Input and State Constrained Nonlinear Predictive Control
Application to a Levitation System

Joanna Zietkiewicz
Institute of Control and Information Engineering, Poznan University of Technology, Piotrowo 3a, 60-965 Poznan, Poland

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Abstract: The subject of the article concerns a constrained predictive control with feedback linearization (FBL) applied for multiple-input and multiple-output (MIMO) system. It relies on finding a compromise in every step between feasible and optimal linear quadratic (LQ) control by minimization of one variable. Behaviour of model signals in function of minimized variable is investigated, in order to assure the optimality of the solution. LQ control based applications for feedback linearized models do not meet the problem of choosing weights in linear quadratic cost function. That important problem is solved here by comparison of the cost function with that obtained for the linear approximated system in the operating point. That provides satisfactory behaviour and also justifies the simplified approach relied on minimization of only one variable for MIMO system.

1 INTRODUCTION

Feedback linearization method (Isidori, 1995), (Khalil, 2002) as the exact method for linearizing nonlinear models provides the advantage of possibility of using the linear control theory for nonlinear systems. The LQ control is applied in the interpolation method, proposed in (Poulsen et al., 2001). The method allows us to introduce constraints of variables into control designing. The advantage of this method is that it relies on minimization of only one variable. It was shown, that the method can be used for multiple-input, multiple-output systems, providing that certain conditions are fulfilled (Zietkiewicz, 2012).

Additional problem appears for feedback linearizated models and linear quadratic control and it concerns weights values in LQ cost function for feedback linearized model. In general the signals in the cost function are not described by direct physical values. Experimental choice or methods involving values of variables like Bryson rule (Kang et al., 2014) may not be sufficient. Usually in literature only weights on diagonal are used and the part which penalizes functions of multiplied state and input variables is omitted. In (Poulsen et al., 2001) a constant value was used in designing control to maintain the equilibrium between values in cost function. It was sufficient for given system, but in general equilibrium between every state and input signals has to be consider. The problem is solved in the article by using cost function with signals of original nonlinear model (it can be assumed as the cost function for the model linearized in the operating point). Signals in that function are known as physical values and weights in it is easier to choose. The function is rearranged by approximation of nonlinear dependencies between signals from nonlinear model and feedback linearized model. The dependencies are obtained from feedback linearization procedure. The finally obtained weights in cost function for feedback linearized model assures the equilibrium between variables of the model.

As the example to present the results of the method, model of levitation system is used (mls2em, 2009). Problem of levitation control is the subject of many papers. Many types of control method have been used, including backstepping technique (Yang and Tateishi, 2001), sliding mode control (Al-Muthairi and Zribi, 2004), fuzzy logic control (Ahmad et al., 2010), predictive control (Bachle et al., 2013) with many others ideas and also fusions of methods. The problem is still widely researched, however only part of solutions consider constraints. The model of levitation system used in this paper is well described with basic control results in (Dragos et al., 2012).

Simulations of proposed method for the levitation model (mls2em, 2009) was performed in matlab environment. Results show that signals fulfill constraints and appropriately chosen weight values cause fast performance. The result of FBL application to

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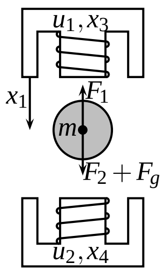

Figure 1: levitation system.

MIMO model is the decomposition to several linear models. The maintained equilibrium between values in cost functions of those linear models provides also equilibrium in the influence of minimized variable on that models. That justifies also the ability of the use of only one minimized variable for MIMO systems.

2 LEVITATION SYSTEM

The object of control is the levitation system shown in figure 1. It consists of two electromagnets and a ball placed beetween them.

The dynamics of the object can be represented by equations

$$\begin{aligned}
\dot{x}_1 &= x_2\\
\dot{x}_2 &= -\frac{F_{emP1}}{mF_{emP2}}x_3^2 e^{-\frac{x_1}{F_{emP2}}} + g + \frac{F_{emP1}}{mF_{emP2}}x_4^2 e^{-\frac{x_d - x_1}{F_{emP2}}}\\
\dot{x}_3 &= \frac{f_{iP2}}{f_{iP1}}(k_i u_1 + c_i - x_3)e^{\frac{x_1}{f_{iP2}}}\\
\dot{x}_4 &= \frac{f_{iP2}}{f_{iP1}}(k_i u_2 + c_i - x_4)e^{\frac{x_d - x_1}{f_{iP2}}}
\end{aligned} \tag{1}$$

where: x_1[m]- position of the ball, x_2[m/s]- velocity of the ball, x_3, x_4[A] - currents of the upper and lower magnets coils respectively, u_1, u_2 are the unitless control signals. The values of constants are presented in table 1.

