

# A High-level Petri Nets Approach for Multi-Objective Optimization in Pipeline Networks

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**Keywords:** High-level Petri Nets, Pipeline Networks, Dynamic Reliability, Dijkstra's Algorithm, Multi-objective Optimization.

**Abstract:** High-level Petri nets are a powerful modelling language appropriate to represent massive, dynamic, and complicated systems like pipeline networks. Finding the optimal path in these networks is not an easy task, especially when we are concerned with multi-objective problems such as in the present study: minimizing path's length and maximizing valves' dynamic reliability which depends on time and conditions of use. This work aims, firstly to calculate the dynamic reliability of the valves engaged in the path's search according to their behavior, and secondly, to transform the multi-objective optimization problem into a shortest path problem through a scalarization method and then to find an optimal path using the Dijkstra's algorithm developed with a High-level Petri nets. This contribution is applied in the transport of oil but it is potentially applicable in many other areas.

## 1 INTRODUCTION

Pipeline networks - for oil, natural gas, chemicals, water, etc. - represent an important part of critical infrastructures for many countries, impact in many areas of our daily lives and are essential to all industries. Particularly crude oil, which is (with natural gas) the most important raw material for energy production, is found in locations far away from where they are processed or refined into fuels, and these processing locations are also far away from where they are consumed. While many forms of transportation are used to move this product to marketplaces, pipelines remain the safest, most reliable, efficient and economical way to move this natural resource.

An oil pipeline network is intricate and can be installed above the ground, under the ground, or underwater. Pipelines operate all-day, everyday with the help of powerful pumps, oil additives that move the oil with less resistance and the laws of physics. Pipeline networks are also used to store oil and to connect the different means of transport (tankers, trucks, trains, etc.) via a set of loading arms. The oil transfer is carried out by selecting an alignment (i.e. path) of pipelines linking the two elements of interest and enabling oil flow by opening the valves in the alignment and closing all adjacent valves in order to isolate it to avoid oil mixture.

Although every pipeline company is working to properly manage her facilities and to achieve incident-free operations, accidents do happen with serious economic, financial, environmental and technological impacts. Having relevant information about the state of these pipelines along the time (monitoring) allows estimating its behavior in the future (prognosis) and can significantly improve safety policies. Dynamic reliability is one of the efficient indicators to assess performance degradation during pipeline network's life. It takes into account the failure rate versus time, as well as impact of operating constraints (Dominique et al., 2007). In industry there exists the growing perception that on one hand constant failure rates - respecting Markovian hypothesis - are no more sufficient to characterize resource confidence (Loman and Wendai, 2002), and on the other hand some influence factors determine the evolution of failure distribution laws (Devooght and Lewins, 1997) and denote the natural impact of the environment on duration of equipment's life.

Using reliable alignments and respecting the time delivery constraints of pipeline companies, are the challenges that this work aims to face, leading to a multi-objective optimization problem. For that, firstly the concept of dynamic reliability is applied to the riskiest component to failure in the oil pipeline network: the valve (SINTEF, 2002); secondly, a scalari-

sation of multi-objective optimization problem is defined to formulate a single objective optimization problem of shortest path resolved by Dijkstra's algorithm and developed in a High-level Petri net (HLP-net) framework taking into account the existing alignments. In order to validate the proposed algorithm which finds the shortest reliable path, a representative example inspired from a real case of a seaport for oil export is applied. The remainder of the paper is organised as follows. Section 2 describes the considered problem and the proposed solution. In Section 3, a case study is described, a brief overview on the HLP-nets used in this paper is presented and the related HLP-nets model is detailed. Section 4 presents the results of the experimentation of the model. Finally, Section 5 sums up the paper and presents some ideas for future works.

## 2 MULTI-OBJECTIVE OPTIMIZATION PROBLEM

Multiple, often conflicting objectives arise naturally in most real-world optimization scenarios such as the problem treated in this paper: the search of the alignment that guarantees simultaneously

- maximum dynamic reliability on valves, and
- minimum distance of pipes in pipeline networks.

These networks are characterized by complex facilities with multiple components (tubes, pumps, valves, etc.) submitted to heavy working conditions that lead usually to a decrease in their reliability. Valves' reliability is most affected and without proper control it may lead to accidents with severe environmental and economical consequences.

