Complete Stiffness Model for a Serial Robot

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Abstract: The paper addresses a problem of robotic manipulator calibration. The main contributions are in the area of the elastostatic parameters identification. In contrast to other works, the considered approach takes into account elastic properties of both links and joint. Particular attention is paid to generation of the complete and irreducible stiffness model that is suitable for the identification. To solve the problem, physical and algebraic model reduction methods are proposed. They are based on taking into account the physical properties of the manipulator elements and structure of the corresponding observation matrix. The advantages of the developed approach are illustrated by an application example that deals with elastostatic calibration of an industrial robot.

1 INTRODUCTION

Industrial robots are gradually finding their niche in manufacturing, replacing less universal and more expensive CNC-machines. Application area of robots is constantly growing, they begin to be used not only for the assembly and pick-and-place operations, but also for the machining. The latter requires special attention to the accuracy of the model, which is used to control the manipulator movements. Furthermore, for this process, the robot is usually subject to essential external loading caused by the machining force that may lead to non-negligible deflections of the end-effector (Dépincé and Hascoët, 2006) and, accordingly, degrade the quality of the final product. This issue becomes extremely important in the aerospace industry, where the accuracy requirements are very high. In this case, the manipulator stiffness modeling and corresponding error compensation technique are the key points (Karan and Vukobratović, 1994; Kövecses and Angeles, 2007), where in addition to accurate geometric model a sophisticated elastostatic one is required.

In practice, the robot positioning accuracy can be improved by means of either on-line or off-line error compensation techniques (Abele, Schützer et al., 2012; Chen, Gao et al., 2013). Usually geometric errors (such as offsets and link lengths) can be efficiently compensated by modifying internal parameters of the robot controller (Mooring, Roth et al., 1991). In contrast, the compliance errors have to be compensated via modification of the controller inputs. In such a case, an off-line error compensation technique is aimed at adjusting the target trajectory in accordance with the errors to be compensated and the geometric model used in the robot controller (Klimchik, Pashkevich et al., 2013). It is evident that the efficiency of the latter approach is quite sensitive to the model completeness and the accuracy of its parameters.

To achieve desired degree of accuracy, the manipulator model should be calibrated for each particular manipulator (Meggiolaro, Dubowsky et al., 2005). In modern robotics, there exist a number of techniques that allow user to identify geometric and elastostatic parameters of either serial or parallel manipulators. In general, classical calibration procedure contains four basic steps: modeling, measurement, identification and implementation (Roth, Mooring et al., 1987). The first step is aimed at development of a model, which is accurate enough and also is suitable for the identification (i.e. without redundant parameters that can cause the convergence breakdown). At the following step, the measurements data are obtained. These data can be gotten

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using open-loop and closed-loop methods (Takeda, Shen et al., 2004; Nubiola and Bonev, 2013). The identification step is aimed at tuning the model parameters in accordance with the experimental data. The last step, implementation, deals with modification the robot control software in accordance with the identified parameters.

In the manipulator stiffness modeling, there are currently three main approaches: the Finite Element Analysis (FEA), the Matrix Structural Analysis (MSA), and the Virtual Joint Method (VJM). As follows from our experience, the VJM method (Alici and Shirinzadeh, 2005; Klimchik, Chablat et al., 2014) provides reasonable trade-off between the model accuracy and computational complexity and will be further used in this paper. It is based on the extension of the traditional rigid model by adding the virtual joints describing the elastic deformations of the links, joints and actuators.

It should be mentioned that calibration of the elasto-static model is much more difficult compared to the geometric one. For a simple case, when only elasticity of the actuated joints is taken into account, an efficient approach has been proposed in (Dumas, Caro et al., 2011), but this simplification does not allow describing some important deflections of the end-effector. More sophisticated model describing both the joint and link elasticity can be developed use CAD-based technique proposed in our previous work (Klimchik, Pashkevich et al., 2013). However, this model includes huge number of parameters that cannot be identified separately using conventional measurement data describing the end-effector deflections caused by external force/torque. It means that from mathematical point of view, this technique may produce redundant models that are not suitable for calibration.

Similar problem is also known in geometric calibration where the concept of *complete-irreducible-continues* model has been introduced (Khalil and Dombre, 2004). However, in elastostatic calibration there is an additional difficulty caused by huge number of model parameters (258 for 6 dof manipulator) and essential difference in their magnitudes. For this reason, this paper deals with developing stiffness model suitable for identification and proposes model reduction methods that allow obtaining reliable results in industrial environment.

It is worth mentioning that the adopted approach deals with quasi-static modeling that is motivated by the considered application area. In particular, it is assumed that trajectory tracking compensation does not takes into account dynamic effects and load disturbances over frequencies.

To address the above mentioned problem, the remainder of the paper is organized as follows. Section 2 presents the stiffness modeling background and problem statement. In Section 3, the developed model reduction methods are presented. Section 4 contains application examples that illustrate advantages of the proposed technique. And finally, Section 5 summarizes the main contributions of the paper.

2 THEORETICAL BACKGROUND AND PROBLEM STATEMENT

Let us consider an elastostatic model of a general serial manipulator, which consists of a fixed "Base", a serial chain of flexible "Links", a number of flexible actuated joints "Ac" and an "End-effector" (0). In order to describe the stiffness of the considered manipulator, let us apply the virtual joint method (VJM), which is based on the lump modeling approach. According to this approach, the rigid model should be extended by adding localized spring describing links elasticity. Besides, in order to take into account the stiffness of the control loop, the virtual springs should be included in the actuated joints.

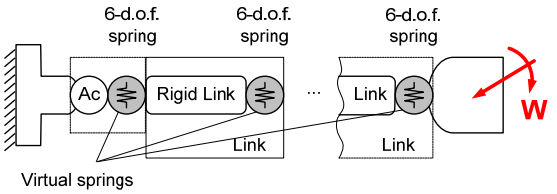

Figure 1: VJM model of serial robot.