Table 1: Constant values of the levitation system.

parameter value	description
$m = 0.0571$ kg	ball mass
$g = 9.81$ m/s^2	gravity constant
$F_{emP1} = 1.7521 * 10^{-2}$ H	electromagnet parameter
$F_{emP2} = 5.8231 * 10^{-3}$ H	electromagnet parameter
$f_{iP1} = 1.4142 * 10^{-4}$ ms	actuator parameter
$f_{iP2} = 4.5626 * 10^{-3}$ m	actuator parameter
$k_i = 2.5165$ A	actuator parameter
$c_i = 0.0243$ A	actuator parameter
$d = 0.075$ m	distance
$b_d = 0.06$ m	ball diameter
$x_d = 0.015$ m	$d - b_d$

Several variables in the system should restrict given constraints:

$$\begin{array}{lllllll}
0 & \text{m} & \leq & x_1(t) & \leq & x_d & \text{m}\\
0.03884 & \text{A} & \leq & x_3(t), x_4(t) & \leq & 2.38 & \text{A}\\
0.00498 & & \leq & u_1(t), u_2(t) & \leq & 1 &
\end{array} \tag{2}$$

The objective of control is to shift the ball from one given point to another by the use of control signals u_1, u_2 with the restricion of constraints.

2.1 Feedback Linearization

For the output $y = x_1$ system has releative degre $l = 3$ and the dimmension of state space model is 4. Feedback linearization can be performed for square MIMO models, with the same number, n, of inputs and outputs. In considered model there are two control inputs and only one output. Additional output has to be chosen; for the variable x_4 as output relative degree is $l_2 = 1$. In this way sum of relative degrees equals the model dimmension, therefore this variable is chosen as the second output $y_2 = x_4$.

Basic method for obtaining feedback linearization is by differentiating the output (or outputs) l times to obtain canonical form. For nonlinear MIMO models FBL results in obtaining n decoupled linear models. After using the FBL procedure for the levitation system (1) with defined above outputs, new variables $\mathbf{z}$ of decoupled linear models are the functions of nonlinear model variables $\mathbf{x}$. That invertable functions are called diffeomorphism:

$$\begin{aligned}
z_1 &= x_1\\
z_2 &= x_2\\
z_3 &= -\frac{F_{emP1}}{mF_{emP2}}x_3^2 e^{-\frac{x_1}{F_{emP2}}} + g + \frac{F_{emP1}}{mF_{emP2}}x_4^2 e^{-\frac{x_d - x_1}{F_{emP2}}}\\
z_4 &= x_4
\end{aligned} \tag{3}$$

The control variables are

$$\begin{aligned}
u_1 = &\left(\frac{x_2 x_3}{F_{emP2}} - \frac{mF_{emP2}}{F_{emP1}}\frac{v_1}{x_3}e^{\frac{x_1}{F_{emP2}}}\right)\frac{f_{iP1}}{2k_i f_{iP2}}e^{-\frac{x_1}{f_{iP2}}} + \frac{x_3 - c_i}{k_i} +\\
&\left(v_2 + \frac{x_2 x_4}{2F_{emP2}}\right)\frac{f_{iP1}}{f_{iP2}}\frac{x_4}{k_i x_3}e^{\frac{2x_1 - x_d}{F_{emP2}}}e^{-\frac{x_1}{f_{iP2}}}\\
u_2 = &\frac{x_4 - c_i}{k_i} + \frac{f_{iP1}}{f_{iP2}}\frac{v_2}{k_i}e^{\frac{x_1 - x_d}{f_2}}
\end{aligned} \tag{4}$$

In this way we obtain two linear systems in canonical form (5, 6) with new state variables z_i and new control inputs v_1, v_2. The output variables are unchanged. Dependence of output on input is linear for both systems.

