Relay Based PID Auto-tuning Applied to a Multivariable Level Control System

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Abstract: In industrial applications involving control systems, PID controllers are present in the great majority of them, mostly because of a very simple architecture and easy tuning. For tuning them, the relay method is also very simple to use and, usually, reach some very satisfactory results, once combined with the appropriate strategy, like Ziegler-Nichols, Cohen-Coon, CHR, and others. Using this methodology, this paper presents a relay based PID auto-tuning applied to a multivariable coupled tanks system.

1 INTRODUCTION

Science has traditionally been concerned with describing nature using mathematical symbols and equations. More recently, engineers have introduced (additional) control variables and adjustable parameters to the mathematical models. Control engineers want to monitor and control engineering systems with controllers, which process information from both desired responses and sensor signals and affect the behaviour of the system. The field of control engineers covers the study of dynamical systems and optimization. If the system is not performing to expectations, they want to detect this under-performance from sensors and generate performance enhancing feedback signals to the actuators (Tay et. al., 1998).

In the world of control systems, the proportional-integral-derivative (PID) controller has several important functions: it provides feedback, it has the ability to eliminate steady state error through integral action and it can anticipate the future through derivative action. PID controllers are sufficient for many control problems, particularly when process dynamics are benign and the performance requirements are modest. In process control, more than 95% of the control loops are of PID types, most loops are actually PI control. (Aström and Hägglund, 1995). In industrial applications, PID control is a very popular control strategy due to its simple architecture and easy tuning. Despite their widespread use and considerable history, PID tuning is still an active area of research, both academic and industrial. (Cong and Liang, 2009).

Aiming for the performance enhancement, some methods for automatic tuning can be used. By automatic tuning (or auto-tuning), we mean a method where the controller is tuned automatically on demand from a user. Typically, the user will either push a button or send a command to the controller. An automatic tuning procedure consists of three steps: generation of a process disturbance, evaluation of the disturbance response and calculation of controller parameters. This is the same procedure that an experienced operator uses when tuning a controller manually. The process must be disturbed in some way in order to determine the process dynamics. This can be done in many ways, e.g., by adding steps, pulses, or sinusoidal signals to the process input. The evaluation of the disturbance response may include a determination of a process model or a simple characterization of the response. (Aström and Hägglund, 1995).

Having well-tuned controllers, with auto-tuning strategies and tools to track their performance over time and the ability to retune them, become an item almost mandatory to maintain processes with high productivity and low cost, not to mention the quality of the final product. Researches in the industrial controllers' market show that the tuning and/or auto-tuning function as the most valued by users, alongside its own PID algorithm and the

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communication protocols (VanDore, 2006).

Among other, the method of step with the relay feedback (or simply, the relay method) was one of the first auto-tuning methods to be marketed and have remained attractive due to its simplicity and robustness. In addition, many researches have been conducted to enhance its capability and efficiency. Furthermore, the PID tuning formulas have been refined in order to improve the controller performance for various processes, such as those with transport delay and oscillations.

Within this scenario, this paper will show an auto-tuning software used in a multivariable system. These systems are widely used in the process industry and academy and many recent papers deal with them. Saeed et. al. (2010) use a predictive PID control for a quadruple tank; Tzouanas and Stevenson (2013) manage the temperature and level control of a multivariable water tank process; Ahmed et. al. (2010) bring the discussion of a Fuzzy model-based predictive control applied to multivariable level control and De Keyser et. al. (2013) validate a multivariable relay-based PID autotuner also using a quadruple tank.

This paper will describe the system used for the experiments; explain the auto-tuning method build and evaluate the controller performance before and after its tuning in order to compare the method efficacy.

2 COUPLED TANK SYSTEM

For the development of this work, it was used a coupled tank system simulator, based on a real (experimental) two-tank system from Quanser (Figure 1).

Figure 1: Real Coupled Water Tank System from *Quanser*.