For the considered manipulator the force-deflection relation for given robot configuration $\mathbf{q}$ is defined by the Cartesian stiffness matrix $\mathbf{K}_C$ as

$$\mathbf{W} = \mathbf{K}_C \cdot \Delta\mathbf{t} \tag{1}$$

In these equations, the end-effector displacement $\Delta\mathbf{t}$ is treated as the model input and the external wrench $\mathbf{W}$ is the model output. The stiffness matrix $\mathbf{K}_C$ can be computed as follows

$$\mathbf{K}_C = \left(\mathbf{J}_\theta \cdot \mathbf{K}_\theta^{-1} \cdot \mathbf{J}_\theta^{\,T}\right)^{-1} \tag{2}$$

Here, the Jacobian matrix $\mathbf{J}_\theta$ depends on the manipulator configuration $\mathbf{q}$ and can be computed as a partial derivative of the end-effector location with

respect to a set of desired virtual joint coordinates $\boldsymbol{\theta}$. Expression (2) allows us to compute the Cartesian stiffness matrix assuming that the matrix $\mathbf{K}_\theta = diag(\mathbf{K}_\theta^{(1)}, \mathbf{K}_\theta^{(2)}, ...)$, defining elastostatic properties of the manipulator links/joins is given. However, in practice, the matrices $\left\{\mathbf{K}_\theta^{(i)}, i = 1,2,...\right\}$ are unknown and should be identified from relevant experiments.

To estimate the desired matrices describing elasticity of the manipulator components (i.e., compliances of the virtual springs), the elastostatic model (1) should be rewritten as

$$\Delta\mathbf{t} = \sum\nolimits_{i=1}^{n}\left(\mathbf{J}_\theta^{(i)}\mathbf{k}_\theta^{(i)}\mathbf{J}_\theta^{(i)T}\right)\cdot\mathbf{W} \qquad (3)$$

where the matrices $\mathbf{k}_\theta^{(i)} = (\mathbf{K}_\theta^{(i)})^{-1}$ denote the link/joint compliances that should be identified via calibration, and the matrices $\mathbf{J}_\theta^{(i)}$ are corresponding sub-Jacobians obtained by the fractioning of the aggregated Jacobian $\mathbf{J}_\theta^T = [\mathbf{J}_\theta^{(1)^T}, \mathbf{J}_\theta^{(2)^T}, ...]$.

In the case when the matrices $\mathbf{k}_\theta^{(i)}$ are known the end-effector deflections $\Delta\mathbf{t}$ caused by external loading $\mathbf{W}$ can be compensated by means of either on-line or off-line error compensation techniques (Lo and Hsiao, 1998; Klimchik, Bondarenko et al., 2014). Usually main geometric errors (such as offsets and link lengths) can be efficiently compensated by modifying internal parameters of the robot controller (Mooring, Roth et al., 1991). In contrast, the compliance errors have to be compensated via modification of the controller inputs. Relevant on-line compensation strategy requires external measurement system that continuously provides the end-effector coordinates, which are compared with the computed ones and the differences are used for adjusting the input trajectory (Lu and Lin, 1997). However, suitable measurement systems are quite expensive and often cannot ensure tracking the reference point in a whole robot workspace. Moreover, behavior of some technological processes hampers the end-effector observability (cutting chip in milling, for instance) and may damage the measurement equipment. In such a case, an off-line error compensation technique looks more reasonable; it is aimed at adjusting the target trajectory in accordance with the errors to be compensated and the geometric model used in the robot controller (Chen, Gao et al., 2013; Klimchik, Pashkevich et al., 2013).

However for the majorit of industrial robots the values of compliance matrices $\mathbf{k}_\theta^{(i)}$ should be identified from the dedicated experimental study. So, for the identification purposes, this expression should be transformed into more convenient form, where all desired parameters (elements of the matrices $\mathbf{k}_\theta^{(i)}, i = 1,2,...$) are collected in a single vector $\boldsymbol{\pi} = (k_{\theta 11}^{(1)}, k_{\theta 12}^{(1)}, ...k_{\theta 66}^{(n)})$. It yields the following linear equation

$$\Delta\mathbf{t} = \mathbf{A}_\pi \cdot \boldsymbol{\pi} \qquad (4)$$

where

$$\mathbf{A}_\pi = [\mathbf{J}_1\mathbf{J}_1^T\mathbf{W}, \mathbf{J}_2\mathbf{J}_2^T\mathbf{W}, ..., \mathbf{J}_m\mathbf{J}_m^T\mathbf{W}] \qquad (5)$$

is so-called observation matrix that defines the mapping between the unknown compliances $\boldsymbol{\pi}$ and the end-effector displacements $\Delta\mathbf{t}$ under the loading $\mathbf{W}$ for the manipulator configuration $\mathbf{q}$. In the observation matrix $\mathbf{A}_\pi$ the subscript defines the parameters set for which the observation matrix is computed. Here, the vectors $\mathbf{J}_i$ are the columns of the matrix $\mathbf{J}_\theta$, i.e. $\mathbf{J}_\theta = [\mathbf{J}_1, \mathbf{J}_2, ..., \mathbf{J}_m]$.