$$\begin{aligned}
\dot{z}_1 &= z_2\\
\dot{z}_2 &= z_3\\
\dot{z}_3 &= v_1\\
y &= z_1
\end{aligned} \tag{5}$$

$$\begin{aligned}\dot{z}_4 &= v_2 \\ y_2 &= z_4\end{aligned} \tag{6}$$

The condition for proper feedback linearization of MIMO systems is that the decoupling matrix $\mathbf{E}(\mathbf{x})$ has to be invertible (in order to obtain functions for inputs u_1, u_2 with nonzero values in denominators)

$$\mathbf{E}(\mathbf{x}) = \begin{bmatrix} L_{g_1}L_{\mathbf{f}}^2 h_1 & L_{g_2}L_{\mathbf{f}}^2 h_1 \\ L_{g_1}h_2 & L_{g_2}h_2 \end{bmatrix} \tag{7}$$

where elements of the matrix are defined by Lie derivatives. Here:

$$det(\mathbf{E}(\mathbf{x})) = -2\frac{F_{emP1}}{mF_{emP2}} k_i^2 \frac{f_{iP2}^2}{f_{iP1}^2} x_3 e^{\frac{x_d}{f_{iP2}} - \frac{x_1}{F_{emP2}}} \tag{8}$$

which is always different from 0, because always $x_3 > 0$ (in order to compensate gravity force).

3 CONTROL ALGORITHM

The control algorithm consists of linear quadratic method and predictive control in order to include constraints. Predictive control is used on discrete model therefore every linear model (5, 6) is discretizing with step T_s. To enable tracking the referense signal $\mathbf{w}$ every linear system is augmented by additional variable z^d,

$$z_{k+1}^d = z_k^d + w_k - y_k, \tag{9}$$

Then the linear model can be described by:

$$\begin{gathered}\begin{bmatrix} \mathbf{z} \\ z^d \end{bmatrix}_{k+1} = \begin{bmatrix} \mathbf{A} & 0 \\ -\mathbf{C} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{z} \\ z^d \end{bmatrix}_k + \begin{bmatrix} \mathbf{B} \\ 0 \end{bmatrix} \mathbf{v}_k + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w_k, \\ y_k = \begin{bmatrix} \mathbf{C} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{z} \\ z^d \end{bmatrix}_k .\end{gathered} \tag{10}$$

and the control signal

$$\hat{v}_k^* = -\mathbf{L} \begin{bmatrix} \mathbf{z} \\ z^d \end{bmatrix}_k + \mathbf{L} \begin{bmatrix} \mathbf{C} & 0 \end{bmatrix}^T w_k. \tag{11}$$

On the other hand one can design such control that provides feasible solution, fulfilling constraints. The interpolation method rely on finding compromise for those two solutions.

3.1 Interpolation Method

Interpolation method relies on changing reference signal:

$$\tilde{w}_{k+i} = w_{k+i} + \alpha_k p_{k+i|k}, \tag{12}$$

Where $p_{k+i|k}$ is the $k+1$ element of vector $\mathbf{p}_{H|k}$ so chosen that it guarantees fulfilling constraints if $\alpha = 1$. Then the prediction model is described by

$$\hat{\mathbf{z}}_{k+i+1|k}^w = \Phi \hat{\mathbf{z}}_{k+i|k}^w + \Gamma(w_{k+i} + \alpha_k p_{k+i|k}), \tag{13}$$

where

$$\Phi = \begin{bmatrix} \mathbf{A} - \mathbf{B}\mathbf{L}_\mathbf{z} & -\mathbf{B}L_d \\ -\mathbf{C} & 1 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} \mathbf{B}\mathbf{L}_\mathbf{z}\mathbf{C}^T \\ 1 \end{bmatrix}, \tag{14}$$

where $\mathbf{L}_\mathbf{z}$ and L_d are the first part (for the $\mathbf{z}$ variable) and the second part (for additional variable z^d) of vector $\mathbf{L}$ obtained from lq control. Control signal equals:

$$v_{k+i} = -\mathbf{L}\mathbf{z}_{k+i}^w + \mathbf{L}\,[\mathbf{C}\,0]^T\, w_{k+i} + \alpha_k \mathbf{L}\,[\mathbf{C}\,0]^T\, p_{k+i}. \tag{15}$$

Then the control designing in every step relies on minimization of α_k with respect of above equations and constraints on horizon. In every step k vector $\mathbf{p}_{H|k}$ can be improved (shifted nearer optimal lq control) by substitution $\mathbf{p}_{H|k+1} = \alpha_k \mathbf{p}_{H|k}$, α_k guarantees that solution is feasible.

