A Mixed-Integer Mathematical Programming Model for Integrated Planning of Manufacturing and Remanufacturing Activities

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Abstract: This paper considers a hybrid remanufacturing and manufacturing system on a closed-loop supply chain. The system manufactures a set of new products characterized by a multi-level structure through multi-stage assembly operations. The required raw or basic parts can be acquired new from suppliers or provided as new by a de-manufacturing facility which performs a remanufacturing process from acquired old products returned by customers. The quality of returned products has impact on the quantity of recovered basic parts which can be assumed as good as new, and on the duration of the remanufacturing process. The considered problem is to determine the production lots for the system machines as well as the quantity of new basic parts and retuned products to be acquired in order to satisfy a deterministic demand in the time buckets of the planning period. The performance criterion to be minimized includes the acquisition costs for the new and returned items, inventory and production costs, recovering and disposal costs, and tardiness costs. A mixed-integer programming model is proposed and its effectiveness is demonstrated by experiments on a case study.

1 INTRODUCTION

In the recent years, both academic and practitioner interest has been focused on closed loop or reverse logistic supply chain management (Guide Jr. and Van Wassenhove, 2009), due to the increasing attention given to environmental issues, in particular relevant to the depletion of natural resources, as well as the new environmental legislations and economic concerns. In this connection, the increasing awareness of customers for environmental sustainable products also drives industries to adapt their resource consumption and supply strategies.

For these reasons, product recovery became a relevant factor that must be appropriately managed in closed loop supply chains, both at planning and operational level (Guide Jr., 2000). In general, product recovery consists in several alternative options aiming at exploiting end-life finished products returned from customers, so drastically reducing the flow of such items to land-fill or incineration (Thierry et al., 1995). In particular, returned products can be repaired or refurbished to be reintroduced in the market, as well as remanufactured (several example can be found in (Fleischmann et al., 1997)). With remanufacturing, old products are disassembled in order to recover reusable components and materials so that they can be used for new product fabrication (Zhang et al., 2004).

In this paper, a remanufacturing and manufacturing system is considered. The system produces several kinds of finished products from the assembly of new or recovered basic parts, which can be made available from a disassembly process. Specifically, the planning of manufacturing and remanufacturing activities must be determined in order to satisfy the demand of finished products over a specified planning horizon. The production is performed by a set of machines or production lines that are dedicated to the execution of the different phases of the production cycle of products, whereas a single demanufacturing facility is assumed through which the returned products are disassembled into raw or basic parts that are restored to as-new state. Usable raw parts provided by the disassembly process are stored in inventory together with new parts that are acquired, whereas not usable ones are sent to disposal. Returned products can be acquired at a cost depending on their quality level; this latter, in turn, influences the number of obtainable recovered raw/basic parts but not their quality that is assumed equivalent to that of new parts (Lund, 1985). The planning is performed at an aggregate level on daily or weekly time buckets, so that only production capacity restrictions are taken into account, whereas setups for the machines are not

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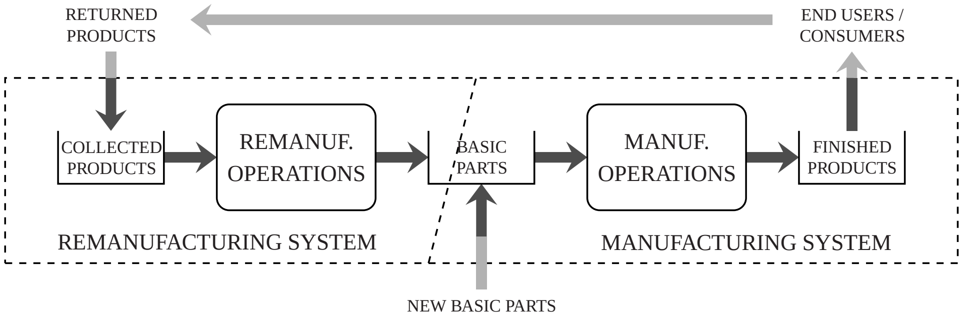

Figure 1: The system model.

explicitly considered. The demand of finished products in the time buckets of the planning horizon is assumed known and an upper bound on the number of the available returned products is considered. Plans should minimize costs due to the inventory of basic parts, semi-finished and finished products, as well as costs for basic parts and returned products acquisition, for disassembly and production, and for late satisfaction of the demand with respect to the due dates.