The two-tank system consists of a pump with a water basin and two tanks of uniform cross sections. Such an apparatus forms an autonomous closed and recirculating system. The two tanks, mounted on the front plate, are configured such that the flow from the first (upper) tank can flow into the second (lower) tank. Flow from the second tank flows into the main water reservoir. In each one of the two tanks, liquid is withdrawn from the bottom through an outflow orifice (i.e. outlet). The outlet pressure is atmospheric. The water level in each tank is measured using a pressure-sensitive sensor located at the bottom of the tank. Additionally, a vertical scale (in centimeters) is also placed beside each tank for visual feedback regarding each tank's water level.

For this experiment, however, it was intended to use a more complex system. For that, there was the availability of a simulator with some different features compared to the Quanser system. It is, for instance, a five-tank and five-pump system (Figure 2) in which each tank receive liquid from both the pump and the upper tank (in exception for the first tank that has only the influence of its pump).

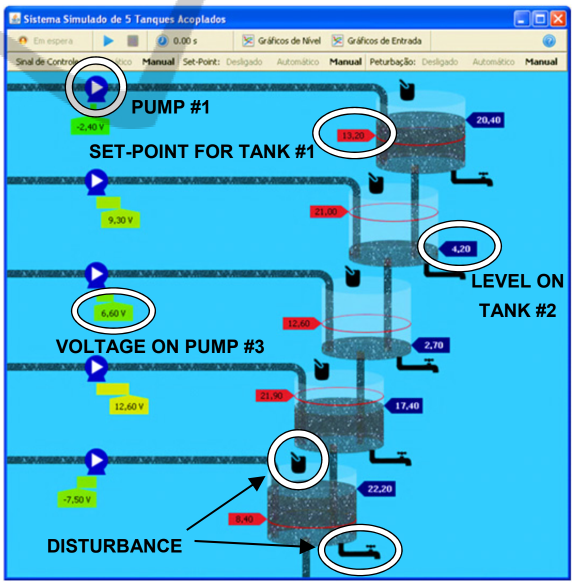

Figure 2: Five-tank system simulator.

The simulator also provides ways for monitoring and manipulating an experiment, like changing the set-points, adding disturbances, showing the system's process variables (the tanks' levels, in cetimeters) and manipulated variables (pumps' voltages), as well as others real systems characteristics, like noise and transport delay.

With this set of features, the simulator is a multivariable coupled tank system that can be used for various applications. For this paper objective it will be used for testing and validating tuning methods in multivariable systems, since it's dynamic is based on a real system.

3 PID CONTROLLERS TUNING STRATEGIES

The main objective of tuning control loop is to identify the process resulting dynamic to some control efforts and, based on performance requirements, define the necessary PID algorithm dynamics in order to eliminate errors. Those algorithm dynamics can be defined in many different forms, depending of the tuning strategy adopted. Some of these strategies will be shown in the next sections.

3.1 Ziegler and Nichols

Ziegler and Nichols (1942) developed two empirical tunings strategies: one based on the system's step response in open loop and another based on the system's critical gain (K_u) and critical period (T_u) when subjected to a sustained oscillation (such as the relay experiment) resulting at the equations in Table 1 for tuning a PID controller.

Table 1: Ziegler and Nichols tuning strategy.

Controller	K_p	T_i	T_d
P	$0.5K_u$	-	-
PI	$0.45K_u$	$T_u/1.2$	-
PID	$0.6K_u$	$T_u/2$	$T_u/8$

Later, Campos and Teixeira (2006) suggest using some slack factors or "detuning" to the Ziegler-Nichols PID tuning strategy due to the uncertainties of the order of 5% to 20% of the estimated process dynamics. The usage of these factors result at (1) and (2).

$$K_p = \frac{K_{pzn}}{1.25} \quad (1)$$

$$T_i = 2.5\, T_{izn} \quad (2)$$

3.2 CHR

Developed at the *Massachusetts Institute of Technology*, by K. L. Chien, J. A. Hrones and J. B. Reswick, it was the first tuning strategy to use an approximate first order model with dead time representing the behaviour of higher order systems (Chien et al., 1952). This work was also the pioneer in the determination of rules for differentiated fit for servo and regulatory characteristics.