### 2.1 Pipe Reliability

Dynamic reliability is an efficient indicator to assess performance degradation during system's life. It depends on time and operating conditions. Pipe reliability depends on the dynamic reliability on one side of the corresponding valve to open and on the other side of all the valves that isolate it from the rest of the network (i.e. to close).

**Definition 1.** Pipe reliability  $R_{Pipe}(t, Z)$  is calculated as the product of dynamic reliability of  $n$  involved valves  $v$  among which one is to open and all the others are to close.

$$R_{Pipe}(t, Z) = R_{v.to.open}(t, Z) \times \prod_{i=0}^{n-1} R_{v.to.close_i}(t, Z). \quad (1)$$

**Definition 2.** The dynamic reliability  $R(t, Z)$  defined by the conventional expression depending of the dynamic failure rate  $\lambda(t, Z)$ ,

$$R(t, Z) = e^{-\int_{t_0}^t \lambda(\tau, Z) d\tau} \quad (2)$$

where

- $t_0$  and  $t$  are respectively the initial instant of functioning and the date of the failure occurrence,
- $Z$  represents the set of influence factors. Those choice depend on the application.

**Definition 3.** The dynamic failure rate  $\lambda(t, Z)$  depends on time  $t$  and influence factors  $Z$ ,

$$\lambda(t, Z) = \lambda_0(t) \times g(Z) \quad (3)$$

**Definition 4.** The failure rate base  $\lambda_0(t)$ , which illustrated in Fig. 1, is modeled by the Weibull distribution with two parameters  $\beta$  and  $\eta$ .

$$\lambda_0(t) = \frac{\beta}{\eta} \times \left(\frac{t}{\eta}\right)^{\beta-1} \quad (4)$$

where:

- $\beta$ : the shape parameter, unitless;
- $\eta$ : the scale parameter in units of time.

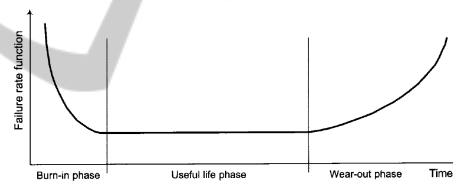


Figure 1: Bath-tub shape of the failure rate base.

**Definition 5.** The influence function  $g(Z)$  represents system's external and internal characteristics,

$$g(Z) = e^{B \times Z} = e^{\sum_{k=1}^m b_k \times z_k} \quad (5)$$

where:

- $m$ : the number of influence factors taken in the model;
- $B = (b_1, \dots, b_m)$ : coefficients' vector of the Cox model (Cox, 1972);
- $Z = (z_1, \dots, z_m)$ : influence factors' vector.

#### 2.1.1 Estimation of the Dynamic Reliability Parameters

The coefficients of the dynamic reliability function were determined through the calculi detailed in (Dominique et al., 2007). The input data needed to identify coefficients were obtained from a database of measurements generated from OREDA database reliability (SINTEF, 2002) which is a data collection

from various industries. In fact, OREDA provides a representation of the failure rate with a normal distribution characterized by its mean and standard deviation. So, firstly, we randomly generated a sufficient number of failure rate values with MATLAB software environment (Hahn and Valentine, 2013). We use these values of failure rate to find the correspondent time and influence factors using the Naval Surface Warfare Center (NSWC) approach (Tyrone and Jones, 2011).

## 2.2 Scalarizing the Multi-objective Optimization Problem into a Shortest Path Problem

We want to minimize the path distance  $D_p$

$$D_p = \min \sum_{i=1}^n l_i \quad (6)$$

where

- $n$ : the number of engaged pipes in the path.
- $l_i$ : length of the  $i^{th}$  engaged pipe.

and to maximize the path reliability  $R_p$

$$R_p(t, Z) = \max \prod_{i=1}^n R_{Pipe_i}(t, Z) \quad (7)$$

where

- $n$ : the number of engaged pipes in the path.
- $R_{Pipe_i}$ : pipe reliability of the  $i^{th}$  engaged pipe.

(6) and (7) define a multi-objective problem, that can be approached by scalarizing it.