The problem faced in this paper then consists in determining the size of the production lots on the considered machines, together with the quantities of new basic parts and returned products to be acquired in the various time buckets. Dynamic lot sizing problems without considering product returns have been widely considered in literature. Recently, in (Teunter et al., 2006) the dynamic lot sizing problem with returns has been introduced. The problem considered here is a multi-item, multi-stage lot sizing problem where setups are assumed negligible or included in the processing times (similar assumptions with no returns are made in (Chen and Ji, 2007) or in (Tonelli et al., 2013)). On the other hand, this problem can be also considered as a flexible assembly scheduling problem with component availability constraints (Kolisch and Hess, 2000), where in general more than a single machine can be used to perform an operation. In addition, as the planning decisions that must be taken include also the quantity of returned products to acquire, this problem is similar to the one in (Shi et al., 2011), where the authors aim at determining the price for the returns considering an uncertain demand.

2 THE SYSTEM MODEL

In the considered class of production systems, manufacturing activities transform basic parts into finished products through the execution of transformation or assembly operations, and remanufacturing activities recover basic parts from returned products and restore them to as-new state. The system model is schematized in Figure 1. The finished products are bought and used by consumers, and, after some time, some of them are returned for remanufacturing. The remanufacturing system collects returned products, if available, on the basis of its needs, and starts its remanufacturing operations to recover basic parts. It is assumed that each operation which recovers parts lasts a variable amount of time, depending on the quality of the returned product. Once recovered, the basic parts are as new and are inserted into the inventory of basic parts of the manufacturing system. The system produces finished products in accordance with an external demand to be satisfied.

In this work it is assumed that all the values characterizing the dynamics of the system are deterministic. The external demand, which varies over the time, is known and deterministic, as well as the maximum number of products which are returned in the various periods of time. Moreover, it is assumed that an unlimited number of new basic parts can be bought in order to satisfy the production requirements. Nevertheless, the demand could be not promptly satisfied due to the finite capacity of resources.

2.1 Planning of Activities

The objective of this paper is to define a mathematical model for integrated planning of manufacturing and remanufacturing activities. In this respect, a discrete-time dynamics is taken into account. Let t be the generic time bucket (or, simply, bucket) where manufacturing and remanufacturing operations are accomplished (usually, day or week, depending on the kind of production); $t = 1, \dots, T$, being T the planning horizon. The planning of activities consists in determining, for each bucket, the following quantities:

- amount of components and finished products to be manufactured,
- quantity of new basic parts to be bought,

- amount of basic parts to be recovered from returned products,
- quantity of returned products to be acquired

so that a performance criterion, which basically takes into account production, tardiness, and inventory costs for the manufacturing part and purchasing, recovery, and disposal costs for the remanufacturing part, is minimized. Such an integrated planning problem is modelled as a mixed-integer mathematical programming problem.

2.2 Manufacturing Operations

The structure of the manufacturing operations is defined by the bill of materials (BOM), as the one illustrated in Figure 2 in which 3 types of finished products are carried out starting from a set of 8 kinds of basic parts.

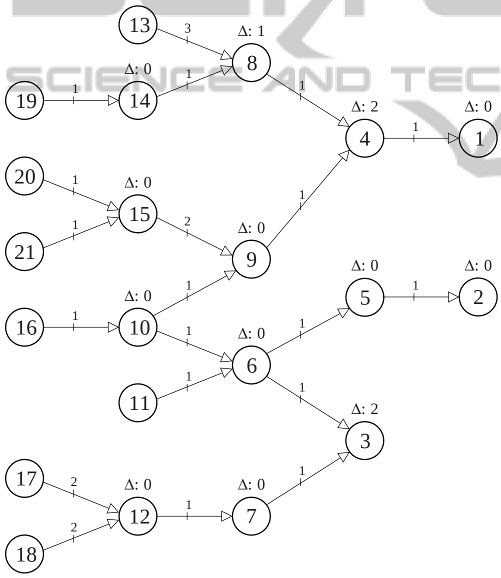

Figure 2: Bill of materials, with lead times and required parts (example).

The following sets and values characterizing the manufacturing system are defined:

- F, C, and B are, respectively, the set of finished products, the set of intermediate components manufactured at various level of the BOM, and the set of basic parts;
- $D_{p,t}$ is the (external) demand of product $p \in F$ in time bucket $t \in \{1,\dots,T\}$;
- $\gamma_{q,p}$ is the quantity of parts $q \in B \cup C$ which are needed to produce one unit of part $p \in C \cup F$;
- Δ_p is the lead time for the production of component or product $p \in C \cup F$; $\Delta_p = \{0,1,\dots\}$, where $\Delta_p = 0$ means that p is available in the same interval in which the production of p starts;
- $i_{p,0}$ is the quantity of parts $p \in B \cup C \cup F$ in the inventory at the beginning (initial inventory level).