Taking into account that the calibration experiments are carried out for several manipulator configurations defined by the actuated joint coordinates $\mathbf{q}_j, j = \overline{1,m}$, the system of basic equations for the identification can be presented in the following form

$$\Delta\mathbf{t}_j = \mathbf{A}_{\pi j} \cdot \boldsymbol{\pi} + \boldsymbol{\varepsilon}_j; \qquad j = \overline{1,m} \qquad (6)$$

where $\boldsymbol{\varepsilon}_j$ describes the measurement noise impact. Further, using these notations and assigning proper weights for each equation, the identification can be reduced to the following optimization problem

$$F = \sum\nolimits_{j=1}^{m}(\mathbf{A}_{\pi j}\boldsymbol{\pi} - \Delta\mathbf{t}_j)^T\boldsymbol{\eta}^T\boldsymbol{\eta}(\mathbf{A}_{\pi j}\boldsymbol{\pi} - \Delta\mathbf{t}_j) \to \min_{\boldsymbol{\pi}} \qquad (7)$$

where $\boldsymbol{\eta}$ is the matrix of weighting coefficients that normalizes the measurement data. In practice, the matrix $\boldsymbol{\eta}$ is used for two main purposes: (i) to avoid the problem of non-homogeneity due to different units of equations in the system (6) (position and orientation components, in this particular case) and (ii) to give higher weights for the measurements whose precision is obviously higher. An example of such an approach has been given in (Klimchik, Wu et al., 2013). Relevant minimization (7) yields the following solution

$$\hat{\boldsymbol{\pi}} = \left(\sum\nolimits_{j=1}^{m}\mathbf{A}_{\pi j}^T\boldsymbol{\eta}^T\boldsymbol{\eta}\mathbf{A}_{\pi j}\right)^{-1}\cdot\left(\sum\nolimits_{j=1}^{m}\mathbf{A}_{\pi j}^T\boldsymbol{\eta}^T\boldsymbol{\eta}\Delta\mathbf{t}_{\pi j}\right) \qquad (8)$$

If the measurement noise is Gaussian (as it is assumed in conventional calibration techniques), expression (8) provides us with an unbiased estimates

for which $E(\hat{\boldsymbol{\pi}}) = \boldsymbol{\pi}$.

It is clear that expression (8) gives reliable estimates of the parameters $\boldsymbol{\pi}$ if and only if the matrix $\sum_{j=1}^{m} \mathbf{A}_{\pi j}^{T} \boldsymbol{\Sigma}^{-T} \boldsymbol{\Sigma}^{-1} \mathbf{A}_{\pi j}$ is invertible. It leads to the problem of the parameter *identifiability* that have been studied by a number of authors for the problem of geometrical calibration (Pashkevich, 2001; Khalil and Dombre, 2004). Relevant techniques are based on the *information matrix rank analysis*. However, they cannot be directly applied for the case of elastostatic calibration.

Let us assume that the vector of desired elastostatic parameters $\boldsymbol{\pi}$ should be identified from the set of the linear equations (4) whose least square solution is defined by the expression (8). Depending on the matrix set $\{\mathbf{A}_{\pi j}\}$, corresponding system of linear equations can be solved for $\boldsymbol{\pi}$ either uniquely or may have infinite number of solutions. In general, if the information matrix is rank-deficient, a general solution of the system (6) can be presented in the following form

$$\hat{\boldsymbol{\pi}} = \mathbf{A}_{\Sigma}^{+} \cdot \mathbf{B}_{\Sigma} + \left(\mathbf{I} - \mathbf{A}_{\Sigma}^{+} \mathbf{A}_{\Sigma}\right) \cdot \boldsymbol{\lambda} \qquad (9)$$

where the superscript "+" denotes the Moore–Penrose pseudo-inverse, $\mathbf{A}_{\Sigma} = \sum_{j=1}^{m} \mathbf{A}_{\pi j}^{T} \boldsymbol{\eta}^{T} \boldsymbol{\eta} \mathbf{A}_{\pi j}$, $\mathbf{B}_{\Sigma} = \sum_{j=1}^{m} \mathbf{A}_{\pi j}^{T} \boldsymbol{\eta}^{T} \boldsymbol{\eta} \Delta \mathbf{t}_{\pi j}$ and $\boldsymbol{\lambda}$ is an arbitrary vector of the same size as $\boldsymbol{\pi}$. So, all desired parameters contained in the vector $\boldsymbol{\pi}$ can be divided into three non-overlapping groups (Pashkevich, 2001):

G1: *Identifiable* parameters that can be obtained from (9) in unique way;

G2: *Non-identifiable* parameters that cannot be computed uniquely from (9) and do not influence on the right-hand side of the equation (4);

G3: *Semi-identifiable* parameters that are also cannot be computed uniquely but have influence on the right-hand side of the equation (4).

To present typical examples of the parameters belonging to the groups G1, G2 and G3, it is possible to use the ideas similar to geometrical calibration. For instance, the elastostatic parameters of the actuated joints and adjacent links are redundant in their totality and belong to the group G3. Besides, if the loading direction cannot be altered, a number of parameters belong to the group G2 and cannot be identified from the corresponding experimental data. So, complete and irreducible model should contain all parameters from the group G1 and partially parameters of the group G3.

Hence, to obtain reliable stiffness model that is suitable for calibration, it is necessary to develop dedicated model reduction techniques and relevant rules allowing us to minimize the number of parameters to be estimated and to reconstruct the original VJM-based model from these data taking into account mathematical relations between the model parameters caused by their physical sense.

3 MODEL REDUCTION

3.1 Physical Approach

Straightforward stiffness modeling approach provides the exhaustive but redundant number of parameters to be identified. For instance, each links is described by a 6×6 matrix that includes 36 parameters that are treated as independent ones. However, as follows from physics, number of pure physical parameters is essentially lower. Hence, there are strong relations between these 36 parameters but this fact is usually ignored in elastostatic calibration. Besides, due to fundamental properties of conservative system, the desired compliance matrices should be strictly symmetrical and positive-definite. In addition, for typical manipulator links, the compliance matrices are sparsed due to the shape symmetry with respect to some axis, but this property is not taken into account also in the identification of the elastostatic parameters.

To use the advantages of the compliance matrix properties and to increase the identification accuracy, three simple methods can be applied that allow us to reduce the number of parameters to be computed in the identification procedure (8). They can be treated as the physics-based model reduction techniques and formalised in the following way.