The initial vector $\mathbf{p}_H$ for nonlinear systems can be obtained by minimization

$$J_k = \sum_{i=0}^{\infty} \left[(\mathbf{z}_{k+1+i|k}^w)^T \mathbf{Q}_K \mathbf{z}_{k+1+i|k}^w + \mathbf{v}_{k+i|k}^T R_K \mathbf{v}_{k+i|k} \right] \tag{16}$$

that is

$$J_k = \sum_{i=0}^{\infty} \left[(\mathbf{z}_{k+1+i|k}^w)^T \mathbf{Q}_j \mathbf{z}_{k+1+i|k}^w + \mathbf{p}_{k+i|k}^T R_j \mathbf{p}_{k+i|k} + 2\mathbf{z}_{k+1+i|k}^T \mathbf{N}_j \mathbf{p}_{k+i|k} \right] \tag{17}$$

where

$$\begin{aligned}\mathbf{Q}_j &= [\mathbf{Q}_K + \mathbf{L}^T R_K \mathbf{L}] \quad R_j = \mathbf{L}[\mathbf{C}\,0] R_K \mathbf{L}[\mathbf{C}\,0] \\ \mathbf{N}_j &= -\mathbf{L}^T R \mathbf{L}[\mathbf{C}\,0]\end{aligned} \tag{18}$$

after taking control:

$$v_k = -\mathbf{L}\mathbf{z}_k + \mathbf{L}[\mathbf{C}\,0]\mathbf{p}_k, \tag{19}$$

for the system

$$z_{k+1}^w = \Phi \mathbf{z}_k^w + \Gamma p_k \tag{20}$$

then

$$p_k = -\mathbf{K}\mathbf{z}_k^w \tag{21}$$

where $\mathbf{L}$ is the optimal gain vector for lq control without constraints.

3.2 Dependence on α

Problem of minimization α on horizon with respect to constraints can be difficult, because of the nonlinear functions obtained by feedback linearization, (3, 4). We can use simple numerical procedure but the dependence of constained values on α has to be monotonical. As the α is increased, the absolute of constrained variables should be decreased (shold be shifted away from constraints). This is not straitforward for every systems and calculations can be difficult as the (3, 4) functions are continouos but the predictive control is working on discrete model. The behaviour of constrained values can be however observed through simulations. For the levitation systems variables in function of time and α are presented in figures 2 - 5.

As it can be seen every variable goes away from constraints as α is increasing therefore the interpolation method can be used.

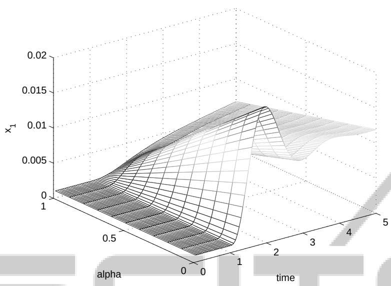

Figure 2: ball position x_1.

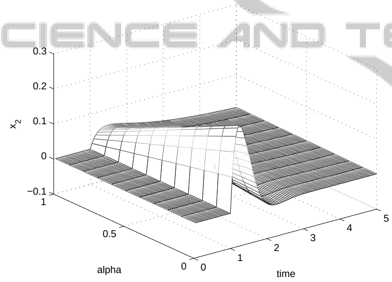

Figure 3: ball velosity x_2.

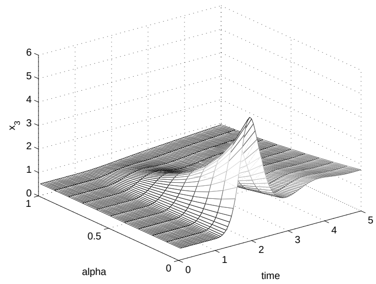

Figure 4: x_3.

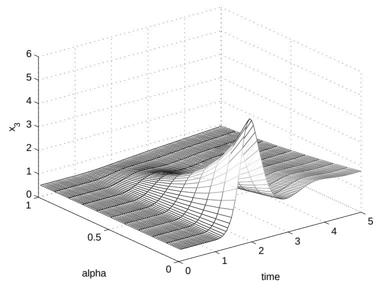

Figure 5: x_3.