Scalarizing a multi-objective optimization problem aims to formulate a single-objective optimization problem such that optimal solutions to the single-objective optimization problem are optimal Pareto solutions to the multi-objective optimization problem (Ching-Lai and AbuSyed, 1979). There exist many scalarization methods (Pagani and Pellegrini, 2009), but we will define a new method more appropriate for our case, as follows.

**Definition 6.** The scalar cost  $SC_i(t, Z, l_i)$  of the  $i^{th}$  pipe is defined as the pipe length divided by the pipe reliability:

$$SC_i(t, Z, l_i) = \frac{l_i}{R_{Pipe_i}(t, Z)}. \quad (8)$$

This scalarization allowed us to transform our multi-objective optimization problem into a shortest path problem. With time, pipe reliability decreases causing the increase of scalar costs; this will prevent

the algorithm to select paths that although short in distance have lower reliability.

Different algorithms have been proposed to find the optimal routes in graphs. Dijkstra's algorithm is probably the best known; it is a graph search algorithm that solves the single-source shortest path problem for a graph with nonnegative edge costs, producing a shortest path tree (Dijkstra, 1959).

**Definition 7.** Shortest reliable path  $SRP(t, Z, L)$ , according to Dijkstra's algorithm, will be calculated from the sum of the scalar costs of the  $n$  pipes engaged in the path.

$$SRP(t, Z, L) = \sum_{i=1}^n SC_i(t, Z, l_i) \quad (9)$$

where

$$L = \sum_{i=1}^n l_i. \quad (10)$$

### 2.2.1 Brief Review of the Dijkstra's Algorithm as Used

Let the pipe where we are starting be called an initial pipe. Let the cost to a pipe  $Y$  be the  $SRP(t, Z, L)$  from the initial pipe to it. Dijkstra's algorithm will assign some initial cost value and will try to improve them step-by-step as follows:

1. Assign to the initial pipe a cost value.
2. Set initial node as current.
3. For current pipe, consider all its unvisited and available (i.e. is not in use by alignments) neighbors and calculate their cost (from the initial pipe).
4. When we are done considering all neighbors of the current pipe, mark it as visited.
5. Set the unvisited node with the smallest cost (from the initial pipe) as the next "current pipe" and continue from step 3).

With these steps, the shortest reliable path from the starting point to the destination can be effectively achieved. And thus this algorithm has been widely used in routing systems, namely in oil pipelines networks: (Rojas-D'Onofrio et al., 2011; Kadri and Zouari, 2014) using respectively automaton and HLP-nets model.

## 3 CASE STUDY

### 3.1 Oil Pipeline Network Description

The case study is a simplified pipeline network representative of an oil-exporting seaport (see Fig. 2). It

is composed by a set of pipes linking a set of tanks storing oil to a set of loading arms placed at the docks of the seaport. Loading arms are connected to tankers that receive the oil and transport it to different destinations. Valves and pumps are the only elements that can be controlled. Valves are, most of the time, in one of two different states: opened or closed, whereas pumps can be on or off. The transitions between these states can be considered instantaneous when compared with the time spent in any of the states.

Alignments are established using valves: some valves are opened along a path linking the elements, whereas some other valves are closed around the path, isolating it from the rest of the pipe network.

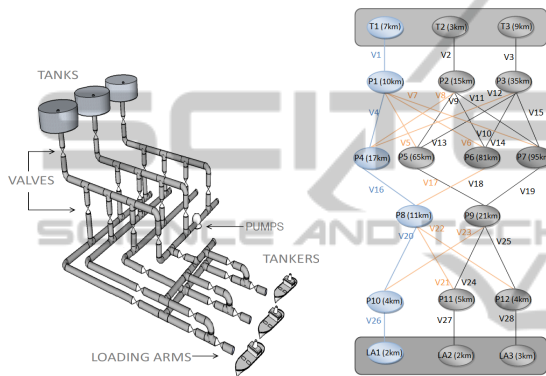


Figure 2: Oil seaport example. Figure 3: Its undirected graph model.

Fig. 3 shows the example as an undirected graph in which arcs represent the valves. The nodes represent pipes with lengths. blue nodes represent an alignment, blue arcs its opened valves and orange arcs its closed valves. To satisfy any request, this work aims to find the shortest reliable path that minimizes *SRP* in (9).