M1:Symmetrisation. For all compliance matrices $\mathbf{k}$ to be identified, replace the pairs of symmetrical parameters $\{k_{ij}, k_{ji}\}$ by a single one $k_{ij}, i < j$.

For each link, this reduction procedure is equivalent to re-definition of the model parameters vector in the following way

$$\boldsymbol{\pi} = \mathbf{M} \cdot \boldsymbol{\pi}' \qquad (10)$$

where the binary matrix $\mathbf{M}$ of size 36×21 describes the mapping from the original to reduced parameter space. It can be proved that corresponding basic expression for the identification (4) can be rewritten as

$$\Delta \mathbf{t} = \mathbf{A}_{\pi'} \cdot \boldsymbol{\pi}' \qquad (11)$$

where $\mathbf{A}_{\pi'} = \mathbf{A}_{\pi} \cdot \mathbf{M}$ denotes the reduced observation matrix. The later can be also computed as

$$\mathbf{A}_{\pi'} = [\mathbf{J}_\theta \boldsymbol{\omega}_1 \mathbf{J}_\theta^T \mathbf{w},\ \mathbf{J}_\theta \boldsymbol{\omega}_2 \mathbf{J}_\theta^T \mathbf{w},\ ... \mathbf{J}_\theta \boldsymbol{\omega}_{21} \mathbf{J}_\theta^T \mathbf{w}] \quad (12)$$

where $\boldsymbol{\omega}_1, \boldsymbol{\omega}_2, ...$ denote the binary matrices of size 6×6 for which non-zero elements (i.e. equal to 1) are located in the following way: for the parameter π_l corresponding to the matrix elements $k_{ij},\ i \le j$, the non-zero elements are $\omega_{ij} = \omega_{ji} = 1$. It is clear that this idea allows us to reduce the number of links compliance parameters from 36 to 21 (and from 258 to 153 for the entire 6 d.o.f. manipulator).

M2:Sparcing. For all compliance matrices $\mathbf{k}$ to be identified, eliminate from the set of unknowns the parameters k_{ij} corresponding to zeros in the stiffness matrix template $\mathbf{k}^0$ derived analytically for the manipulator link with similar shape.

To obtain a desired template matrix, is convenient to use any realistic link-shape approximation. For example using the trivial beam (Timoshenko and Goodier, 1970), the desired template can be presented as

$$\mathbf{k}^0 = \left[\begin{array}{ccc|ccc} * & 0 & 0 & 0 & 0 & 0 \\ 0 & * & 0 & 0 & 0 & * \\ 0 & 0 & * & 0 & * & 0 \\ \hline 0 & 0 & 0 & * & 0 & 0 \\ 0 & 0 & * & 0 & * & 0 \\ 0 & * & 0 & 0 & 0 & * \end{array}\right] \quad (13)$$

where the symbol "*" denotes non-zero elements. It allows further reducing the number of the unknown parameters from 21 to 8, taking into account only essential ones from physical point of view. It can be also proved that the template (13) is valid for any link whose geometrical shape is symmetrical with respect to three orthogonal axes. But it is necessary to be careful if this property is not kept strictly.

It should be stressed that the actuated joint compliances cannot be identified separately. So, they should be included in the compliance matrix of the previous link by means of modification of the corresponding diagonal elements. This idea does not contradict to the physical nature of the problem even if the actuated joint compliance dominates corresponding compliance in the link stiffness matrix.

M3:Aggregation. Eliminate from the set of model parameters the ones that corresponds to joint compliances before which there is an elastic link; in terms of parameters identifiability the compliance of those joints cannot be split from the links.

Summarizing these methods, it should be mentioned that the above presented approach essentially reduce the number of parameters to be identified (by the factor 4.5) but they do not violate such basic properties as the mode completeness, i.e. the ability to describe any deflection caused by the external loading. Below, these reduced set of the original model parameters $\boldsymbol{\pi}$ will be referred to as $\boldsymbol{\pi}'$. However, the obtained reduced model may still have some redundancy in the frame of entire manipulator, where the virtual springs of adjacent joints/actuators cause similar impact on the end-effector deflections under the loading.

As it known from the geometrical calibration, in spite of the fact that redundant model is suitable for direct and inverse computations, it cannot be used in identification since the observation matrix does not have sufficient rank. Similar problem arises in elastostatic calibration where some stiffness matrix elements of adjacent links/joints are coupled and cannot be identified separately. Let us present an algebraic technique allowing to overcome this difficulty.

3.2 Algebraic Approach

The physical approach described in the previous sub-section allows us essentially reducing the number of model parameters. However, it does not guarantee that the obtained model is suitable for calibrations (i.e. that the model is non-redundant and the number of parameters is equal to the observation matrix rank). In practice, the following inequality is often satisfied: $rank(\mathbf{A}_{\pi'}) < \dim(\boldsymbol{\pi}')$. To overcome the problem, this sub-section presents some algebraic tools aimed at further reduction of the model parameter set from $\boldsymbol{\pi}'$ to $\boldsymbol{\pi}''$, which ensures full identifiability:

$$rank(\mathbf{A}_{\pi'}) = rank(\mathbf{A}_{\pi''}) = \dim(\boldsymbol{\pi}'') \quad (14)$$

These tools are based on the partitioning of the parameters set $\boldsymbol{\pi}'$ into three non-overlapping groups (identifiable, non-identifiable and semi-identifiable), which are either eliminated from the model or reduced to ensure the equality (14).