4 PROBLEM OF WEIGHTS IN LINEAR QUADRATIC COST

As the objective of the control is to track the reference trajectory and the constraints are fulfilled by the interpolation method it is possible to choose the weights in LQ cost function and apply the algorithm. The simulations presented in figures 6-8 are obtained with weigth in quadratic cost function so chosen that for every state variabe z_i (values on diagolnal of $\mathbf{q}$) and control v_i (the value $\mathbf{R}$) equals 0.1 for both linear systems (5, 6). The model is augmented, so for the additional state variable which realize tracking the weight equals 100. Unortunately obtained charts presents very slow motion of the controlled ball.

Problem of choosing weights for the **z** state and **v** control is more difficult than for model of known signals of physical meaning. Finding weights for variables **x** and **u** is easier as that signals usually repre-

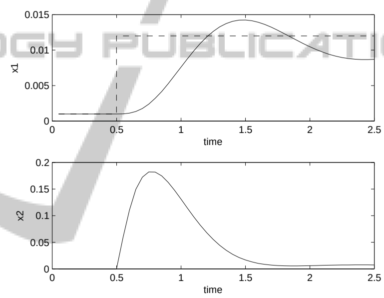

Figure 6: ball position x_1 and velocity x_2.

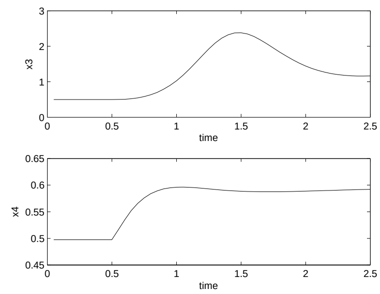

Figure 7: x_3, x_4.

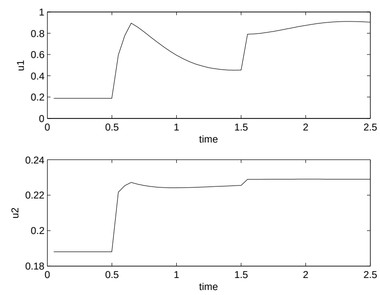

Figure 8: control signals u_1, u_2.

sent known physical variables. When one wants to use that knowledge for the linearizad model one meets the problem of the nonlinear, multivariable functions obtained from FBL (3, 4).

The way that can be used here is to confront the cost function designed for the original nonlinear system (for example for the purpose of linear quadratic conttrol for system linearized in the point of work by Jacobian linearization) with the function we need to use for new linear system. In the example of levitation system the first cost function has form:

$$J = \sum_{k=1}^{\infty} \tilde{\mathbf{x}}_k^T \mathbf{Q}_x \tilde{\mathbf{x}}_k + \tilde{\mathbf{u}}_k^T \mathbf{R}_x \tilde{\mathbf{u}}_k \tag{22}$$

where

$$\begin{array}{l} \tilde{\mathbf{x}}(t) = \mathbf{x}(t) - \mathbf{x}_s \\ \tilde{\mathbf{u}}(t) = \mathbf{u}(t) - \mathbf{u}_s \end{array} \tag{23}$$

and $\mathbf{x}_s, \mathbf{u}_s$ describe steady point for desired value of y (the further value in vector $\mathbf{w}$). The system should be shifted (by using variables (23)) that $\tilde{\mathbf{x}}_s = 0, \tilde{\mathbf{u}}_s = 0$, but after feedback linearization the shift weights sholud be considered in the weights values. If the $\mathbf{Q}_x$ and $\mathbf{R}_x$ are diagonal matrices then the cost function for given k step is

$$F_x = q_1 x_1^2 + q_2 x_2^2 + q_3 x_3^2 + q_4 x_4^2 + r_1^2 u_1^2 + r_2^2 u_2^2 \tag{24}$$

The function should be approximated with similar function for linear system. In levitation example there are two linear systems, therefore the two cost functons can take form

$$F_{z1} = [z_1 z_2 z_3] \mathbf{Q}_{z1} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + v_1^2 R_{z1} + 2[z_1 z_2 z_3] \mathbf{N}_{z1} v_1 \tag{25}$$

$$F_{z2} = z_4^2 \mathbf{Q}_{z2} + v_2^2 R_{z2} + 2 z_4 \mathbf{N}_{22} v_2 \tag{26}$$