As in Ziegler and Nichols, this strategy also results in a set of equations (Table 2) to define the controller parameters, based on the first-order model's gain K, dead-time θ and time constant τ.

Table 2: Tuning by CHR strategy.

Controller	K_p	T_i	T_d
P	$\frac{0.3\tau}{K\theta}$	-	-
PI	$\frac{0.6\tau}{K\theta}$	4θ	-
PID	$\frac{0.95\tau}{K\theta}$	2.375θ	0.421θ

3.3 Cohen and Coon

The desired result of the Cohen and Coon (1953) strategy was to tune higher dead time processes, i.e., with uncontrollable factor (θ / τ) greater than 0.3. The tuning equations are shown in Table 3.

Table 3: Cohen and Coon strategy.

Controller	K_p	T_i	T_d
P	$\frac{\left(1.03 + 0.35\left(\frac{\tau}{\theta}\right)\right)\tau}{K\theta}$	-	-
PI	$\frac{\left(1.9 + 0.083\left(\frac{\tau}{\theta}\right)\right)\tau}{K\theta}$	α	-
PID	$\frac{\left(1.35 + 0.25\left(\frac{\tau}{\theta}\right)\right)\tau}{K\theta}$	β	γ

Where:

$$\alpha = \frac{\left(0.9 + 0.083\left(\frac{\tau}{\theta}\right)\right)\theta}{1.27 + 0.6\left(\frac{\tau}{\theta}\right)}, \beta = \frac{\left(1.35 + 0.25\left(\frac{\tau}{\theta}\right)\right)\theta}{0.54 + 0.6\left(\frac{\tau}{\theta}\right)},$$

$$\gamma = \frac{0.5\theta}{1.27 + 0.6\left(\frac{\tau}{\theta}\right)}$$

3.4 IAE, ITAE

A research group from *Louisiana State University* (Lopez et al., 1967) developed, in the 60's, a methodology for minimizing performance criteria based on IAE (Integral Absolute Error) and ITAE (Integral Time Absolute Error). From solving a problem of multi-objective optimization, they obtained a set of rules for adjusting the parameters of the PID controller for different characteristics of a first order model with dead time, as in Table 4.

Table 4: Tuning strategy based on IAE and ITAE.

Controller	K_p	T_i	T_d
PI - IAE	$\frac{0.984}{K}\left(\frac{\tau}{\theta}\right)^{0.986}$	α	-
PI - ITAE	$\frac{0.859}{K}\left(\frac{\tau}{\theta}\right)^{0.977}$	β	-
PID - IAE	$\frac{1.435}{K}\left(\frac{\tau}{\theta}\right)^{0.921}$	γ	ε
PID - ITAE	$\frac{1.357}{K}\left(\frac{\tau}{\theta}\right)^{0.947}$	δ	ϵ

Where:

$$\alpha = \frac{\tau}{0.608\left(\frac{\tau}{\theta}\right)^{0.707}}, \beta = \frac{\tau}{0.647\left(\frac{\tau}{\theta}\right)^{0.680}},$$

$$\gamma = \frac{\tau}{0.878\left(\frac{\tau}{\theta}\right)^{0.749}}, \delta = \frac{\tau}{0.842\left(\frac{\tau}{\theta}\right)^{0.738}},$$

$$\varepsilon = \tau\left(0.482\left(\frac{\theta}{\tau}\right)^{1.137}\right), \epsilon = \tau\left(0.381\left(\frac{\theta}{\tau}\right)^{0.995}\right)$$

3.5 Internal Model Control (IMC)

The adjustment rules for the IMC strategy are recommended for the controllability factor (θ / τ) > 0.125 (Rivera et al., 1986). They considered different process dynamics and obtained PID controllers for each one depending on the performance parameter λ. When the process dynamics can be described by a first-order model with transport delay, the proposed tuning strategy is shown in Table 5.

Table 5: IMC strategy.