To introduce relevant algebraic technique, let us apply the SVD decomposition and present the aggregated observation matrix $\mathbf{A}_{\pi'}$ as the product of three matrices $\mathbf{U} \cdot \boldsymbol{\Sigma} \cdot \mathbf{V}^T$ (orthogonal, diagonal and orthogonal, respectively):

$$\mathbf{A}_{\pi'} = [\mathbf{U}_1, ... \mathbf{U}_m] \cdot \left[\begin{array}{c|c} diag(\sigma_1, .. \sigma_r) & \mathbf{0}_{r \times n'} \\ \hline \mathbf{0}_{m' \times r} & \mathbf{0}_{m' \times n'} \end{array}\right] \cdot \begin{bmatrix} \mathbf{V}_1^T \\ \cdots \\ \mathbf{V}_n^T \end{bmatrix} \quad (15)$$

Here $\mathbf{U}=[\mathbf{U}_1,...\mathbf{U}_m]$ and $\mathbf{V}=[\mathbf{V}_1,...\mathbf{V}_v]$ are orthogonal matrices of the size $m\times m$ and $n\times n$ respectively whose columns are denoted as $\mathbf{U}_i$ and $\mathbf{V}_j$; the second factor $\mathbf{\Sigma}$ is a rectangular diagonal matrix of the size $m\times n$ containing r positive real numbers $\sigma_1,...\sigma_r$ in descending order; $m=\dim(\Delta\mathbf{t}_a)$ is the number of rows in the observation matrix (i.e. number of equations used for the identification), $n=\dim(\boldsymbol{\pi}')$ is current number of the model parameters, and r is the rank of the aggregated observation matrix, $m'=m-r$, $n'=n-r$. It is clear that r defines the maximum number of parameters that can be identified using given set of manipulator configurations $\{\mathbf{q}_i\}$ and corresponding wrenches $\{\mathbf{W}_i\}$.

Further, after substitution (15) into (11) and left-multiplication by $\mathbf{U}^T$, the original system of m identification equations (4) can be rewritten as

$$\left[\begin{array}{ccc|c}\sigma_1 & \cdots & 0 & \\ \vdots & \ddots & \vdots & \mathbf{0}_{r\times n'} \\ 0 & \cdots & \sigma_r & \\ \hline \multicolumn{3}{c|}{\mathbf{0}_{m'\times r}} & \mathbf{0}_{m'\times n'}\end{array}\right]\cdot\begin{bmatrix}\mathbf{V}_1^T\\ \cdots\\ \mathbf{V}_n^T\end{bmatrix}\cdot\boldsymbol{\pi}'=\begin{bmatrix}\mathbf{U}_1^T\\ \cdots\\ \mathbf{U}_m^T\end{bmatrix}\cdot\Delta\mathbf{t}_a \quad (16)$$

where the number of equation is equal to n and perfectly corresponds to the vector $\boldsymbol{\pi}'$ dimension (it is obvious that $n\le m$). Taking into account particularities of the sparse matrix $\mathbf{\Sigma}$ (with r non-zero elements only), it is possible to rewrite the system (16) in the form

$$\begin{aligned}&\sigma_i\cdot\mathbf{V}_i^T\cdot\boldsymbol{\pi}'=\mathbf{U}_i^T\cdot\Delta\mathbf{t}_a; \qquad i=1,2,...,r\\ &0\cdot\mathbf{V}_i^T\cdot\boldsymbol{\pi}'=\mathbf{U}_i^T\cdot\Delta\mathbf{t}_a; \qquad i=r+1,...,n\end{aligned} \quad (17)$$

where the second group of $m'=m-r$ equations should be excluded from further consideration because relevant residuals do not depend on the parameters of interest $\boldsymbol{\pi}'$ (since they are multiplied by zero matrix). It can be proved that $\mathbf{U}_i^T\cdot\Delta\mathbf{t}_a\equiv\mathbf{0}$ for $i>r$ if the measurement vector $\Delta\mathbf{t}_a$ does not contain noise. It is also worth mentioning that for real identification problems (with the measurement noise), the second group of equations produces constant residuals that cannot be minimised in the least-square objective (7) by varying the vector of unknown parameters $\boldsymbol{\pi}'$.

Hence, for the identification of n parameters included in the vector $\boldsymbol{\pi}'$, a system of r linear equations have been obtained that cannot be solved uniquely in a general case. Its partial solution can be found by dividing on $\sigma_i>0$ each of r linear equations $\sigma_i\cdot\mathbf{V}_i^T\cdot\boldsymbol{\pi}'=\mathbf{U}_i^T\cdot\Delta\mathbf{t}_a$ and further straightforward multiplication of the left and right sides by the matrix $[\mathbf{V}_1,\mathbf{V}_2,...,\mathbf{V}_r]$, which yields

$$[\mathbf{V}_1,...,\mathbf{V}_r]\begin{bmatrix}\mathbf{V}_1^T\\ ...\\ \mathbf{V}_r^T\end{bmatrix}\cdot\boldsymbol{\pi}'=[\mathbf{V}_1,...,\mathbf{V}_r]\begin{bmatrix}\mathbf{U}_1^T/\sigma_1\\ ...\\ \mathbf{U}_r^T/\sigma_r\end{bmatrix}\cdot\Delta\mathbf{t}_a \quad (18)$$

Using the first set of r equations of system (17) one can obtain partial solution of system (16)

$$\boldsymbol{\pi}'_0=\sum_{i=1}^{r}\left(\sigma_i^{-1}\,\mathbf{V}_i\cdot\mathbf{U}_i^T\right)\cdot\Delta\mathbf{t}_a \quad (19)$$

This allows us to present the general solution (9) as the sum of this partial solution and an arbitrary vector from the subspace with the basis $\mathbf{V}_{r+1}$, $\mathbf{V}_{r+2},...\mathbf{V}_n$

$$\hat{\boldsymbol{\pi}}'=\boldsymbol{\pi}'_o+\sum_{i=r+1}^{n}\lambda_i\mathbf{V}_i \quad (20)$$

where λ_i, $i=\overline{r+1,n}$ are arbitrary real values.