The manufacturing operations are carried out by machines or production lines with eligibility constraints, i.e., a part (both intermediate components and finished products) can be produced by using one of a set of compatible machines. Then, the following sets, parameters and variables are defined:

- M is the set of machines (or production lines);
- M_p is the set of machines compatible with the production of component or product $p \in C \cup F$; $M_p \subseteq M$;
- P_m is the set of components and products that can be produced on machine $m \in M$; $P_m \subseteq C \cup F$;
- $\Phi_{m,t}$ is the total production capacity of machine $m \in M_p$ in time bucket $t \in \{1,\dots,T\}$.

The unitary costs to produce components and products, to store parts, and to buy new basic parts are the following:

- $u^{\mathrm{P}}_{p,m,t}$ is the cost to produce a unit of product $p \in C \cup F$ on machine $m \in M_p$ in time bucket $t \in \{1,\dots,T\}$;
- $u^{\mathrm{T}}_{p,h,t}$ is the cost due to a late production of a unit of product $p \in F$, when it is due at time bucket $t \in \{1,\dots,T\}$ but it is produced in time bucket $h \in \{t+1,\dots,T+1\}$; in the proposed model it is assumed:
$$u^{\mathrm{T}}_{p,h,t} = u^{\mathrm{L}}_{p} \cdot (h-t)^{\beta} \tag{1}$$
where
  - u^{L}_{p} is the cost paid when a unit of product $p \in F$ which is required in a certain bucket is produced in the subsequent bucket (it is the unitary tardiness cost);
  - β is a given parameter;
- u^{U}_{p} is the cost for not producing a unit of product $p \in F$ in the planning horizon;
- $u^{\mathrm{IP}}_{p,t}$ is the inventory cost which is paid for storing a unit of part $p \in B \cup C \cup F$ in time bucket $t \in \{1,\dots,T\}$;
- $u^{\mathrm{N}}_{p,t}$ is the cost for acquiring a unit of new basic part $p \in B$ in time bucket $t \in \{1,\dots,T\}$.

Finally, the variables which characterize the manufacturing system are the following:

- $x_{p,m,t}$ (integer variable) is the quantity of components or products $p \in C \cup F$ produced on $m \in M_p$ in the interval from time bucket $t \in \{1,\dots,T\}$ to bucket $t + \Delta_p$ (i.e., the quantity of components or products available at the end of bucket $t + \Delta_p$);

- $z_{p,t}$ (integer variable) is the quantity of finished products $p \in F$ produced before bucket $t \in \{1,\ldots,T\}$ (thus available in the inventory) or completed in t used for (partially) satisfy the demand of p in bucket t;
- $l_{p,h,t}$ (integer variable) is the quantity of products $p \in F$ which are produced in a bucket $h \in \{t+1,\ldots,T+1\}$ (subsequent to bucket $t \in \{1,\ldots,T\}$) and used to (partially) satisfy the demand of p in bucket t (backorder); the demand produced in bucket $T+1$ corresponds to not satisfied demand;
- $a^{\mathrm{N}}_{p,t}$ (integer variable) is the quantity of new basic parts $p \in B$ acquired in time bucket $t \in \{1,\ldots,T\}$;
- $i_{p,t}$ (integer variable) is the quantity of parts $p \in B \cup C \cup F$ in the inventory at the end of bucket $t \in \{1,\ldots,T\}$.

2.3 Remanufacturing Operations

The remanufacturing operations are defined by specifying, for each of the products that can be returned, the list of basic parts that can be retrieved. Thus, in this work, an inverse BOM is not defined for the remanufacturing system. A remanufacturing process for the system characterized by the BOM in Figure 2 can be that illustrated in Figure 3 in which the returned products of type 1 and 2 are disassembled to provide basic parts of type 11, 16, 19, 20, and 21.

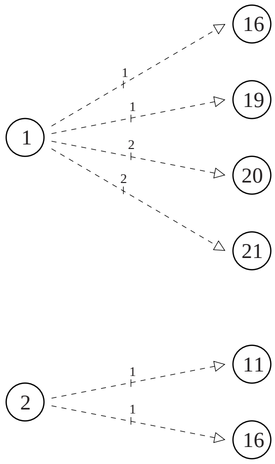

Figure 3: Remanufacturing process, with retrieved quantities (example).

