\tau(t) + \phi_0(t) + \chi_0(t))\theta(t) + \\ &+ (u(t) + f_L(t) + \chi_L(t))y(L,t) + \\ &+ \int_0^L (f(s,t) + \chi(s,t))y(s,t)\mathrm{d}s \end{aligned} \tag{4}$$

Here $\chi_0(t)$, $\chi_L(t)$, $\chi(s,t)$ are the virtual forces accounting for viscous damping forces given by

$$\begin{aligned} &\chi_0(t) = -c_0\dot{\theta}(t)\ ,\ \chi_L(t) = -c_L y_t(L,t)\ , \\ &\chi(s,t) = -c(s)y_t(s,t) \end{aligned} \tag{5}$$

Define the functional

$$I(t_1,t_2) := \int_{t_1}^{t_2} (E_k(t) - E_p(t) + W_m(t))\mathrm{d}t \tag{6}$$

and, following the approach of the variational calculus (Akhiezer, 1981), introduce the following variations

$$y(s,t) = \bar{y}(s,t) + \varepsilon\eta(s,t) \tag{7}$$

where $\bar{y}(s,t)$ corresponds to an extremal. Let $I_\varepsilon(t_1,t_2)$ be the functional (6) written along the variations (7). The necessary condition for the extremum is given by

$$I'_\varepsilon(0) = \left.\frac{\mathrm{d}I}{\mathrm{d}\varepsilon}\right|_{\varepsilon=0} = 0 \tag{8}$$

After some standard manipulation (Răsvan, 2014) we find the following model boundary value problem

$$\begin{aligned} &-\rho(s)y_{tt} - c(s)y_t + (T(s)(y_s - \theta))_s + f(s,t) = 0 \\ &y_s(0,t) = \theta(t) \\ &-J_H\ddot{\theta} - c_0\dot{\theta} + \int_0^L T(s)(y_s(s,t) - \theta)\mathrm{d}s + \\ &+\tau(t) + \phi_0(t) = 0 \\ &-my_{tt}(L,t) - c_L y_t(L,t) - T(L)(y_s(L,t) - \theta) + \\ &+u_L(t) + d_L(t) = 0 \end{aligned} \tag{9}$$

The model is somehow nonstandard due to the fact that it contains both linear and angular motion coordinates. However, if the flexible rod material has homogeneous properties i.e. $\rho(s)$, $c(s)$ but especially $T(s)$ are constant i.e. independent of the space coordinate then equations (9) become closer to the traditional ones, as follows

$$\begin{aligned} &-\rho y_{tt} - cy_t + Ty_{ss} + f(s,t) = 0 \\ &y_s(0,t) = \theta \\ &-J_H\ddot{\theta} - c_0\dot{\theta} - TL\theta + T(y(L,t) - y(0,t)) + \\ &+\tau(t) + \phi_0(t) = 0 \\ &-my_{tt}(L,t) - c_L y_t(L,t) + T\theta - Ty_s(L,t) + \\ &+u_L(t) + d_L(t) = 0 \end{aligned} \tag{10}$$

The model is clearly described by a boundary value problem for the string equation, the boundary conditions being derivative.It is worth mentioning that in standard cases c - the distributed damping - is negligible and there is no thrust control for the payload mass at $s = L$ i.e. $u_L(t) \equiv 0$.

In the following we shall write down the so called *energy identity* for (9): we multiply the first equation by y_t and perform some integration by parts with respect to s from 0 to L and take into account the boundary conditions to obtain

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}t}\cdot\frac{1}{2}\Big\{J_H(\dot{\theta}(t))^2 + m(y_t(L,t))^2 + \\ &+\int_0^L [\rho(s)y_t(s,t)^2 + T(s)(y_s(s,t) - \theta(t))^2]\mathrm{d}s\Big\} + \\ &\quad +c_0(\dot{\theta}(t))^2 + c_L(y_t(L,t))^2 + \int_0^L c(s)y_t(s,t)^2\mathrm{d}s - \\ &\quad -(\tau(t) + \phi_0(t))\dot{\theta}(t) - \\ &\quad -(u_L(t) + f_L(t))y_t(L,t) - \int_0^L f(s,t)y_t(s,t)\mathrm{d}s \equiv 0 \end{aligned} \tag{11}$$

