Why Using the Alpha-stable Distribution in Neuroimage?

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Abstract: The main goal and overall objective of this contribution is to attract the attention of the potentialities and wide range of applications of the α-stable distribution in biomedical applications, specifically in neuroimaging. The α-stable density is a heavy-tailed, non-symmetric distribution with similar desirable properties to the Gaussian. Indeed, the Gaussian distribution is a particular case of the α-stable family. The Gaussian distribution is used ubiquitously in brain image processing. For this reason, we believe that the α-stable density can be potentially used as an alternative to the Gaussian distribution in several biomedical applications regarding brain imaging. Some of the proposed applications of the α-stable distribution considered in this work are the development of brain image processing approaches with applications to intensity normalization of SPECT images, MRI segmentation and feature extraction for the diagnosis of Parkinsonian's syndrome.

1 INTRODUCTION

Non-Gaussian statistical signal processing is important when signals deviate from the ideal Gaussian model. α-stable distributions are amongst the most important non-Gaussian models. They share defining characteristics with the Gaussian distribution, such as the stability property and central limit theorem, and include the Gaussian distribution as a limiting case.

Heavy-tailed modeling using the alpha-stable distribution has been successfully applied in many fields of research. In engineering related methods: radar processing, telecommunications, acoustics, network modeling, queuing theory or ICA/blind source separation. In economics and finance: modeling asset returns, option pricing and commodity price modelling. In computer science, physics, astronomy, chemistry, geology, geophysics and genetics.

The history of research on this particular distribution family is old starting with the work of Paul Lévy in 1925. The applications of alpha-stable distributions have been limited however until much later when (Mandelbrot, 1963) suggested they could be used to model financial time series data. Thereafter, the alpha-stable distribution has been frequently found in analysis of critical behaviour and financial data (Voit, 2003).

The α-stable distribution in engineering gained popularity and attracted the attention of engineers worldwide after the work published by Shao in Proceedings of the IEEE (Shao and Nikias, 1993). Applications of the α-stable distribution in engineering are still a productive field of research nowadays. Nevertheles, the α-stable distribution has been used just in a few cases recently published in neuroimaging applications.

On contrast, the Gaussian distribution is used ubiquitously in neuroimaging. For this reason, we believe alpha-stable density can be potentially used as an alternative to the Gaussian distribution.

2 SOME PROPERTIES OF THE α-STABLE DISTRIBUTION

The characteristic function $\varphi(\omega)$ of an α-stable distribution $f_{\alpha,\beta}(y|\gamma,\delta)$ is given by:

$$\varphi(\omega) = \begin{cases} e^{-|\gamma\omega|^{\alpha}[1-i\,\mathrm{sign}(\omega)\beta\tan(\frac{\pi\alpha}{2})]+i\delta\omega}, (\alpha \neq 1) \\ e^{-|\gamma\omega|[1+i\,\mathrm{sign}(\omega)\frac{2}{\pi}\beta\log(|\omega|)]+i\delta\omega}, (\alpha = 1) \end{cases} \quad (1)$$

where the parameters of the stable distribution are: $\alpha \in (0,2]$ is the characteristic exponent which sets the level of impulsiveness, $\beta \in [-1,+1]$ is the skewness parameter, ($\beta = 0$, for symmetric distributions and $\beta = \pm 1$ for the positive/negative stable family respectively), $\gamma > 0$ is the scale parameter, also called dispersion, and δ is the location parameter.

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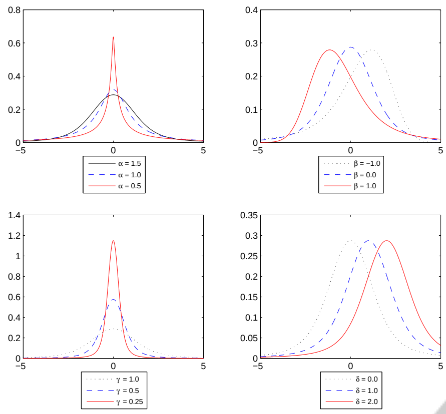

Figure 1: α-stable probability density function with reference parameters $\alpha = 1.5$, $\beta = 0$, $\gamma = 1$ and $\delta = 0$ with changing: (a) Characteristic exponent α. (b) Skewness parameter β. (c) Dispersion γ. (d) Location parameter δ.

Figure 1 shows the α-stable probability density function for different values of the parameters. We use the distribution with parameters $\alpha = 1.5$, $\beta = 0$, $\gamma = 1$ and $\delta = 0$ as reference. This figure also explain the name of the parameters: α controls the degree of impulsiveness. When α decreases, the degree of impulsiveness increases. β controls the skewness and its sign, if the asymmetry is on the left or the right. γ controls the concentration of the samples along the bulk of the distribution. Lower values of γ correspond with higher concentration of the samples. Lastly, different values of δ produce the same probability density function but shifted in the x-axis.