Controller	K_p	T_i	T_d
PI	$\frac{2\tau+\theta}{2K\lambda}$	$\tau + \frac{\theta}{2}$	-
PID	$\frac{2\tau+\theta}{K(2\lambda+\theta)}$	$\tau + \frac{\theta}{2}$	$\frac{\tau\theta}{2\tau+\theta}$

4 RELAY METHOD

The limitations of the Ziegler and Nichols tuning method led Astrom and Hagglund to propose the use of a relay in the system to be tuned, creating the method shown on Figure 3 (Astrom and Wittenmark, 1988).

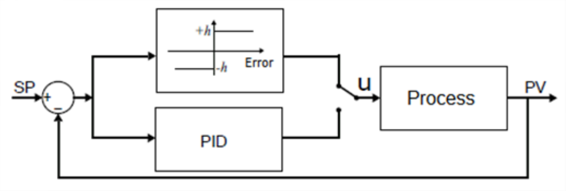

Figure 3: Relay method on closed loop.

The purpose of this method is to cause limited and controlled oscillations in the process and, from its response (Figure 4), estimate the system's frequency response. From the output of amplitude "a" caused by the relay, the critical gain can be estimated, as in (3).

$$K_u = \frac{4h}{\pi a} \tag{3}$$

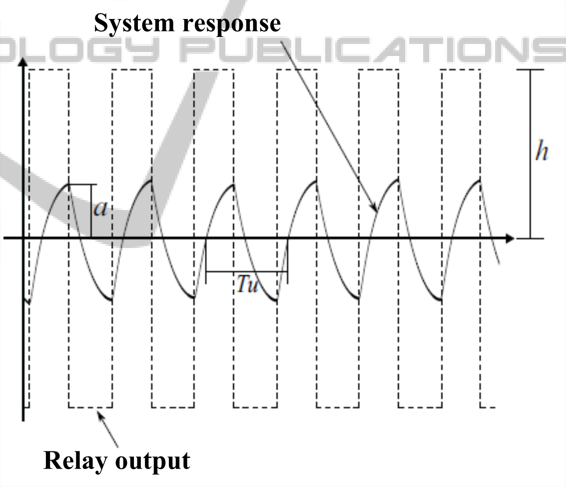

Figure 4: System response from the relay method.

The critical period (Tu) is the oscillation period of the relay itself. With this information on the process dynamics (Ku and Tu), any tuning strategy (like the Ziegler and Nichols, for example) can be used to obtain the values for the PI/PID controller.

Improving the relay to bypass the problem of unwanted switching caused by noise, Aström and

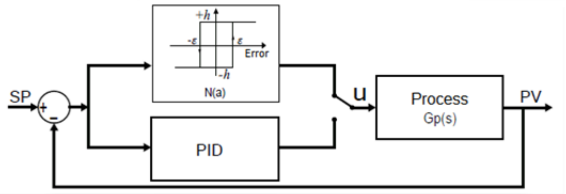

Figure 5: Relay with hysteresis.

Hägglund (1984) proposed the use of hysteresis in the relay (Figure 5).

The relay switch is ruled by the following rule, where ε is the hysteresis width:

- $If\ [error(t) \geq \ \varepsilon], then\ u(t) = h$
- $If\ [error(t) < \ -\varepsilon], then\ u(t) = -h$
- $If\ [-\varepsilon < \ error(t) < \ \varepsilon], then\ u(t) = u(t-1)$

On the developed software, six relay switches were considered enough to make all necessary calculations.

In practical applications, the hysteresis should be selected based on the noise, for example, two times greater than its amplitude (Hang et al., 2002; Coelho and dos Santos Coelho, 2004; Campos and Teixeira, 2006) for the establishment of the limit cycle.

The critical frequency (ω_u) is given by (4).

$$\omega_u = \frac{2\pi}{T_u} \tag{4}$$

The descriptive function of relay with hysteresis, designated by $N(a)$ is given by (5). This form comes from approaching the fundamental component of the Fourier series.

$$N(a) = \frac{4h}{\pi a} \angle \phi \tag{5}$$

where $\phi = -\sin^{-1}(\varepsilon/a)$.