We shall end this subsection by some additional comments and explanations concerning the premises of the obtained model. Our starting references had been at the beginning (Icart et al., 1992; Cherkaoui and Conrad, 1992); the model used there appeared to be more of pure mathematical interest while for the stabilizing control displayed some drawbacks (at least with respect to our purpose - “guess” and use of a “natural” control Liapunov function); on the other hand, the analogy of several models arising from various fields (overhead crane and marine riser, oilwell drillstring) turn to be stimulating for seeking for another model - obtained from physical premises. The reference (de Queiroz et al., 2000) contained such unified models under the framework of the control Liapunov functionals and the model of the flexible arm was adopted but based on the Hamilton principle. As mentioned in (de Queiroz et al., 2000), the model there strongly relies on the model in (Junkins and Kim, 1993). It must be mentioned however that the models of the aforementioned reference were obtained mainly for aeronautical structures and the physics might have been different e.g. centrifugal effects of stiffening and softening. Neglecting such “side effects” led e.g. to formula (2). There are also other specific simplifying approaches in our model but we discuss only the choice of the arm modeling as a rod what gave the string equation while there existed other options as described in (Russell, 1986) among which the Euler Bernoulli beam is mostly preferred. Last but not least, we have been guided in our option by the aim to obtain a rigorous ground for the boundary value problems thus obtained: complicated models, due to their nonlinearities and/or discontinuities are not easy to give a basic theory allowing to go beyond the formal level. It will become clear that even our models cannot be treated in all cases and open problems persist.

B. In the following we shall discuss the equilibria - constant trajectories - of system (9) and their “inherent stability”i.e. with blocked control signals; with respect to this we shall consider all perturbations identically 0

$$f(s,t) \equiv 0 \;,\; \phi_0(t) \equiv 0 \;,\; d_L(t) \equiv 0 \qquad (12)$$

and take $\tau(t) \equiv \bar{\tau}$, $u_L(t) \equiv \bar{u}_L$. Letting the time derivatives be identically 0 the following equations for the steady state are obtained

$$\begin{aligned} &(T(s)(\bar{y}_s - \bar{\theta}))_s \equiv 0 \;;\; \bar{y}_s(0) - \bar{\theta} = 0 \\ &\int_0^L T(s)(\bar{y}_s(s) - \bar{\theta})\mathrm{d}s + \bar{\tau} = 0 \\ &-T(L)(\bar{y}_s(L) - \bar{\theta}) + \bar{u}_L = 0 \end{aligned} \qquad (13)$$

to obtain $\bar{u}_L = 0$, $\bar{\tau} = 0$, $\bar{y}(s) = \bar{\theta}s + \bar{y}(0)$; taking into account (2) and the significance of $\nu(s,t)$ it follows that $\bar{y}(0) = 0$; however $\bar{\theta}$ remains undetermined what is quite natural since it is a cyclic coordinate.

The occurrence of a cyclic coordinate requires a deeper investigation of the possible steady states. With respect to this we introduce the so called coordinates of the *symmetric Friedrichs form for the partial differential equations* as follows

$$y_t(s,t) := v(s,t) \;,\; T(s)(y_s(s,t) - \theta(t)) := w(s,t) \qquad (14)$$

also $\dot{\theta} = \Omega$ to obtain the following boundary value problem

$$\begin{aligned} &\rho(s)v_t + c(s)v - w_s = f(s,t) \\ &w_t - T(s)(v_s(s,t) - \Omega(t)) = 0 \\ &w(0,t) = 0 \\ &J_H\dot{\Omega} + c_0\Omega - \int_0^L w(s,t)\mathrm{d}s = \tau(t) + \phi_0(t) \\ &mv_t(L,t) + c_L v(L,t) + w(L,t) = u_L(t) + d_L(t) \end{aligned} \qquad (15)$$

Taking again identically zero perturbations $f(s,t)$, $\phi_0(t)$, $d_L(t)$ and blocked (constant) input signals $\bar{\tau}$ and $\bar{u}_L$, a new type of equilibria is obtained, namely

$$\bar{v}(s) = \bar{\Omega}s + \bar{v}(0) \;,\; \bar{w}(s) = \int_0^s (\Omega\sigma + \bar{v}(0))c(\sigma)\mathrm{d}\sigma \qquad (16)$$

where $\bar{v}(0)$ and Ω are the unique solution of a linear system. If these steady state variables are non-zero this would signify a uniformly rotating arm (!) with some steady state elastic deformations unless we impose again $\bar{u}_L = 0$, $\bar{\tau} = 0$. This is the only interesting case form the engineering point of view.