Hence, as follows from analysis of (17) and (19), depending on the properties of the matrix $\mathbf{V}$, all model parameters $\boldsymbol{\pi}'$ can be partitioned into three groups: G1 – *identifiable* parameters that are uniquely defined by the equation (20) and do not depend on the arbitrary values λ_i, for these parameters the corresponding row of the sub-matrix $[\mathbf{V}_{r+1},...,\mathbf{V}_m]$ is equal to zero; G2 – *non-identifiable* parameters that do not effect the residuals of system (17), for these parameters the corresponding row of the sub-matrix $[\mathbf{V}_1,...,\mathbf{V}_r]$ is equal to zero; G3 – *semi-identifiable* parameters that effect the residuals but cannot be identified uniquely, couplings between these parameters is defined by the vectors $\mathbf{V}_i$, $i=\overline{1,r}$. Thus, based on this decomposition, the algebraic-based model reduction techniques can be formalised in the following way:

M4a:Partitioning. Divide the reduced set of the model parameters $\boldsymbol{\pi}'$ into three non-overlapping groups G1, G2 and G3 in accordance with the following rules applied to all π'_i, $i=\overline{1,\dim(\boldsymbol{\pi}')}$:

Rule 1: Include the parameter π'_i into the group G1 if the *i*th row of the sub-matrix $[\mathbf{V}_{r+1},...,\mathbf{V}_m]$ is equal to zero;

Rule 2: Include the parameter π'_i into the group G2 if the *i*th row of the sub-matrix $[\mathbf{V}_1,...,\mathbf{V}_r]$ is equal to zero;

Rule 3: If the parameter π'_i is not included in G1 or G2, include it in the group G3.

M4b:**Elimination.** Eliminate from the set of unknowns (model parameters) non-identifiable parameters that correspond to group G2.

After application of these methods, the current set of model parameters $\boldsymbol{\pi}'$ is reduced to the sub-set $\{\boldsymbol{\pi}' \setminus \boldsymbol{\pi}_{G2}\}$ that does not influence the rank of the observation matrix, i.e. $rank\left(\mathbf{A}_{\boldsymbol{\pi}' \setminus \boldsymbol{\pi}_{G2}}\right) = rank\left(\mathbf{A}_{\boldsymbol{\pi}'}\right)$. Nevertheless, relevant model may be redundant yet, i.e. $rank\left(\mathbf{A}_{\boldsymbol{\pi}' \setminus \boldsymbol{\pi}_{G2}}\right) < \dim\left(\{\boldsymbol{\pi}' \setminus \boldsymbol{\pi}_{G2}\}\right)$. It should be noted that $rank\left(\mathbf{A}_{\boldsymbol{\pi}_{G1}}\right) = \dim\left(\boldsymbol{\pi}_{G1}\right)$ while $rank\left(\mathbf{A}_{\boldsymbol{\pi}_{G3}}\right) < \dim\left(\boldsymbol{\pi}_{G3}\right)$. So, another, and the most difficult problem that arises after M4, is to define the sub-set of identifiable parameters inside of $\boldsymbol{\pi}_{G3}$ (the remaining ones should be set to constant values).

It is clear that the above mentioned problem has infinite number of solutions. Let us presents an algorithm that is able to split the set of parameters $\boldsymbol{\pi}_{G3}$ into the non-overlapping groups of coupled parameters $\boldsymbol{\pi}^j_{G3}$ and then choose identifiable one from the group based on their physical scene:

M5a:Splitting. Split the set of semi-identifiable model parameters $\boldsymbol{\pi}_{G3}$ into the non-overlapping groups of coupled parameters $\boldsymbol{\pi}^j_{G3}$ for which the following conditions are satisfied:

(a) $\boldsymbol{\pi}_{G3} = \boldsymbol{\pi}^1_{G3} \bigcup \boldsymbol{\pi}^2_{G3} \bigcup .... \boldsymbol{\pi}^m_{G3}$, $\boldsymbol{\pi}^i_{G3} \bigcap \boldsymbol{\pi}^j_{G3} = \varnothing$ $\forall\ i \neq j$;

(b) $rank\left(\mathbf{A}_{\boldsymbol{\pi}^i_{G3}}\right) = rank\left(\mathbf{A}_{\boldsymbol{\pi}^i_{G3} \setminus \boldsymbol{\pi}^i_{G3}(j)}\right)$ $\forall\ j = 1 : \dim(\boldsymbol{\pi}^i_{G3})$

(c) $rank\left(\mathbf{A}_{\boldsymbol{\pi}^i_{G3}}\right) < rank\left(\mathbf{A}_{\boldsymbol{\pi}^i_{G3} \cup \boldsymbol{\pi}^j_{G3}(k))}\right)$ $\forall\ i \neq j, k = 1 : \dim(\boldsymbol{\pi}^j_{G3})$

In practice, when this grouping is not evident, it is possible to use numerical technique, which is based on the SVD-decomposition of the reduced observation matrix $\mathbf{A}_{\boldsymbol{\pi}_{G3}}$. Using similar notation, the matrix $\mathbf{V}$ can be presented as $\mathbf{V} = [\mathbf{V}_1, \mathbf{V}_2, ...]$ in accordance with the rank of $\mathbf{A}_{\boldsymbol{\pi}_{G3}}$. So, the couplings between the elements are defined by the sub-matrix $[\mathbf{V}_1, ..., \mathbf{V}_r]$. One of the easiest ways to find the desired couplings is to compute the matrix

$$\mathbf{L} = \left([\mathbf{V}_1, ..., \mathbf{V}_r]^*\right)^{-1} \cdot \begin{bmatrix} \mathbf{V}_1^T \\ ... \\ \mathbf{V}_r^T \end{bmatrix} \quad (21)$$

where the symbol "*" denotes operation of the row selection that conserve the matrix rank. The latter leads to a full-rank square matrix presented above as the first term of (21). It should be noted that this operation is not unique, nevertheless, it allows to obtain the couplings between the model parameters described by the sparse matrix $\mathbf{L}$. Then, the desired groups of parameters can be easily detected after transformation $\mathbf{L}$ into the block-diagonal form.