The cost functions for linear system should be close to the function for nonlinear system, that is for the levitation example

$$F_x \approx F_{z1} + F_{z2}. \tag{27}$$

The method that can be used is the linear approximation in the point (y_s, x_s, u_s) of two functions - diffeomorphism $\mathbf{x}(\mathbf{z})$ and control $\mathbf{u}(\mathbf{z}, v)$ (obtained from (3, 4)) for every variable x_i, v_j. For the levitation system, where

$$\begin{array}{c} x_1(z_1, z_2, z_3, z_4) \\ x_2(z_1, z_2, z_3, z_4) \\ x_3(z_1, z_2, z_3, z_4) \\ x_4(z_1, z_2, z_3, z_4) \\ u_1(z_1, z_2, z_3, z_4, v_1, v_2) \\ u_2(z_1, z_2, z_3, z_4, v_1, v_2) \end{array} \tag{28}$$

we can obtain linear approximation

$$\begin{array}{r} x_i \approx \frac{dx_i}{dz_1}|_s z_1 + \frac{dx_i}{dz_2}|_s z_2 + \frac{dx_i}{dz_3}|_s z_3 + \frac{dx_i}{dz_4}|_s z_4 \\ u_j \approx \frac{du_j}{dz_1}|_s z_1 + \ldots + \frac{du_j}{dz_4}|_s z_4 + \frac{du_j}{dv_1}|_s v_1 + \frac{du_j}{dv_2}|_s v_2 \end{array} \tag{29}$$

where index s indicates, that in the obtained functions variables from vectors $\mathbf{z}$ and $\mathbf{v}$ should be replaced by values in desired point s $(\mathbf{z}_s, \mathbf{v}_s)$. Variables x_i and u_j obtained in this way can be substitute into function F_x in order to obtain cost function with variables $\mathbf{z}$ and $\mathbf{v}$. It can happen (that was for the levitation system) that for MIMO systems we can obtain fragments of function that will not appear in the cost functions for linear systems (F_{z1}, F_{z2}) (for example function of (z_1, v_2)) and that fragments have to be simply omitted. The weights for cost functions for LQ method for linear systems are available from F_z functions.

The procedure was proceeded for the levitation example, where values of weights for nonlinear variable were following: $q_1 = 100, q_2 = 0.1, q_3 = 0.1, q_4 = 1, r_1 = 0.1, r_2 = 0.1$. Simulation has been prepared for the step of discretization $T_s = 0.05s$ and the horizon $h = 20$. Obtained results are presented on figures 9-11

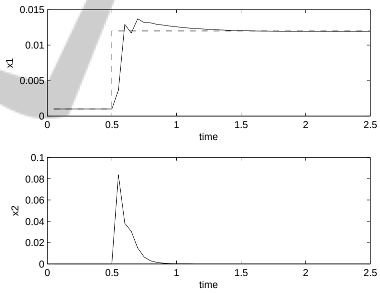

Figure 9: ball position x_1 and velocity x_2.

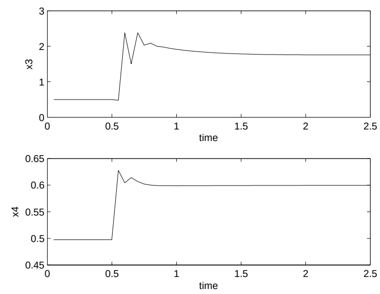

Figure 10: x_3, x_4.

5 CONCLUSIONS

Obtained results show that the method is valid for the system, constraints are fulfilled (it is visible on charts of x_3 and u_1 which are close to constraints; when sim-

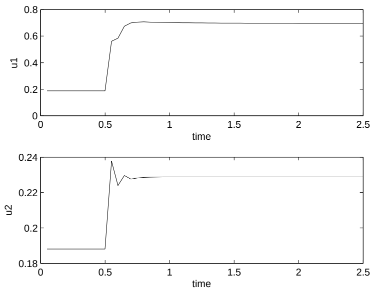

Figure 11: control signals u_1, u_2.

ulations with too short horizon were implemented, the constraints of the varaiables were violated). The minimized variable α, obtained in every step, was used the same for both linear models (5, 6). Simulations show how important is to choose proper weights values. For the first choice, when only diagonal values for input and state was plased, performance of output was very slow. Weights obtained by aproximation and comparison to function with signals of physical meaning improved the response.

As the weights are obtained through approximation, for hardly nonlinear models it can be convinient to change that values for situations, when system changes the working point. Nonetheless the algorithm is based on exact linearization, therefore if proportions of signals are similar to that of signals in approximation point the control law should be sufficient for once calculated weights.

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