In order to obtain some information about the stability of the equilibrium thus determined we shall examine the free system (15); this is a linear boundary value problem and the stability of the identically zero equilibrium might be examined using the Laplace transform at least at the formal level since we have no information about its admissibility for the solutions of (15). Taking into account the space varying parameters and the quite complicated boundary conditions, the associated transforms might be not easy to tackle. But we have at our disposal the energy identity (11) which suggests the following quadratic Liapunov functional (written in the variables of (15))

$$\begin{aligned} \mathcal{V}(X,Y,\phi(\cdot),\psi(\cdot)) = \frac{1}{2}\Big\{ & J_H X^2 + mY^2 + \\ & + \int_0^L (\rho(s)\phi^2(s) + \psi^2(s))\mathrm{d}s \Big\} \end{aligned} \qquad (17)$$

defined on $\mathbb{R} \times \mathbb{R} \times \mathcal{L}^2(0,L) \times \mathcal{L}^2(0,L)$ with the restriction $Y = \phi(L)$. The energy identity also gives the expression of the derivative of (17) along the solutions of the free system (15) i.e. with $u_L(t) \equiv 0$, $\tau(t) \equiv 0$, $\phi_0(t) \equiv 0$, $d_L(t) \equiv 0$, $f(s,t) \equiv 0$

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{V}(\Omega(t), v(L,t), v(\cdot,t), w(\cdot,t)) = \\ &= -c_0\Omega^2(t) - c_L v^2(L,t) - \int_0^L c(s)v^2(s,t)\mathrm{d}s \end{aligned} \tag{18}$$

Since V is positive definite and its derivative - negative semi-definite, the equilibrium at the origin is stable (Liapunov stable in the sense of the norm defined by the Liapunov functional (17) itself). The Liapunov functional being an energy, it is only natural to obtain a derivative functional which is negative semi-definite only - this is well known for energy like Liapunov functions arising in the early stages of the stability theory. We may use the Barbashin Krasovskii LaSalle invariance principle (at least in a formal way, but we may hope to be in one of the cases when it holds for abstract dynamical systems also (Saperstone, 1981)) and find that the only invariant set where the kernel of the derivative function is contained is the zero solution. This zero solution corresponding to the arm with the payload stopped in some resulting position is thus stable.

This result (still at the formal level) suggests the following remarks

i) the free system (15) has a quite non-robust stability since it depends on the natural dampings only and these dampings are very weak even negligible in practice;

ii) the system having a cyclic variable, it is but natural to have the arm with the payload stopped in an arbitrary position depending on the initial one; however this is not acceptable in the practical applications.

The above considerations show that, in order to ensure positioning of the payload and an improved stability for the flexible manipulator, a feedback controller must be used. In the following we shall consider controller synthesis and the properties of the closed loop system.

3 CONTROLLER DESIGN. THE RESULTING CLOSED LOOP SYSTEM

We shall realize the controller design based on the c.l.f. (control Liapunov function) method by making use of a modified version of the functional (17) taking into account the two aforementioned tasks of the controller: precise positioning of the payload and improved stability. For the positioning we shall add to (17) a quadratic term penalizing the error from a given angular reference θ_p.

A. Refer first to the basic controlled system (9); since we discuss stabilization, the persistent perturbations $f(s,t)$, $\phi_0(t)$, $d_L(t)$ are again set to zero. The Liapunov functional suggested by (11) is as follows

$$\begin{aligned} &\mathcal{V}(X,Y,Z,\phi(\cdot),\psi(\cdot)) = \frac{1}{2}\left\{a_0X^2 + J_HY^2 + mZ^2 + \right. \\ &\left. + \int_0^L (\rho(s)\phi^2(s) + T(s)(\psi(s)-X)^2)\mathrm{d}s\right\} \end{aligned} \tag{19}$$

where $a_0 > 0$ is a free design parameter; as mentioned, the term $(\psi(\cdot)-\theta_p)^2$ is penalizing the deviation from the prescribed angular position of the payload. The functional is defined on $\mathbb{R}\times\mathbb{R}\times\mathbb{R}\times\mathcal{L}^2(0,L)\times\mathcal{L}^2(0,L)$ with the restriction $Z = \phi(L)$. On the domain of definition the functional is positive definite. Along the solutions of (9) this functional reads as follows

$$\begin{aligned} &\mathcal{V}(\theta(t)-\theta_p, \dot\theta(t), y_t(L,t), y_t(\cdot,t), y_s(\cdot,t)) = \\ &= \frac{1}{2}\left\{a_0(\theta(t)-\theta_p)^2 + J_H\dot\theta^2(t) + my_t^2(L,t) + \right. \\ &\left. + \int_0^L (\rho(s)y_t^2(s,t) + T(s)(y_s(s,t)-\theta(t))^2)\mathrm{d}s\right\} \end{aligned} \tag{20}$$

Its derivative along the solutions has the following form

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{V}(\theta(t)-\theta_p, \dot\theta(t), y_t(L,t), y_t(\cdot,t), y_s(\cdot,t)) = \\ &= a_0(\theta(t)-\theta_p)\dot\theta(t) - c_0(\dot\theta(t))^2 - c_L(y_t(L,t))^2 - \\ &- \int_0^L c(s)y_t(s,t)^2\mathrm{d}s + \tau(t)\dot\theta(t) + u_L(t)y_t(L,t) \end{aligned} \tag{21}$$