The duration of a remanufacturing operation is a function of the quality of the returned product. The quality is defined through an integer level $v \in \{1,\ldots,V\}$, being V the number of different quality levels that are considered. The best (respectively, the worst) quality corresponds to level 1 (resp., V). The quality of a returned product also affects the successfulness of the remanufacturing activities: it is here assumed that the worst is the quality level of the returned product, the higher is the probability of obtaining basic parts that cannot be restored to as-new state (in the current deterministic model, this aspect is modelled through given percentages of parts that can be recovered).

The following sets and values characterizing the remanufacturing system are defined:

- R is set of finished products that can be returned; $R \subseteq F$;
- R_p is the set of returnable finished products from which the basic part $p \in B$ can be recovered; $c \in IC$, $R_p \subseteq R$
- B_r is the set of basic parts that can be obtained by product $r \in R$ through a remanufacturing process; $B_r \subseteq B$;
- B^{R} is the set of initial components that can be obtained through a remanufacturing process; $B^{\mathrm{R}} = \{p \in B : \exists r \in R : p \in B_r\}$;
- $G^v_{r,p}$ is the maximum amount of finished products $r \in R$ with quality $v \in \{1,\ldots,V\}$ that are returned in time bucket $t \in \{1,\ldots,T\}$;
- $\delta_{r,p}$ is the quantity of basic part $p \in B_r$ that can be recovered from one unit of product $r \in R$;
- $\rho^v_{r,p}$ is the percentage of component $p \in B_r$ that can be recovered "as new" from a returned product of family $r \in R$ with quality $v \in \{1,\ldots,V\}$;
- $\Gamma^v_{r,p}$ is the lead time for recovering basic part $p \in B_r$ from finished product $r \in R$ with quality $v \in \{1,\ldots,V\}$; $\Gamma^v_{r,p} = \{0,1,\ldots\}$, where $\Gamma^v_{r,p} = 0$ means that p is available in the same interval in which the remanufacturing process recovering p from r starts.
- $i^{\mathrm{R},v}_{r,0}$ is the quantity of returned products $r \in R$ with quality $v \in \{1,\ldots,V\}$ in the inventory at the beginning (initial inventory level).

As regards remanufacturing activities, it is assumed the presence of a single dedicated machine (i.e., not used for manufacturing operations) characterized by:

- Ψ_t, the total capacity of the remanufacturing machine in time bucket $t \in \{1,\ldots,T\}$.

The unitary costs to purchase returned product, to recover basic parts, and to disposal bad parts are the following:

- $u^{\mathrm{A},v}_{r,t}$ is the cost for acquiring a unit of product $r \in R$ with quality $v \in \{1,\ldots,V\}$ in time bucket $t \in \{1,\ldots,T\}$;

- $u^{\mathrm{IR}}_{r,t}$ is the inventory cost which is paid for storing a unit of product $r \in R$ in time bucket $t \in \{1,\dots,T\}$;
- $u^{\mathrm{R},v}_{r,p,t}$ is the cost for recovering a unit of basic part $p \in B_r$ from a unit of returned product $r \in R$ with quality $v \in \{1,\dots,V\}$ in time bucket $t \in \{1,\dots,T\}$;
- $u^{\mathrm{D},v}_{r,p,t}$ is the cost for recovering and disposal parts for a unit of basic part $p \in B_r$ from a unit of returned product $r \in R$ with quality $v \in \{1,\dots,V\}$ in time bucket $t \in \{1,\dots,T\}$;

Finally, the variables which characterize the remanufacturing system are the following:

- $a^{\mathrm{B}}_{p,t}$ (integer variable) is the quantity of recovered basic parts $p \in B_r\ \forall r \in R$ that become available in time bucket $t \in \{1,\dots,T\}$;
- $a^{\mathrm{A},v}_{r,t}$ (integer variable) is the quantity of returned products $r \in R$ with quality $v \in \{1,\dots,V\}$ acquired in time bucket $t \in \{1,\dots,T\}$;
- $s^{v}_{r,t}$ (integer variable) is the quantity of returned products $r \in R$ with quality $v \in \{1,\dots,V\}$ that start the remanufacturing process in time bucket $t \in \{1,\dots,T\}$;
- $i^{\mathrm{R},v}_{r,t}$ (integer variable) is the quantity of returned products $r \in R$ with quality $v \in \{1,\dots,V\}$ in the inventory at the end of bucket $t \in \{1,\dots,T\}$.