3 α-STABLE DISTRIBUTION IN NEUROIMAGE

3.1 Intensity Normalization of Brain FP-CIT SPECT Images

Previous studies have demonstrated that when [123I]FP-CIT SPECT reaches equilibrium binding in the brain, a simple unitless ratio of regional radioactivities is proportional to the binding potential (Scherfler et al., 2005; Aarts et al., 2012). Nevertheless, this binding ratio leads to intersubject differences in the histogram of intensity values. As the histogram of intensity values is unimodal, skewed and heavy-tailed, one can model this histogram using an alpha-stable distribution in order to perform intensity normalization of the brain images.

The histogram of intensity values in FP-CIT SPECT images shares some of the properties of the α-stable distribution. Some of these common properties are:

- Heavy probability tails due to the existence of a few regions of the brain with high intensity values.
- Peaked bulk, because most of the voxels in the brain, the non-specific area, have very similar intensity values, except the striatum, which is the area with greater variability depending on the type of image (Parkinson's syndrome (PD) or Normal Control patients (NC)).
- Positive asymmetry, because intensity values are always greater than 0 and the bulk of the distribution reaches lower intensity values compared with the values obtained in the striatum.

The predicted α-stable parameters and the location-scale property can be used to transform the intensity values in each voxel linearly. This transformation ensures that the new histograms in each image have a pre-specified α-stable distribution with desired location and dispersion values.

These features have been exploited in (Salas-Gonzalez et al., 2013b). The histogram of a vector of intensity data with α-stable distribution f with parameters $[\alpha, \beta, \gamma, \delta]$, denoted by $X \sim f_{\alpha,\beta}(y|\gamma, \delta)$ can be easily transformed to another α-stable distribution with parameters $Y \sim f_{\alpha,\beta}(y|\gamma^*, \delta^*)$ by using the following expression (Samoradnitsky and Taqqu, 1994):

$$Y = aX + b \tag{2}$$

where $a = \frac{\gamma^*}{\gamma}$ and $b = \delta^* - \frac{\gamma^*}{\gamma}\delta$.

Figure 2 depicts the histogram of intensity values in FP-CIT SPECT brain images showing the specific and non-specific regions and their locations in a transaxial slice.

3.2 Modelization of the Distribution of Brain Matter

The principal goal of a segmentation process is to partition an image into different regions which are homogeneous with respect to one or more features. Segmentation is an important tool in medical image processing and has been useful in many applications. In MRI, segmentation is performed to divide the entire image into sub-regions such as white matter, gray matter and cerebrospinal fluid spaces of the brain (Balafar et al., 2010). The Gaussian mixture model has been widely applied in brain MRI segmentation, nevertheless, distribution of white and grey matter is more similar to the α-stable distribution, As it has been pointed out in (Salas-Gonzalez et al., 2013a).

Figures 3 and 4 show the histogram of intensity values for a magnetic resonance brain image.

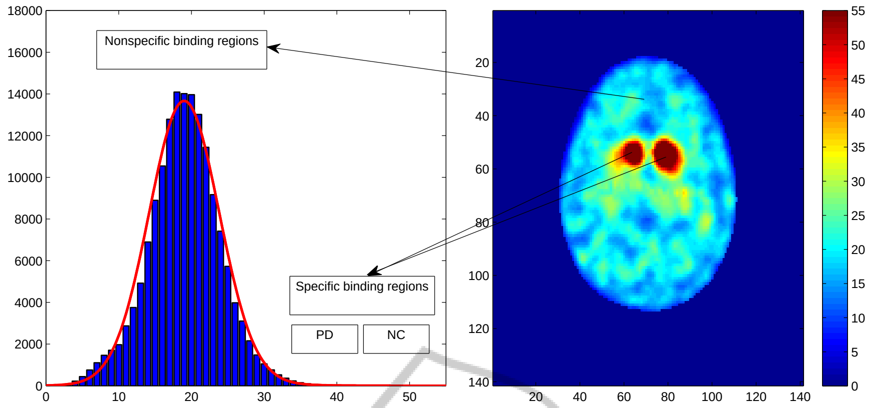

Figure 2: Continuous red line: predicted α-stable density.

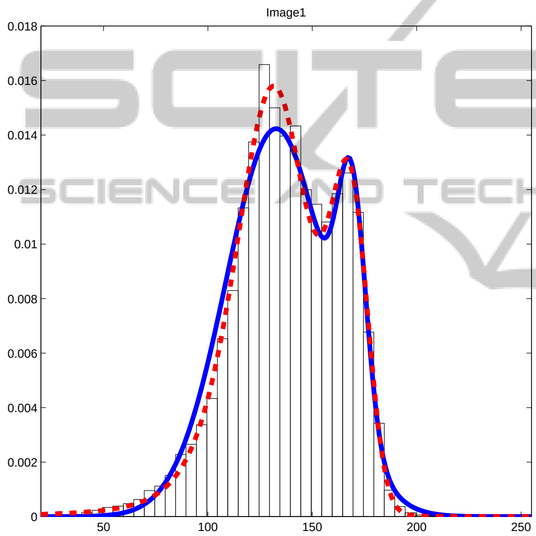

Figure 3: Histogram of intensity values. Red dashed line: predicted alpha-stable mixture density. Blue solid line: Gaussian mixture model with two components.