A rather simple choice for the controller structure would be

$$\begin{aligned} &\tau(t) = -a_0(\theta(t)-\theta_p) - g_0(\dot\theta(t)) , \\ &u_L(t) = -g_L(y_t(L,t)) \end{aligned} \tag{22}$$

where $g_i(\sigma)$, $i := 0, L$ are sector restricted nonlinear functions that is, subject to the following inequalities

$$\underline{g}_i\sigma^2 \le g_i(\sigma)\sigma \le \bar{g}_i\sigma^2 , \; g_i(0) = 0 , \; i := 0, L \tag{23}$$

Observe that these nonlinear functions are arbitrary, the only conditions being (23). This signifies that we assume some robustness properties of the stabilization with respect to some uncertainty of the controllers. In particular these functions can be taken linear thus “pointing” to a PD (proportional derivative) controller at the boundary $s = 0$ and a D (derivative) controller at $s = L$. In the general case of the sector restricted nonlinearities, these are nonlinear controllers. Introducing the controllers' equations (22) in the controlled equations (9), the equations of the closed loop system are obtained as follows (including now the non-zero persistent perturbations

$$\begin{aligned} &\rho(s)y_{tt} + c(s)y_t - (T(s)(y_s - \theta))_s = f(s,t) \\ &y_s(0,t) = \theta(t) \\ &J_H\ddot{\theta} + c_0\dot{\theta} + g_0(\dot{\theta}) + a_0(\theta(t) - \theta_p) - \\ &- \int_0^L T(s)(y_s(s,t) - \theta)\mathrm{d}s = \phi_0(t) \\ &my_{tt}(L,t) + c_L y_t(L,t) + g_L(y_t(L,t)) - \\ &-T(L)(y_s(L,t) - \theta) = d_L(t) \end{aligned} \tag{24}$$

From these equations it is quite clear that the PD controller at $s = 0$ "imposes" to the cyclic variable θ the reference value θ_p and introduces an additional nonlinear/linear damping.The other controller - a nonlinear D controller - introduces an additional nonlinear/linear damping.

B. We shall now discuss in brief some straightforward properties of system (24). Let again the persistent perturbations be identically zero and check first the steady state solutions, which are static equilibria (speaking the language of Rational Mechanics) subject to the steady state equations

$$\begin{aligned} &(T(s)(\bar{y}_s(s) - \bar{\theta}))_s = 0\ ,\ \bar{y}_s(0) = \bar{\theta} \\ &a_0(\bar{\theta} - \theta_p) - \int_0^L T(s)(\bar{y}_s(s) - \bar{\theta})\mathrm{d}s = 0 \\ &T(L)(\bar{y}_s(L) - \bar{\theta}) = 0 \end{aligned} \tag{25}$$

It is a matter of simple and straightforward manipulation to find

$$\bar{\theta} = \theta_p\ ,\ \bar{y}(s) = \theta_p s$$

hence the static equilibrium is unique and its significance is that positioning is without steady state error.

We have to prove however that this steady state is a limit regime and it has asymptotic stability. With respect to this we remind that to the closed loop system (24) there is associated the c.l.f. (control Liapunov functional) (19) which has the form (20) along system (24) and is positive definite. If the persistent perturbations are again set to zero since the aim is equilibrium stability, then the derivative of the Liapunov functional that follows from (21) with the controller choice (22) will be

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{V}(\theta(t) - \theta_p, \dot{\theta}(t), y_t(L,t), y_t(\cdot,t), y_s(\cdot,t)) = \\ &= -\int_0^L c(s)y_t(s,t)^2\mathrm{d}s - (c_0\dot{\theta}(t) + g_0(\dot{\theta}(t)))\dot{\theta}(t) - \\ &-(c_L y_t(L,t) + g_L(y_t(L,t)))y_t(L,t) \leq 0 \end{aligned} \tag{26}$$

Let us first mention that in the real case of a flexible manipulator there is no thrust controller on the payload (unlike in the case of e.g. the marine riser) hence $g_L(\sigma) \equiv 0$. If the sector conditions (23) are such that $c_0\sigma + g_0(\sigma) > 0$ then the derivative function (26) vanishes on the set

$$y_t(s,t) \equiv 0\ ,\ c_0\dot{\theta}(t) + g_0(\dot{\theta}(t)) \equiv 0\ ,\ y_t(L,t) \equiv 0$$

On this set the trajectories of system (24) are defined exactly by the steady state equations (25) hence the largest invariant set contained in the set where the derivative function (26) vanishes is exactly the unique equilibrium. Global asymptotic stability of this equilibrium would follow *provided the invariance principle Barbašin Krasovskii LaSalle is valid in this case*. In some special cases this might be true, see (Saperstone, 1981); one of this cases which is some how alike the considered here corresponds to the overhead crane and is the subject of (d'Andréa Novel et al., 1994), being mentioned in (Răsvan, 2014). In the following the problem will be analyzed within the framework of the neutral functional differential equations attached to the mixed initial boundary valued problems for hyperbolic partial differential equations.