The system will show a continuous limit cycle (marginally stable) when the following condition is satisfied:

$$1 + N(a)G_p(j\omega_u) = 0 \tag{6}$$

$$G_p(j\omega_u) = -\frac{1}{N(a)}$$

The intersection of the Nyquist plot of $G_p(j\omega_u)$ and $-(1/N(a))$ in the complex plane for relay with hysteresis (Figure 6) results in the process critical point. The critical gain K_u, at the critical frequency ω_u, is given by (7).

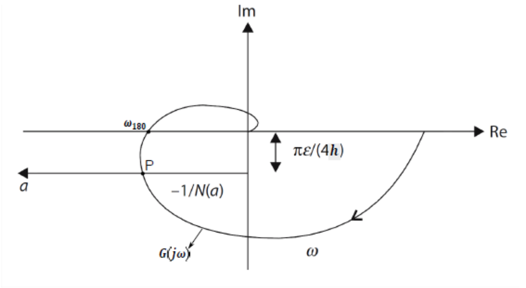

Figure 6: Nyquist plot.

$$K_u = \frac{1}{|G_p(j\omega_u)|} = \frac{4h}{\pi a} \tag{7}$$

If necessary, it's also possible, after identifying the critical point, to define the parameters for a first order system with transport delay using (8) to calculate the time constant τ and (9) to calculate the transport delay θ (Cheng, 2006),

$$\tau = \frac{\sqrt{(K_u K)^2 - 1}}{\omega_u} \tag{8}$$

$$\theta = \frac{\pi - tan^{-1}(\tau\omega_u) + \phi}{\omega_u} \tag{9}$$

In (8), it is assumed that the process static gain (K) is known or can be obtained by means of the step response test ($K = \Delta y/\Delta u$). However, even the relay test data can be used for this purpose, as in (10) (Hang et al., 2002).

$$K = \frac{\int_0^{T_u/2} y(t)\, dt}{\int_0^{T_u/2} u(t)\, dt} \tag{10}$$

With only the values of K_u and T_u obtained from the relay test, the software can already tune the controller using the Ziegler-Nichols strategy. However, with the other parameters (τ, θ and K) it can also use other tables tunings strategies shown on the section 3 of this paper. Therefore, developed software is able (with a single relay method experiment) to determine all these parameters and generate different tuning parameters so that the user can compare them in order to choose the most appropriate one.

5 RESULTS

The tuning software was developed in Java and its communication with the system variables was via OPC (OLE for Process Control), which is a widely used protocol in industry.

Since the system contain five tanks and, therefore, five control loops, the tuning procedure need to follow some predefined rules (Campos, 2001):

- First, tune the top tank control loop (with all other in manual operation), since its dynamic is not affected by the others;
- Set the top control loop in automatic operation (with the calculated tune) and start tuning the second (from top to bottom) control loop, whose dynamic is affected only by the already tuned top tank. The loops below should

remain in manual operation;
- Repeat the last rule for the third, fourth and fifth loop (in this order) always putting the recently tuned loops in automatic operation;
- By the end of the fifth loop, the procedure is complete.

For each loop tuning process, different tune sets are proposed by the software, according to the different strategies used. The user only need to choose the one that fits the best for each case.

5.1 First Tank Control Loop

Before using the tuning software, it was used a controller whose parameters were defined by empiric values, resulting in a poor system response (Figure 7).

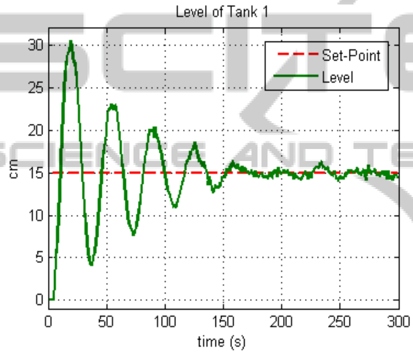

Figure 7: Top control loop before auto-tuning.