For each valve, the studied influence factors  $z_i$  are components of the  $Z$  influence vector

$$Z = (C_s, T, S). \quad (11)$$

where:

$C_s$ : Commutation stress, the total number of changing state from opened to closed or the contrary.

$T$ : The total operating time in opened and in closed states.

$S$ : The last valve state (opened or closed). This state will be compared to the new state of its corresponding valve in order to detect the changing state.

From (5), considering these three influence factors, we can write

$$g(Z) = e^{(b_1 \times C_s + b_2 \times T + b_3 \times S)}. \quad (12)$$

The constant  $b_3$  is also called the gamma factor i.e. probability of failure (to close/open) (Cacheux and Collas, ).

To illustrate our approach, we used the previous example developed using the environment CPN Tools 4 (Westergaard and Kristensen, 2009).

### 3.2 High-level Petri Nets

HLP-nets allow a concisely representation in a unified structure both of the static and the dynamic aspects of the considered system, thanks to its twofold representation - graphical and mathematical. The graphical aspect enables a concise way to design and verify the model, while the mathematical aspect allows formal modelling of these interactions and analysis of the modelled system properties (Jensen and Kristensen, 2009).

Among the several types of HLP-nets (Colored, timed, Stochastic, etc.), we use particularly the following:

**Colored Petri Nets** associate color to each token distinguishing one token from the other and their value can be manipulated and tested with “Meta Language” during arcs, transitions and guards.

**Hierarchical HLP-nets** allow to divide the model into submodels small enough to be easily tracked, and to verify independently each submodel properties.

**Timed Petri Nets** allow to modelling timed information related to the functioning of the proposed model.

**Reset Nets** extend Petri nets with a special type of arc, the reset arc. It does not impose a precondition on firing, and empties the place when the transition fires.

**Petri Nets with Inhibitor Arcs** which imposes the precondition that the transition may only fire when the place connected to it has zero tokens.

**Prioritised Petri Nets** add priorities to transitions, whereby a transition cannot fire, if a higher-priority transition is enabled (i.e. can fire).

These types of HLP-nets revealed adequate and practical to model the optimal path search problem of pipeline networks, as shown in the following.

### 3.3 HLP-nets Model

This section describes the HLP-nets modeling technique used to search the shortest reliable path. In particular, the model developed in this paper is referred

to a seaport pipeline network, but it can be easily applied to any pipeline network.

The model in Fig. 4 presents the upper layer of the hierarchical HLP-nets model describing the generic behavior of the Dijkstra's algorithm applied to a pipeline in operation:

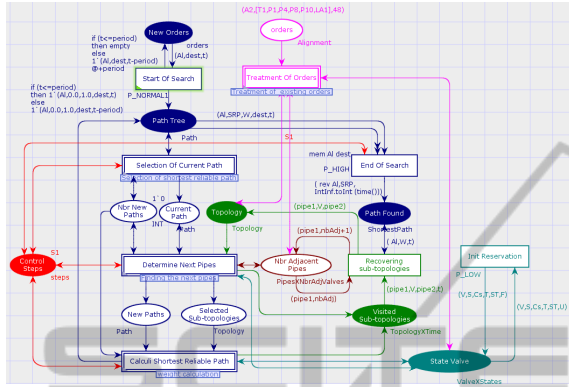


Figure 4: Path search algorithm model.

- The token of the place "New Orders" defines the related information of an order: Source, destination and duration. For each order, the shortest reliable path search is repeated every 48 hours (2 days is the average time to fulfill a normal tanker, but this time can be adapted by the user) in order to maintain the proper alignment choice for long-term orders.
- The sequentiality of the three steps of Dijkstra's algorithm is verified via a token in the place "Control Steps".
- The topology of the pipeline network is defined as a set of tokens in the place "Topology". Every token is a triplet containing a valve connecting each two pipes ( $pipe_1, valve, pipe_2$ ) based on the rule: "between two pipes there is one and only one valve". The order of the pipes in the triplet informs us the direction of oil flow. Note that this simple technique makes the model sufficiently generic to present any pipeline network independently of its size and its shape, and is also flexible for the modification of the pipeline topology: joining or deleting pipes or valves reduces to simply create or delete tokens.
- When search is started, the place "Path Tree" contains the produced shortest reliable paths. We have two cases:

**Destination reached:** when the transition "End Of Search" is fired which is a higher-priority transition,

- the shortest reliable path is placed in the place

"Path Found" with its SRP value and its starting time;

- a reset arc empties the place "Path Tree" and a mechanism of recovering the used tokens from the place "Topology" is activated in order to re-initialize the model.