4 THE BASIC PROPERTIES OF THE DYNAMICAL SYSTEM

Starting from the 60ies of the previous century, some authors who studied hyperbolic partial differential equations in two dimensions, applied to physics and engineering, discovered that integration of the Riemann invariants along the characteristics allow association of a system of differential equations with deviated argument to the mixed initial boundary value problem with unusual (in the sense of non standard that is different from the Neumann or Robin) boundary conditions containing higher order derivatives. This "association" must be understood in the sense that the aforementioned system of differential equations with deviated argument is constructed starting from a solution of the boundary value problem and the structure of the boundary conditions. Moreover, a one to one correspondence between the solutions of the two aforementioned mathematical objects may be established in a rigorous way (see Theorem 1 and all mathematical results obtained for one of these objects may be projected on the other one. Among the papers contributing to this approach, which are enumerated in the Reference list of (Răsvan, 2014), it is worth mentioning the first of that list (Abolinia and Myshkis, 1960) and the less circulated but the most useful for our context (Cooke, 1970) (where the term “derivative boundary conditions” is used).We shall not reproduce here the results of the aforementioned reference but rather apply them to our case.

A. The first step will be to associate the system of functional differential equations, starting from the considered basic system since it has been already mentioned that this method does not apply but to particular cases. Consider the closed loop system (24) under the following assumptions

$$c(s) \equiv 0 \; ; \; \rho(s) \equiv \rho = \text{const} \; , \; T(s) \equiv T = \text{const}$$

We assume also, to simplify the computation formulae, that the distributed perturbations are identically zero i.e. $f(s,t) \equiv 0$. This assumption does not affect generality of the analysis since we are interested mainly in the Liapunov stability of the equilibria. Under these circumstances (24) becomes

$$\begin{aligned} &\rho y_{tt} - T y_{ss} = 0 \; ; \; y_s(0,t) = \theta \\ &J_H \ddot{\theta} + c_0 \dot{\theta} + g_0(\dot{\theta}) + TL\theta + a_0(\theta - \theta_p) - \\ &\qquad - T \int_0^L y_s(s,t) \mathrm{d}s = \phi_0(t) \\ &m y_{tt}(L,t) + c_L y_t(L,t) + T(y_s(L,t) - \theta) = d_L(t) \end{aligned} \tag{27}$$

Introduce now the variables for the symmetric Friedrichs form, with their initial conditions deduced from those of (24)

$$\begin{aligned} &v(s,t) := y_t(s,t) \; , \; w(s,t) := y_s(s,t) \; ; \\ &(y(s,0) = y_0(s) \; , \; y_t(s,0) = v_0(s)) \Rightarrow \\ &\Rightarrow v(s,0) = v_0(s) \; , \; w(s,0) = y_0'(s) \end{aligned} \tag{28}$$

to obtain the new system

$$\begin{aligned} &w_t = v_s \; , \; \rho v_t - T w_s = 0 \; ; \; w(0,t) = \theta \\ &J_H \ddot{\theta} + c_0 \dot{\theta} + g_0(\dot{\theta}) + TL\theta + a_0(\theta - \theta_p) - \\ &\qquad - T \int_0^L w(s,t) \mathrm{d}s = \phi_0(t) \\ &m v_t(L,t) + c_L v(L,t) + T(w(L,t) - \theta) = d_L(t) \end{aligned} \tag{29}$$

Introduce now the Riemann invariants as follows

$$u^{\pm}(s,t) = \frac{1}{2}(v(s,t) \mp \sqrt{T/\rho}\, w(s,t)) \tag{30}$$

with the corresponding initial conditions

$$u^{\pm}(s,0) = \frac{1}{2}(v_0(s) \mp \sqrt{T/\rho}\, y_0'(s)) \tag{31}$$

Here $u^+(s,t)$ stands for the progressive wave and $u^-(s,t)$ for the reflected one. Consequently the initial boundary value problem for the Riemann invariants reads as below

$$\begin{aligned} &u_t^{\pm} \pm \sqrt{T/\rho}\, u_s^{\pm} = \frac{1}{2\rho} f(s,t) \\ &\sqrt{\rho/T}(u^-(0,t) - u^+(0,t)) = \theta \\ &J_H \ddot{\theta} + c_0 \dot{\theta} + g_0(\dot{\theta}) + TL\theta + a_0(\theta - \theta_p) - \\ &\qquad - \sqrt{\rho T} \int_0^L (u^-(s,t) - u^+(s,t)) \mathrm{d}s = \phi_0(t) \\ &m \frac{\mathrm{d}}{\mathrm{d}t}(u^-(L,t) + u^+(L,t)) + c_L(u^-(L,t) - u^+(L,t)) + \\ &+ \sqrt{\rho T}(u^-(L,t) - u^+(L,t)) - T\theta = d_L(t) \end{aligned} \tag{32}$$