Using the above presented idea, the next step can be presented as follows:

M5b:Selection. In each group of parameters $\boldsymbol{\pi}^j_{G3}$, specify $n_j = rank\left(\mathbf{A}_{\boldsymbol{\pi}^j_{G3}}\right)$ parameters that will be treated as identifiable

M5c:Assigning. In each group of parameters $\boldsymbol{\pi}^j_{G3}$, fix remaining $m_j = \dim\left(\boldsymbol{\pi}^j_{G3}\right) - rank\left(\mathbf{A}_{\boldsymbol{\pi}^j_{G3}}\right)$ parameters to some constants; these parameters will be treated as non-identifiable

It should be noted that the sequence of methods M5b and M5c is not strict; identifiable and non-identifiable parameters can be selected and fixed iteratively, using the methods M5b and M5c several times. After application of the methods M5a, M5b and M5c, the set of parameters $\boldsymbol{\pi}_{G3}$ is split into two subsets: the subset of the parameters that will be treated as identifiable $\boldsymbol{\pi}^{id}_{G3}$ and subset that will be treated as non identifiable ones $\boldsymbol{\pi}^{ni}_{G3}$ and will be assigned to some constant values ($\boldsymbol{\pi}^{ni}_{G3} = const$); i.e. $\boldsymbol{\pi}^{id}_{G3} \bigcup \boldsymbol{\pi}^{ni}_{G3} = \boldsymbol{\pi}_{G3}$, $\boldsymbol{\pi}^{id}_{G3} \bigcap \boldsymbol{\pi}^{ni}_{G3} = \varnothing$.

After application of the algebraic approach, the complete set of parameters $\boldsymbol{\pi}'$ is reduced to $\boldsymbol{\pi}''$, It includes all parameters from the group G1 and assigned-to-be-identifiable ones from the group G3. It is clear that the presented algebraic methods do not violate the model completeness, i.e. $rank\left(\mathbf{A}_{\boldsymbol{\pi}''}\right) = rank\left(\mathbf{A}_{\boldsymbol{\pi}'}\right)$.

4 APPLICATION EXAMPLE

To demonstrate benefits of the developed techniques for industrial applications, this section presents some experimental results on the elastostatic calibration of industrial robot Kuka KR-270 employed in high pre-

cession machining of aircraft parts.. For the considered application area, the technological process generates essential interaction between the workpiece and manipulator, which causes non-negligible deflections of the end-effector. To compensate related positioning errors on the control level (via adjusting a target trajectory), an accurate but simple enough elasto-static model is required. In practice, the desired model is not usually provided by robot manufactures and should be obtained from dedicated experimental study. Let us apply the developed technique to get the desired model and to identify its parameters in real industrial environment.

The considered manipulator contains 7 links separated by 6 actuated joints. Taking into account that in general the elastostatic properties of each link are defined by 6x6 stiffness matrix, the complete but obviously redundant model contains 258 parameters. As a result of application model reduction techniques (M1-M5), the number of parameters to be identified has been reduced down to 26. More details on each step are given in 0. Additional restrictions here are caused by the partial-pose measurement technique and the gravity-based loading generating the desired deflections. Relevant experimental setup is presented in 0. Because of such measurement method, 10 elastostatic parameters are not identifiable from the available measurement data. The manipulator configurations for the elastostatic calibration were generated using the design of experiments and previously developed test-pose technique, which is based on the industry-oriented performance measure (Klimchik, Wu et al., 2012). Another particularity of the industrial robot KUKA KR270 that should be taken into account in an accurate elasto-staic model is a gravity compensator that is attached in parallel to the second actuated joint. Its equivalent model was presented in (Klimchik, Wu et al., 2013). In the frame of the complete and irreducible model, the gravity compensator impact is taken into account by introducing a configuration dependant virtual spring in the second joint. More details on this approach are given in (Klimchik, Wu et al., 2013).

In order to ensure higher identification accuracy, the measurement configurations have been selected using the design of experiment theory. In contrast to other works, an industry-oriented performance measure has been used (Wu, Klimchik et al., 2013), which evaluates the robot positioning accuracy after calibration. In total, 15 measurement configurations for 5 different angles q_2 have been generated.

For the comparison purposes, calibration was performed using several elastostatic models that differ in their basic assumptions: (i) complete irreducible stiffness model and (ii) conventional model for the manipulator with rigid links and compliant actuated joints. Here, conventional elastostatic models J1 and J2 take into account the actuated joint compliances only. Both models (complete and reduced ones) have been examined with and without taking into account the effect of the gravity compensator. The obtained results are summarized in 0 showing capability to compensate the compliance errors using different elastostatic models. As follows from them, the lowest compliance errors can be achieved using the model C2 (obtained using the developed model reduction technique), which ensures the positional accuracy 0.21 mm. In contrast, the conventional elasto-static model with rigid links gives accuracy 3.5 times worse comparing to model C2. The difference in the efficiency of the compliance errors compensation between the models J1/J2 and C1/C2 confirms that link compliances have essential impact on the robot positioning accuracy and cannot be neglected in accurate manufacturing. According to experimental results presented in 0, by means of the complete elastostatic model, it is possible to compensate 95% of the compliance errors. In contrast, using the conventional model that takes into account the joint elasticity's merely, only 84% of positioning errors caused by external force can be compensated. This emphasizes the advantage of the proposed model. The histograms of the errors distribution for the model C2 (0) show that the non-compensated compliance errors in all directions are unbiased and almost normally distributed.

Hence, using the developed low-order stiffness model for the compliance error compensation gives essential improvement of the precision for the robotic based milling. It allowed us to compensate more than 95% of deflections caused by external loading and to guarantee the precision of about 0.2 mm under the loading of 2.5 kN (it is comparable with the robot repeatability of 0.06 mm).