3 THE MATHEMATICAL MODEL

The integrated planning of manufacturing and remanufacturing activities is modelled as a mixed-integer mathematical programming (MIP) problem and it is solved through standard methodologies. In the following, the objective function to be minimized and the constraints of the problem are reported.

3.1 The Mathematical Programming Problem

Let w_{P}, w_{T}, w_{U}, w_{I}, w_{N}, w_{A}, w_{R}, and w_{D} be the weights for the objective function components.

3.1.1 Objective Function

$$\begin{aligned}\min \quad & w_{\mathrm{P}}\,\mathcal{C}_{\mathrm{P}} + w_{\mathrm{T}}\,\mathcal{C}_{\mathrm{T}} + w_{\mathrm{U}}\,\mathcal{C}_{\mathrm{U}} + w_{\mathrm{I}}\,\mathcal{C}_{\mathrm{I}} + \\ & + w_{\mathrm{N}}\,\mathcal{C}_{\mathrm{N}} + w_{\mathrm{A}}\,\mathcal{C}_{\mathrm{A}} + w_{\mathrm{R}}\,\mathcal{C}_{\mathrm{R}} + w_{\mathrm{D}}\,\mathcal{C}_{\mathrm{D}}\end{aligned} \tag{2}$$

being

- total cost of production:

$$\mathcal{C}_{\mathrm{P}} = \sum_{p \in C \cup F} \sum_{m \in M_p} \sum_{t=1}^{T} u^{\mathrm{P}}_{p,m,t}\, x_{p,m,t} \tag{3}$$

- total cost for late satisfaction of demand:

$$\begin{aligned}\mathcal{C}_{\mathrm{T}} &= \sum_{p \in F} \sum_{t=1}^{T} \sum_{h=t+1}^{T} u^{\mathrm{T}}_{p,h,t}\, l_{p,h,t} = \\ &= \sum_{p \in F} \sum_{t=1}^{T} \sum_{h=t+1}^{T} u^{\mathrm{L}}_{p}\,(h-t)^{\beta}\, l_{p,h,t}\end{aligned} \tag{4}$$

- total cost for demand not satisfied within the planning period:

$$\mathcal{C}_{\mathrm{U}} = \sum_{p \in F} \sum_{t=1}^{T} u^{\mathrm{U}}_{p}\, l_{p,T+1,t} \tag{5}$$

- total inventory cost:

$$\mathcal{C}_{\mathrm{I}} = \sum_{t=1}^{T} \left(\sum_{p \in B \cup C \cup F} u^{\mathrm{IP}}_{p,t}\, i_{p,t} + \sum_{r \in R} u^{\mathrm{IR}}_{r,t} \sum_{v=1}^{V} i^{\mathrm{R},v}_{r,t} \right) \tag{6}$$

- total cost for the acquisition of new parts:

$$\mathcal{C}_{\mathrm{N}} = \sum_{p \in B} \sum_{t=1}^{T} u^{\mathrm{N}}_{p,t}\, a^{\mathrm{N}}_{p,t} \tag{7}$$

- total cost for purchasing returned products:

$$\mathcal{C}_{\mathrm{A}} = \sum_{r \in R} \sum_{t=1}^{T} \sum_{v=1}^{V} u^{\mathrm{A},v}_{r,t}\, a^{\mathrm{A},v}_{r,t} \tag{8}$$

- total cost for recovery parts from returned products:

$$\mathcal{C}_{\mathrm{R}} = \sum_{r \in R} \sum_{t=1}^{T} \sum_{v=1}^{V} \left(\sum_{p \in B_r} u^{\mathrm{R},v}_{r,p,t}\, \rho^{v}_{r,p}\, \delta_{r,p} \right) s^{v}_{r,t} \tag{9}$$

- total disposal cost for returned products that cannot be recovered:

$$\mathcal{C}_{\mathrm{D}} = \sum_{r \in R} \sum_{t=1}^{T} \sum_{v=1}^{V} \left(\sum_{p \in B_r} u^{\mathrm{D},v}_{r,p,t}\, (1-\rho^{v}_{r,p})\, \delta_{r,p} \right) s^{v}_{r,t} \tag{10}$$

3.1.2 Dynamics of Inventories

- $\forall\, p \in F : \Delta_p > 0,\ \forall\, t = 1,\dots,\Delta_p$:

$$i_{p,t-1} = z_{p,t} + \sum_{h=1}^{t-1} l_{p,t,h} + i_{p,t} \tag{11}$$

- $\forall\, p \in F,\ \forall\, t = \Delta_p + 1,\dots,T$:

$$\sum_{m \in M_p} x_{p,m,t-\Delta_p} + i_{p,t-1} = z_{p,t} + \sum_{h=1}^{t-1} l_{p,t,h} + i_{p,t} \tag{12}$$