We are now in position to apply the algorithm suggested in (Cooke, 1970) and fully described and proved in (Răsvan, 2014). Integration along the characteristics of the Riemann invariants will give

$$\begin{aligned} &y^+(t) := u^+(L,t) \;\Rightarrow u^+(0,t) = y^+(t + L\sqrt{\rho/T}) \; ; \\ &y^-(t) := u^-(0,t) \;\Rightarrow u^-(L,t) = y^-(t + L\sqrt{\rho/T}) \end{aligned} \tag{33}$$

and suggests the following representation formulae

$$\begin{aligned} &u^+(s,t) = y^+(t + s\sqrt{\rho/T}) \; , \\ &u^-(s,t) = y^-(t + (L-s)\sqrt{\rho/T}) \end{aligned} \tag{34}$$

Introducing the new functions

$$\eta^{\pm}(t) = y^{\pm}(t + L\sqrt{\rho/T}) \tag{35}$$

we obtain, after some manipulation the following system of functional equations

$$\begin{aligned} &J_H \ddot{\theta} + c_0 \dot{\theta} + g_0(\dot{\theta}) + TL\theta + a_0(\theta - \theta_p) - \\ &\qquad - T \int_{-L\sqrt{\rho/T}}^0 (\eta^-(t+\lambda) - \eta^+(t+\lambda)) \mathrm{d}\lambda = \phi_0(t) \\ &\sqrt{\rho/T}(\eta^+(t) - \eta^-(t - L\sqrt{\rho/T})) + \theta = 0 \\ &m \frac{\mathrm{d}}{\mathrm{d}t}(\eta^-(t) + \eta^+(t - L\sqrt{\rho/T})) + \\ &+ c_L(\eta^-(t) + \eta^+(t - L\sqrt{\rho/T})) + \\ &+ \sqrt{\rho T}(\eta^-(t) - \eta^+(t - L\sqrt{\rho/T})) - T\theta = d_L(t) \end{aligned} \tag{36}$$

Following the same procedure (Cooke, 1970; Răsvan, 2014), we associate to (36) the initial conditions

$$\eta_0^+(t) = u^+(-t\sqrt{T/\rho}, 0)\ ,$$
$$\eta_0^-(t) = u^-(L + t\sqrt{T/\rho}, 0)\ ;\ -L\sqrt{T/\rho} \le t \le 0 \qquad (37)$$

The converse procedure, based on the representation formulae deduced from (35) namely

$$u^+(s,t) = \eta^+(t + (s-L)\sqrt{\rho/T})\ ,$$
$$u^-(s,t) = \eta^-(t - s\sqrt{\rho/T}) \qquad (38)$$

allows the statement of the following theorem establishing a one-to-one correspondence between the solutions of (32) and the solutions of (36)

Theorem 1. *Let $(u^\pm(s,t), \theta(t))$ be a solution of (32) with some initial conditions $(u^\pm(s,0), \theta(0), \dot\theta(0))$, $0 \le s \le L$. Then $\eta^\pm(t), \theta(t)$ is a solution of (36) with the initial conditions $\eta_0^\pm(t), \theta(0), \dot\theta(0))$, where $\eta_0^\pm(t)$ are defined on $-L\sqrt{T/\rho} \le t \le 0$ by (37).*

Conversely, let $\eta^\pm(t), \theta(t)$ be a solution of (36) with some initial conditions $\eta_0^\pm(t), \theta(0), \dot\theta(0))$, where $\eta_0^\pm(t)$ are defined on $-L\sqrt{T/\rho} \le t \le 0$. Then $(u^\pm(s,t), \theta(t))$ is a solution of (32) with the initial conditions $(u^\pm(s,0), \theta(0), \dot\theta(0))$, where $u^\pm(s,t)$ are defined by the representation formulae (38) and the initial conditions $(u^\pm(s,0), \theta(0), \dot\theta(0))$ with $u^\pm(s,0)$ defined accordingly also from (38).