**Destination not reached:** - the substitution transition "Selection Of Current Path" sets the path with the lowest SRP in the place "Current Path";

- the substitution transition "Determine Next Pipes" creates new paths as concatenation of current path with its unvisited neighbor pipes and stores them in the place "New Paths".
- the substitution transition "Calculi Shortest Reliable Path" computes the SRP of each new path.

### 3.3.1 Submodel "Treatment Of Orders"

The submodel of Fig. 5 models the operation of the pipeline. It contains a place based on external events called "State Valve" which represents the characteristics of each elementary valve such as the initial instant of functioning, the identity, availability, etc., in addition to its influence factors values: the commutation stress ( $C_s$ ), the operating time ( $T$ ), and the last valve state ( $S$ ). Order by the pipeline alignment, the valve will be committed to opened or closed state based on its current state and the specificity of the alignment and its characteristics will be updated.

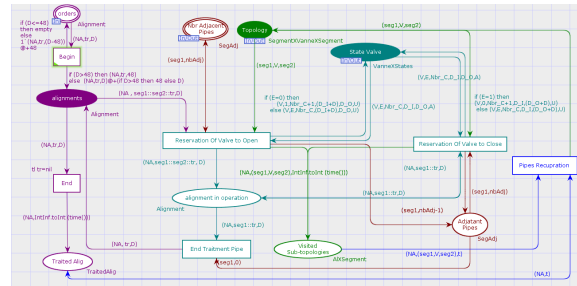


Figure 5: Submodel "Treatment Of Orders".

### 3.3.2 Submodel "Selection Of Current Path"

In this HLP-nets submodel (Fig. 6), the steps of the shortest reliable path selection are:

- from "Path Tree", one path is chosen randomly;
- for each other path, the submodel compares its SRP with that of the chosen one, makes as chosen the path with the lowest SRP and places the other in the place "Paths Tested";

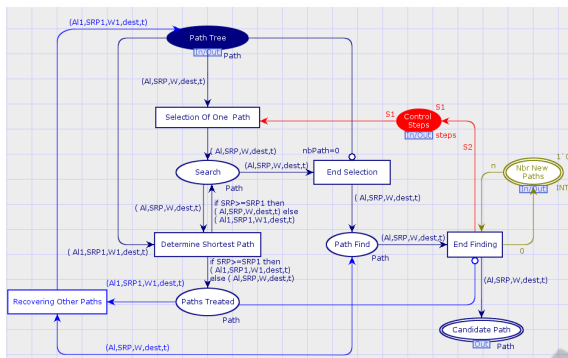


Figure 6: Submodel "Selection Of Current Path".

- when all candidate paths are tested, the one with the lowest *SRP* is placed in the place "Path Find" first and then in the place "Current Path".
- before moving to the next step by firing the transition "End Finding", the transition "Recovering Other Paths" is fired enough times to recover all paths from the place "Paths Tested" to the place "Path Tree".

### 3.3.3 Submodel "Determine Next Pipes"

This HLP-nets submodel (Fig. 7) aims to create paths from the current path and its unvisited neighbor pipes.

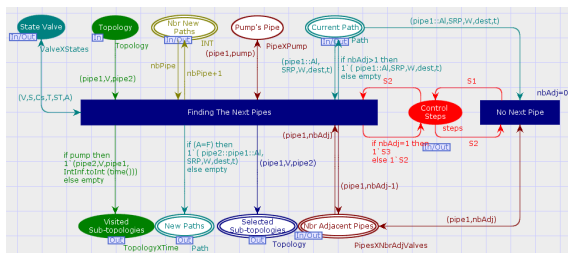


Figure 7: Submodel "Determination of neighbor pipes".