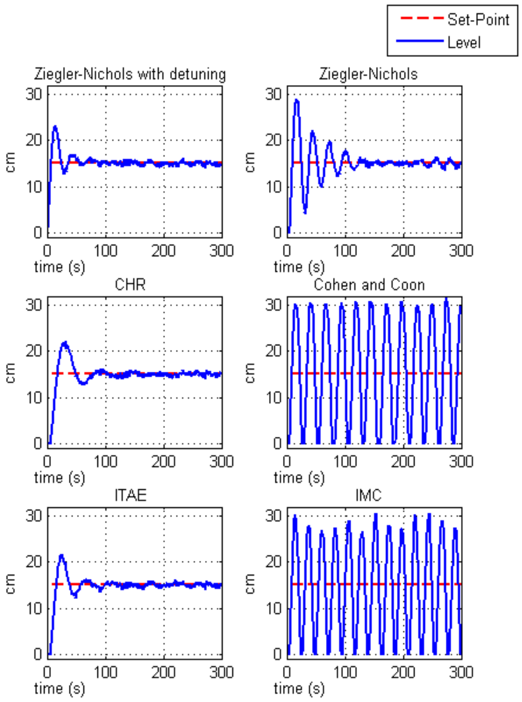

Figure 8: Top control loop after auto-tuning.

The tuning software used, then, the relay test to “study” the system behavior (on a desired operation point of 15cm, tank’s limits average) and propose some tuning sets showing several results (Figure 8) that the user can evaluate. The chosen one for this loop was the Ziegler-Nichols with ‘detuning’ factors, which resulted in a much better system response regarding a performance based an minor overshoot and faster system response. (Figure 8).

5.2 Other Tanks Control Loops

Then, the same procedure was executed for the second control loop, for which the software showed some tuning sets and their results when applied to the system. The chosen set for this loop was also the one made by the Ziegler-Nichols with ‘detuning’ factors strategy (Figure 9).

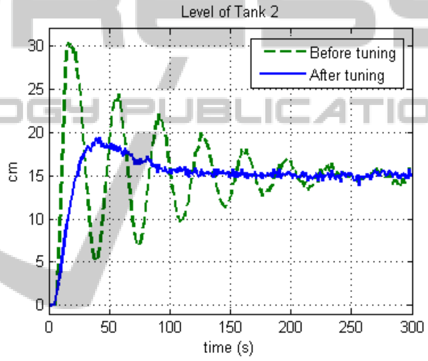

Figure 9: Second control loop before and after tuning.

The third control loop, however, was tuned by the CHR strategy, since it turned out to result in a better system response (Figure 10). The fourth control loop was tuned by the Ziegler-Nichols strategy (Figure 11) and the fifth by the ITAE strategy (Figure 12).

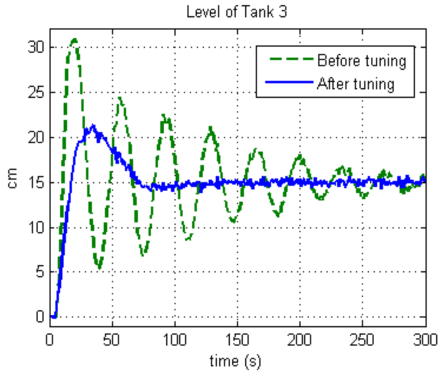

Figure 10: Third control loop tuning.

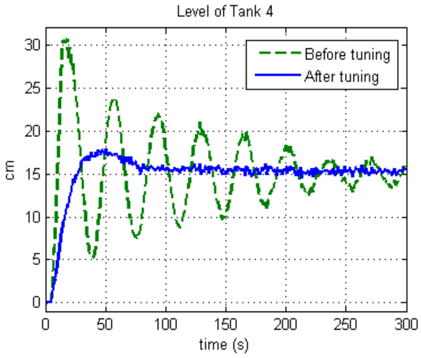

Figure 11: Fourth control loop tuning.