B. Having Theorem 1 at our disposal, we may focus now on system (36) since all results concerning its solutions are automatically projected back on the solutions of (32). Observe first that system (36) is a system of coupled delay differential and difference equations in continuous time. Such systems belong to the broader class of functional differential equations of *neutral type* see e.g. (Hale and Lunel, 1993). It can be seen that system (36) is quasi-linear: it contains a single nonlinear function which is sector restricted - see (23). For such systems existence, uniqueness and well posedness of the Cauchy initial value problem holds - see (Hale and Lunel, 1993), Section 2.8. From these results on neutral functional differential equations we deduce existence, uniqueness and well posedness of the initial boundary value problem (32) at the level of *continuous/discontinuous classical solutions*. This is a first step in overcoming the formal level of the results concerning system (28) and of (27) also, due to various representation formulae leading from these systems to (32) and further, to (36) and conversely.

We focus in the following on the stability problem for the equilibria of (36). The unique equilibrium of (36) corresponding to $\phi_0(t) \equiv 0$, $d_L(t) \equiv 0$ is given by

$$\bar\theta = \theta_p\ ;\ \bar\eta^+ = -\bar\eta^- = \frac{1}{2}\sqrt{T/\rho}\theta_p$$

The next step is to write down the Liapunov functional (20) in the language of the variables of system (36). Some straightforward manipulation based on (30),(33)-(35) will give the following form of the Liapunov functional

$$\mathcal{V}(\theta(t)-\theta_p, \dot\theta(t), \eta^+(\cdot), \eta^-(\cdot)) = \frac{1}{2}\{a_0(\theta(t)-\theta_p)^2$$
$$+ J_H\dot\theta^2(t) + m(\eta^-(t - L\sqrt{\rho/T}) + \eta^+(t))^2 +$$
$$\rho\int_0^L [(\eta^-(t - s\sqrt{\rho/T}) + \eta^+(t + (s-L)\sqrt{\rho/T}))^2 +$$
$$(\eta^-(t - s\sqrt{\rho/T}) + \eta^-(t + (s-L)\sqrt{\rho/T}) -$$
$$\theta(t))^2]\mathrm{d}s\} \qquad (39)$$

which is positive definite. In the same way we rewrite the derivative (26) as follows, taking also into account that $c(s) \equiv 0$ and $g_L(\sigma) \equiv 0$

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{V}(\theta(t)-\theta_p, \dot\theta(t), y_t(L,t), y_t(\cdot,t), y_s(\cdot,t))$$
$$= -(c_0\dot\theta(t) + g_0(\dot\theta(t)))\dot\theta(t) - \qquad (40)$$
$$-c_L(\eta^+(t - L\sqrt{\rho T}) + \eta^-(t))^2 \le 0$$

The set where this derivative vanishes is defined by

$$\dot\theta(t) \equiv 0\ ,\ \eta^+(t - L\sqrt{\rho T}) + \eta^-(t) \equiv 0 \qquad (41)$$

From these conditions and from equations (36) we deduce that the largest invariant set contained in (41) is exactly the unique equilibrium of (36) namely

$$\bar\theta = \theta_p\ ;\ \bar\eta^+ = -\bar\eta^- = (1/2)\sqrt{T/\rho}$$

Application of the Barbašin Krasovskii LaSalle invariance principle can thus give global asymptotic stability of this equilibrium of system (36) and, *via* Theorem 1, global asymptotic stability of the time invariant solution of (32); taking into account the converse expressions of (30), the same property holds for the equilibrium of (29).

Now, the invariance principle for neutral functional differential equations is given by Theorem 12.7.2 of (Hale and Lunel, 1993). However this theorem is valid only for those neutral functional differential equations with strongly stable difference operator. In the case of (36) this difference operator is defined by

$$\mathcal{D}\begin{pmatrix}\eta^+\\ \eta^-\end{pmatrix}(\cdot) = \begin{pmatrix}\eta^+(0)\\ \eta^-(0)\end{pmatrix} - \begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix} \times$$
$$\begin{pmatrix}\eta^+(-L\sqrt{\rho/T})\\ \eta^-(-L\sqrt{\rho/T})\end{pmatrix} \qquad (42)$$

and its stability is established by the location of the eigenvalues of the matrix

$$D = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

with respect to the unit disk of the complex plane. Since these eigenvalues are $\pm \imath$, the difference operator is only stable and Theorem 12.7.2 cannot be applied. As mentioned in (Saperstone, 1981) the strong stability condition might be replaced by an additional condition of smoothing the trajectories of the dynamical system, but such a condition has not been pointed out yet.