**If there are neighbor pipes**, the transition "Finding the next pipes" will be fired enough times that the current path has adjacent pipes in possible directions (i.e. possible oil flow) causing

- the search of all unvisited and available neighbor pipes, and their storage in the place "Selected Sub-topology";
- the replacement of the current path by new ones corresponding to each pipe found concatenated with the current path;
- for all pipes found containing a pump, their tokens representing the other oil flow direction are placed in the place "Visited Sub-topology".

**If there are no neighbor pipes**, the transition "No next pipes" will be fired causing the deletion of

the token representing the current path and the return to the previous step for selecting another path.

### 3.3.4 Submodel "Calculi Shortest Reliable Path"

The objective of this submodel (Fig. 8) is, for every new path created, the computation of its *SRP* value (7).

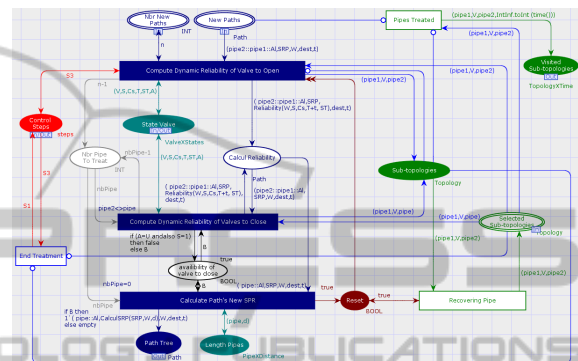


Figure 8: Submodel "Calculi Shortest Reliable Path".

The submodel incorporates a computation procedure to assess the scalar cost ( $SC(l, t, Z)$ ) (6) of each pipe based on its length  $l$  and on the dynamic reliability ( $R(t, Z)$ ) (2) of these valves involved in opening or in closing.

In order to determine the *SRP* for each path in the place "New Paths", the following steps must be achieved:

- from the place "Selected Sub-topology", determination of the new valve invoked to opening and the computing of its ( $R(t, Z)$ );
- determination of its new valves invoked to closing and computation of their  $R(t, Z)$  and the pipe reliability ( $R_{pipe}(t, Z)$ ) from all the remaining tokens in the place "Selected Sub-topology" representing possible neighbor pipes by firing enough times the transition "Calculate Dynamic Reliability of Valves to Close";
- computation of the  $SC(l, t, Z)$  and updating the *SRP* via the transition "Calculate Path's New *SRP*";
- placing the used tokens from the place "Selected Sub-topologies" in the place "Sub-Topologies" and those from the place "Topology" in the place "Adjacent Sub-topologies";
- Deleting paths with valve to close engaged to open with an existing order.

- **if there are still paths:** the transition "*Recovering Pipes*" is fired enough times to recuperate all tokens in the place "*Selected sub-topologies*" from the place "*Sub-topology*".

**If there are no more paths:** the transition "*Pipes Treated*" is fired enough times to place all tokens of the place "*Sub-topology*" in the place "*Next Sub-topologies*".

- At the end, the transition "*End Treatment*" is fired in order to return to the first step.

#### 4 THE CASE STUDY: SIMULATION AND RESULTS

To extract the coefficients of the failure rate base function and of the influence function, we have considered the OREDA database. It indicates, for valves type *Butterfly*, that are widely used in oil pipeline networks their average lifetime is equal to 98,500 hours (i.e. more than 11 years). So, we divided this lifetime between the two considered phases:

**useful life phase** ( $\beta = 1$ ) for the first 71,600 hours (i.e. more than 8 years): it's equivalent to the operational time of these type's valves estimated based on a data collector conducted by OREDA.

**wear-out phase** ( $\beta = 4.748$ ) for the rest.

The **burn-in phase** is not treated because it's not included in the OREDA database: it's assumed that the data collection is started with the useful life phase. The  $\eta$  determined is 98522.167 hours; and the calculated coefficients of influence factors are listed in Table 1.

Table 1: The coefficients of influence factors.