5 CONCLUSION

The paper deals with the problem of the manipulator stiffness modeling. The main attention is paid to the elastostatic parameters identification and model reduction. In contrast to previous works, the manipulator stiffness properties are described by the sophisticated model, which takes into account the flexibilities of all mechanical elements described by 6×6 stiffness matrices. This obviously yields extremely high number of the model parameters that cannot be

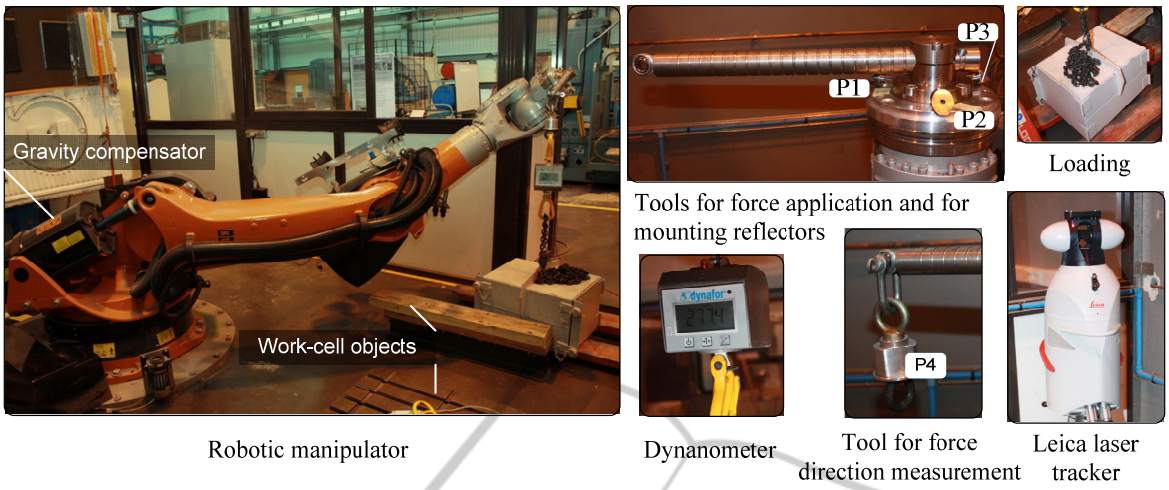

Figure 2: Experimental setup for elastostatic calibration.

Table 1: Summary of the elasto-static model reduction process for industrial robot Kuka KR-270.

Approach	Step	Model description	Number of parameters
Original model		6 joints +7 links (36 parameters per link)	258
Physical	M1: Symmetrisation	6 joints +7 links (21 parameters per link)	153
	M2: Sparcing	6 joints +7 links (8 parameters per link)	62
	M3: Aggregation	7 links (8 parameters per link)	56
Algebraic	M4a:Partitioning M4b:Elimination	G1: Identifiable parameters – 1 G2: Non-identifiable parameters - 10 G3: Semi-identifiable parameters – 45	46
	M5a:Splitting M5b:Selection M5c:Assigning	Selection of 25 independent parameters from 45 semi-identifiable ones	26

Table 2: Efficiency of the compliance errors compensation using complete and reduced models.

Stiffness model	Number of parameters	Compliance errors, mm							
		x-direction		y-direction		z-direction		positional	
		MAX	RMS	MAX	RMS	MAX	RMS	MAX	RMS
Deflections magnitude without compensation		2.51	1.03	3.14	1.02	8.14	1.91	8.18	4.58
Complete model C1	26	0.27	0.10	0.43	0.13	0.38	0.12	0.45	0.22
Complete model C2	30	0.28	0.10	0.45	0.14	0.32	0.11	0.49	0.21
Conventional model J1	5	1.42	0.43	1.73	0.41	0.66	0.23	1.78	0.75
Conventional model J2	9	1.42	0.42	1.73	0.42	0.49	0.19	1.76	0.73

Model C1: Complete irreducible stiffness model without gravity compensator

Model C2: Complete irreducible stiffness model with gravity compensator

Model J1: Conventional model for the manipulator with rigid links and compliant actuators, without gravity compensator

Model J2: Conventional model for the manipulator with rigid links and compliant actuators, with gravity compensator

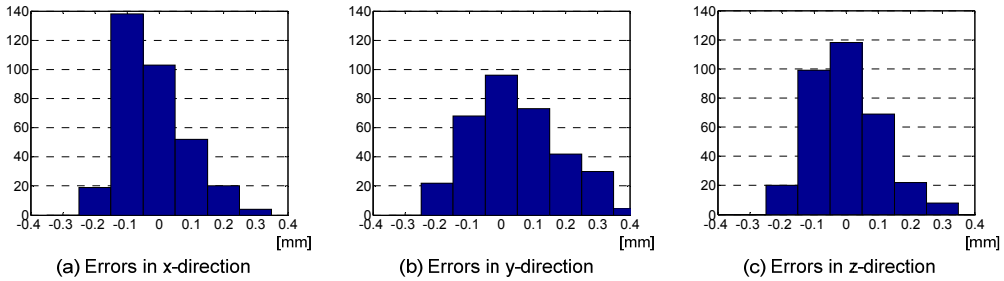

Figure 3: Statistical distribution of compliance errors after compensation

identified separately. To solve the problem, physical and algebraic model reduction methods were developed. They take into account mathematical relations between the elements of the compliance matrices. The advantages of the developed approach are illustrated by an application example that deals with elastostatic calibration of an industrial robot used in aerospace industry.

In future, the problem of the complete model generation from the obtained set of parameters will be in the focus of our attention, i.e. re-construction of the joint compliances and 6x6 link stiffness matrices from the reduced model obtained after the identification. It is clear that this procedure requires additional knowledge on the coupling between the stiffness matrix elements that are induced by their physical nature.

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