If nevertheless the payload mass effects are considered negligible, then the last equation of (36) is replaced by

$$c_L(\eta^-(t) + \eta^+(t - L\sqrt{\rho/T})) + \\ + \sqrt{\rho T}(\eta^-(t) - \eta^+(t - L\sqrt{\rho/T})) - T\theta = d_L(t)$$

Consequently the matrix D of the newly defined difference operator will be replaced by

$$D = \begin{pmatrix} 0 & 1 \\ -\dfrac{c - \sqrt{\rho T}}{c + \sqrt{\rho T}} & 0 \end{pmatrix}$$

whose purely imaginary eigenvalues are now inside the unit disk. The global asymptotic stability follows now in a rigorous way being in fact valid for the equilibrium of the following system

$$\begin{aligned} & w_t = v_s\ ,\ \rho v_t - T w_s = 0\ ;\ w(0,t) = \theta \\ & J_H\ddot{\theta} + c_0\dot{\theta} + g_0(\dot{\theta}) + TL\theta + a_0(\theta - \theta_p) - \\ & \qquad - T\int_0^L w(s,t)\mathrm{d}s = \phi_0(t) \\ & c_L v(L,t) + T(w(L,t) - \theta) = d_L(t) \end{aligned} \tag{43}$$

C. It is interesting and useful to make some comments concerning the global asymptotic stability of system (43). This control-theoretical result has been obtained by using a naturally associated control Liapunov functional suggested by the energy identity. The synthesized stabilizing controller has a twofold role - exact positioning of the payload and increasing the damping factor at the controlled boundary. This simple controller has nevertheless a necessary *robust stability* in the sense that the global asymptotic stability property is valid for an entire class of linear and nonlinear damping functions $g_0(\sigma)$ satisfying the sector condition

$$c_0\sigma + g_0(\sigma) > 0$$

that is system (43) is *absolutely stable*. Moreover, if the approach of K.P. Persidskii is carefully used for the systems of neutral functional differential equations as in the case of delayed functional differential equations (Răsvan, 2012) then this asymptotic stability may be shown as *exponential* - what is basic in engineering systems stability.

5 CONCLUSIONS. OPEN PROBLEMS AND PERSPECTIVES

We have presented throughout this paper an attempt to have a sound mathematical basis for the analysis of the dynamics and the control of a flexible manipulator with distributed parameters - assimilated to a rod. We overview briefly the topics already presented in the Abstract: writing down of the model as an initial boundary value problem with derivative boundary conditions by applying the variational principle of Hamilton; association of a "natural" control Liapunov functional issued from the energy identity; synthesis of a positioning and stabilizing boundary controller; association of a system of functional differential equations of neutral type allowing the rigorous construction of the basic (existence, uniqueness, well posedness) theory as well as of the stability theory based on weak Liapunov functionals and Barbašin Krasovskii LaSalle principle. Worth mentioning that this association allows more, e.g., numerically robust computational approaches based on the method of lines (which turns to be within this framework delay approximation by ordinary differential equations) which are implemented using structures belonging to the techniques of Artificial Intelligence (Danciu, 2013a; Danciu, 2013b; Danciu and Răsvan, 2014).

A number of possible extensions and open problems have been pointed out even throughout the paper exposition. We give here a brief account of some of them. First, the approach *via* neutral functional differential equations appears as feasible for systems without distributed damping and with uniform parameters. In other cases they are much more complicated (Abolinia and Myshkis, 1960) and an additional analysis is necessary. With respect to this it is worth recalling some aspects that have been discussed in Section 2.

Mathematical modeling starting from Physics and Theoretical Engineering is almost always associated with the attempt to be as complete and exhaustive as possible regardless how complicated the model results. However, a good well fitted simplifying assumption may transform that primary model into a tractable one (we have in mind the Prandtl assumption that made the Navier Stokes equations solvable). In the present case we have been guided by the similarities between the models occurring in such various engineering fields as flexible manipulator arms, risers and overhead cranes, drillstrings (Bobaşu et al., 2012; Saldivar et al., 2013; Răsvan, 2013; Răsvan, 2014). These similarities at their turn arise from considering identical expressions for the energies and the same type of forces or torques acting on these systems. Worth mentioning that such simplifying assumptions as adoption of formula (2) have also their role as well as the string model for the beams occurring in the physical models; other models of beams presented in (Russell, 1986) such as Euler Bernoulli, Rayleigh, Timoshenko may provide other sources for research development.

Even in the analyzed case a lot of problems occurred when considering basic (existence and uniqueness) as well as stability theory and we were able to solve only some particular cases. Here the discussion is quite interesting and deserves some attention. The experience of the authors shows that all encountered models arising from Mechanics and tackled by associating neutral functional differential equations generate difference operators - see Section 4 - that are stable but not strongly stable. On the other hand the standard theory of these equations (Hale and Lunel, 1993) is based on the strong stability assumption. Therefore relaxation of this assumption is another urgent task. Finally, development of the qualitative theory to include the case of persistent perturbations (dissipativeness) and stability of the forced oscillations is also of obvious interest. The authors consider these assertions as underlying a genuine research program since the considered model of this paper applies to other fields also.

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