Coefficient	Value
$b_{C_s}$	$-36.621 \times 10^{-5}$
$b_T$	$-6.103 \times 10^{-5}$
$b_{S\_to\_open}$	1.118
$b_{S\_to\_close}$	1.125

The HLP-nets model of the Dijkstra's algorithm exploits the pipeline network to find the shortest reliable path. We will reconsider the oil seaport example of Fig. 2 and Fig. 3 to answer to the order of transporting the oil from tank  $T_1$  to the loading arm  $LA_1$  during 96 hours (4 days). First, let us assume that the pipeline has already been used to satisfy previous orders and initialized with the values of Table 2 and the existing alignment of Fig. 3 will end in 48 hours.

Table 2: Valves coefficient.

Valve	S	$C_s$	T (hour)	Valve	S	$C_s$	T (hour)
V1	open	693	33264	V15	open	1377	66096
V2	open	365	17520	V16	open	455	21840
V3	close	148	7104	V17	close	582	27936
V4	open	1607	77136	V18	close	494	71712
V5	open	473	22704	V19	open	1123	53904
V6	open	629	30192	V20	open	399	19152
V7	open	205	9840	V21	open	583	27984
V8	open	451	21648	V22	open	121	5808
V9	close	388	18624	V23	open	1025	49200
V10	close	536	25728	V24	close	38	1824
V11	open	1297	62256	V25	open	747	35856
V12	close	318	15264	V26	open	173	8304
V13	open	823	39504	V27	open	23	1104
V14	open	919	44112	V28	close	952	45696

To reach the destination point  $LA_1$  from tank  $T_1$ , the developed algorithm determines minimal paths based on pipes' length and valves' dynamic reliability engaged to open and to close. The results computed by the proposed model are given in Table 3.

Table 3: Results.

Shortest reliable path	$SRP(t, Z, L)$	time
$T_1, P_1, P_7, P_9, P_{10}, LA_1$	492.5712	0
$T_1, P_1, P_6, P_8, P_{10}, LA_1$	390.0216	48

When an alignment is chosen to satisfy one or more orders, it will be used throughout the period necessary. If this period is too long it would impact the dynamic reliability of its valves and some orders have been completed and others began. Consequently, the proposed model will favor another alignment to finish the current order or for similar orders and that is what happened in the studied example. For an order with a duration of four days, an optimal alignment is obtained for the first 48 hours and another for the remaining two days.

During the simulation of the shortest path search, a set of monitors can be integrated into the HLP-nets model to estimate, based on dynamic reliability, valves' lifetime and their maintenance timeout.

Furthermore, the simulation allows the validation of certain properties of the studied system:

- **Flexibility:** the modification of pipeline network topology (i.e. maintenance, extension, etc.) does not influence the developed model because the related information of the network topology is specified only in the tokens.
- **Boundedness:** the number of paths in the place "*Path Tree*" which represents the calculated shortest reliable paths, is bounded because it is less than or equal to the number of all possible paths in the network which is also bounded.

The use of hierarchical model has major advantages. Indeed, added to the ability to make easier the mod-

elling of the complex system, such alternative allows to independently verifying each submodel properties and having more compact and understandable models. Using each submodel makes easier the validation of every step of Dijkstra's algorithm.

## 5 CONCLUSIONS

This research work addressed the problem of multi-objective optimization in oil pipeline network: the calculation of the shortest reliable path is taking into account pipes' length and valves' dynamic reliability which varies with time and with conditions of use.

The study proposed, firstly, the definition of dynamic reliability and the determination of its parameters; secondly, the adoption of the scalarization method in order to transform the multi-objective optimization problem into a shortest path problem solved by Dijkstra's algorithm and based on a HLP-nets model.

The implementation of this algorithm in the framework of HLP-nets has the important advantage of avoiding the curse of dimensionality that can appear in combinatorial optimization problems concerning real pipeline networks. With this framework the problem dimension is independent of the graph's nodes' number, since the dimension impacts only in the tokens' number.

Another important issue is the global network management when several orders must be satisfied simultaneously and several non-intercepting alignments must be used at the same time. This introduces additional constrains in the minimal path search that must be accommodated in the HLP-nets model to prevent conflicts in pipe selection and concatenation.

Maintenance operations must be programmed to prevent an exaggerated decrease in valves and pumps reliability. The dynamic reliability, used in this work, can eventually be used as a trigger to maintenance plans that must be taken into consideration and represent another additional constraint in the problem.

These are interesting directions for future research.

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