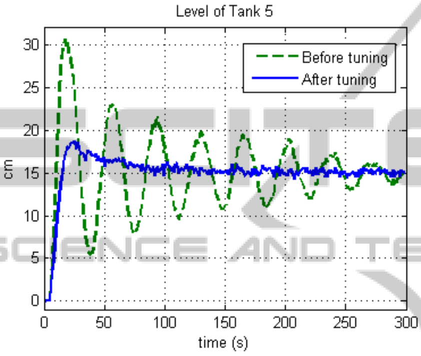

Figure 12: Fifth control loop tuning.

5.3 Other Tests

The tuning results show improvements at the controllers' performances, however, those improvements are shown only at the specific operation point (15 cm) that was used for the tuning procedure. In order to really evaluate their performances, it's recommended to test the system with an experiment that takes it to different points, i.e., a set of different set-points for each control loop (Figure 13).

An also very important experiment to be made is a disturbance test, i.e., an experiment that can show how the system will respond when some disturbance is applied (Figure 14).

6 CONCLUSIONS

Control engineers, for having to deal with hundreds of control loops, have the need for methods that could be easily incorporated in the industry for tuning and/or auto-tuning of PID controllers. Thus, the software developed and presented in this paper for relay based PID tuning fits promisingly, since the results showed that the procedure introduced by Aström and Hägglund (1984), even after two decades of evolution of tuning strategies, is still very satisfactory, with its advantages in simplicity and

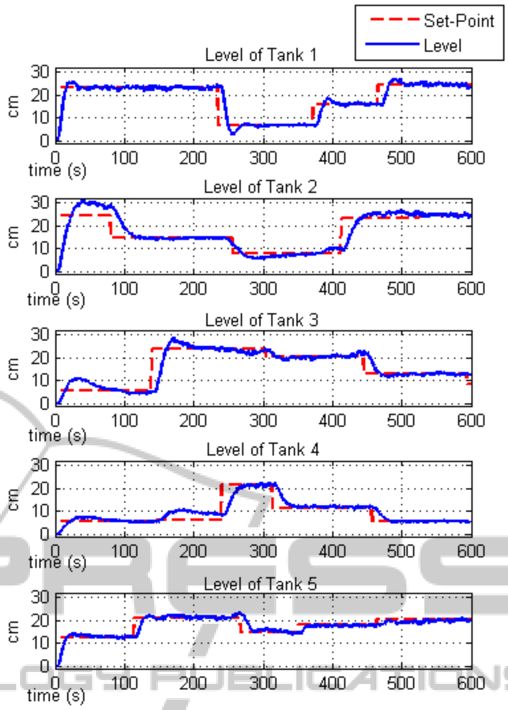

Figure 13: Test for multiple set-points.

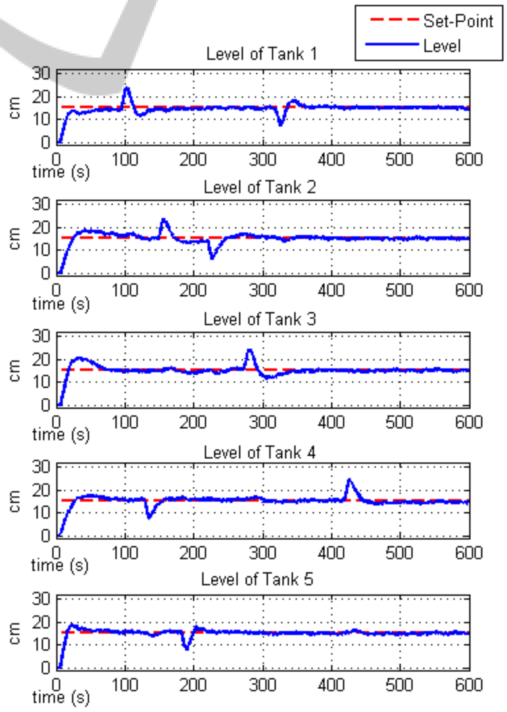

Figure 14: Test for disturbance.

robustness. As future prospects, one can think of an even more automatic tuning method, for example, using some performances measures like Mean Squared Error (MSE), Integral Square Error (ISE) or Integral Time Absolute Error (ITAE). They can help the software to evaluate the several tuning strategies results by itself and make a decision of which one is the best for the system.

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