- The individual histograms of white and grey matter have a shape which is similar to the α-stable distribution.
- Even if in most cases the global histogram of intensity values are unimodal, because their WM and GM components are very mixed, the mixture of α-stable model is robust enough to find the hidden components successfully.
- Many MRI segmentation procedures are based on a preliminary modelling of the histogram of WM and GM, using a Gaussian mixture model, we believe that the α-stable distribution is a good alternative due to its theoretical and practical properties.
- The α-stable mixture model allows us to calculate the relative likelihood for each voxel in the image.

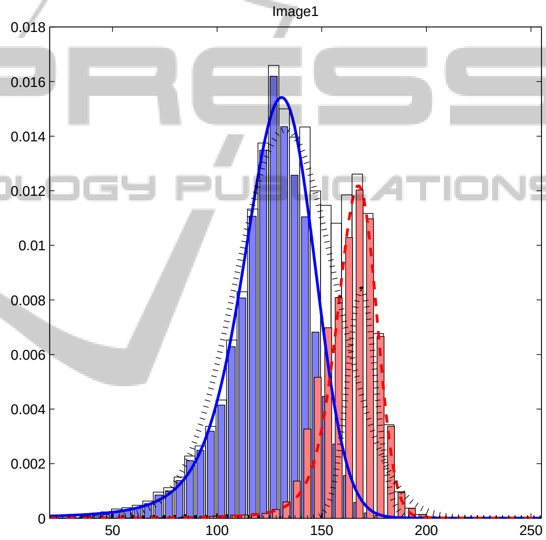

Figure 4: Red and blue lines: predicted alpha-stable components. Dotted black line: Gaussian components. Solid histogram: Ground truth (according to the Internet brain segmentation repository IBSR- data).

Figure 5: Transaxial slices represented using a linear grayscale map. First column: probability to belong to GM (p_{GM}). White voxels, $p_{GM} = 1$. Black voxels, $p_{GM} = 0$. Second column: GM voxels according to the manual segmentation procedure. Third column: p_{WM}. White voxels, $p_{WM} = 1$. Black voxels, $p_{WM} = 0$. Fourth column: WM regions according to manual segmentation procedure.

These quantities are very useful and could be used in more complex Bayesian segmentation models in future works.

3.3 Feature Extraction for Parkinson's Disease Diagnosis

After intravenous injection, 123I-FP-CIT SPECT binds to the dopamine transporters in the striatum. It has been found that patients with Parkinson's disease exhibit a decreased uptake of the tracer (Bhidayasiri, 2006; Hauser and Grosset, 2012). An accurate diagnosis of Parkinson's disease is important because it enables us to monitor disease progression and the therapeutic effects of the treatment. This motivates the development of automated techniques for quantification which neither depend on time consuming operator-intensive work, nor expert skills in manually locating the regions of interest in the brain (Papathanasiou et al., 2006). We believe the α-stable distribution and its properties can be used to develop a computer aided diagnosis system as a decision-making aid for Parkinson's syndrome diagnosis for automatic classification.

The discriminative area to perform diagnosis of Parkinson's syndrome in FP-CIT SPECT brain images is located in a specific region of the brain, the striatum. According to the histogram of the intensity values, this area also controls the degree of impulsiveness in the histogram. The degree of impulsiveness is related to the characteristic exponent, which also models the Paretian behaviour of the tails in alpha-stable distributions. This property can be exploited to assess differences between images belonging to normal controls and patients with Parkinsonian syndrome, measuring the Paretian behaviour in the tail of the distribution to extract the discriminant features. Then, these features could be used for statistical classification using support vector machines or classification trees.

4 NOVELTY, ADVANTAGES AND DISADVANTAGES

The main advantage of the α-stable methods is that they are generalizations of the Gaussian distribution which is widely used in neuroimaging methods, therefore they are expected to perform better than Gaussian, or equally when the Gaussian assumption holds. The main disadvantage of the α-stable distribution is, mainly, the non existence of a closed form for its probability density function, therefore, numerical methods needs to be used to evaluate it.

The originality of the goals and methods envisaged in this paper is demonstrated by the fact that the α-stable distribution has not been previously used in neuroimaging apart from two very recent works published in 2013 (Salas-Gonzalez et al., 2013a; Salas-Gonzalez et al., 2013b). We believe is timely to extend these recently published methods, exploiting additional and useful properties of the α-stable distribution in the study of signal processing methods for brain tomographic applications.

5 CONCLUSION

The Gaussian distribution and mixture of Gaussian model are ubiquitous in brain imaging literature; nevertheless, the Gaussian distribution, and the mixture of Gaussians are particular limiting cases of the alpha-stable distribution, and the mixture of alpha-stable model. Sometimes, brain-imaging data present a certain degree of asymmetry and/or impulsiveness and therefore, it can be modelled more accurately using alpha-stables. For this reason, the alpha-stable distribution is expected to work better than those approaches in the literature assuming Gaussian distribution of the data.

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