- $\forall p \in F, \forall t = 1, \ldots, T$:

$$z_{p,t} + \sum_{h=t+1}^{T+1} l_{p,h,t} = D_{p,t} \tag{13}$$

- $\forall p \in C : \Delta_p > 0, \forall t = 1, \ldots, \Delta_p$:

$$i_{p,t-1} = \sum_{q \in C \cup F} \sum_{m \in M_q} \gamma_{p,q} x_{q,m,t} + i_{p,t} \tag{14}$$

- $\forall p \in C, \forall t = \Delta_p + 1, \ldots, T$:

$$\sum_{m \in M_p} x_{p,m,t-\Delta_p} + i_{p,t-1} = \sum_{q \in C \cup F} \sum_{m \in M_q} \gamma_{p,q} x_{q,m,t} + i_{p,t} \tag{15}$$

- $\forall p \in B, \forall t = 1, \ldots, T$:

$$i_{p,t-1} + a^{\mathrm{N}}_{p,t} + a^{\mathrm{B}}_{p,t} = \sum_{q \in C \cup F} \sum_{m \in M_q} \gamma_{p,q} x_{q,m,t} + i_{p,t} \tag{16}$$

- $\forall p \in B^{\mathrm{R}}, \forall t = 1, \ldots, T$:

$$a^{\mathrm{B}}_{p,t} = \sum_{r \in R : p \in B_r} \left(\sum_{v=1}^{V} \rho^v_{r,p} \delta_{r,p} s^v_{r,t-\Gamma^v_{r,p}} \right) \tag{17}$$

- $\forall p \in B, p \notin B^{\mathrm{R}}, \forall t = 1, \ldots, T$:

$$a^{\mathrm{B}}_{p,t} = 0 \tag{18}$$

- $\forall r \in R, \forall t = 1, \ldots, T, \forall v = 1, \ldots, V$:

$$i^{\mathrm{R},v}_{r,t-1} + a^{\mathrm{A},v}_{r,t} = s^v_{r,t} + i^{\mathrm{R},v}_{r,t} \tag{19}$$

- $\forall r \in R, \forall t = 1, \ldots, T, \forall v = 1, \ldots, V$:

$$a^{\mathrm{A},v}_{r,t} \leq G^v_{r,t} \tag{20}$$

3.1.3 Capacity constraints

- $\forall m \in M, \forall t = 1, \ldots, T$:

$$\sum_{p \in P_m} \sum_{h=0}^{\Delta_p} x_{p,m,t-h} \leq \Phi_{m,t} \tag{21}$$

- $\forall t = 1, \ldots, T$:

$$\sum_{r \in R} \sum_{p \in B_r} \left(\sum_{v=1}^{V} \sum_{h=1}^{\Gamma^v_{r,p}} \rho^v_{r,p} \delta_{r,p} s^v_{r,t-h} \right) \leq \Psi_t \tag{22}$$

This problem, in which all the involved quantities are restricted to integer values, is NP-hard as it generalizes the deterministic capacitated planning problem with no setup costs which is NP-hard itself, as proved in (Florian et al., 1980).

4 EXAMPLE

The MIP problem described in the previous section has been implemented in a C# procedure and solved by the Cplex 12.5 MIP solver. To test the correctness of the proposed approach, an example which is reported in the following has been defined.

Consider an integrated manufacturing-remanufacturing system, whose bill of materials and remanufacturing process are those reported in Figures 2 and 3, respectively. The system produces three types of finished products. Two out of three types of products can be returned (1 and 2) and the remanufacturing process can provide five out of eight types of basic parts (11, 16, 19, 20, and 21; part 16 can be recovered from both products of type 1 and products of type 2). Four machines are in the system: machine 1 can produce parts (finished products and intermediate component) 3, 4, 6, 8, 9, 12, 15; machine 2 can produce parts 6, 8, 9, 12, 15; machine 3 can produce parts 1, 2, 5, 7; machine 4 can produce parts 5, 7, 10, 14; the capacity of each machine varies with time, in the range $[200, 400]$ parts/day. A fifth machine, whose capacity is 100 parts/day, is dedicated to the remanufacturing activities. The system must satisfy an external demand over a period of $T = 12$ days; the values of such a demand are reported in Table 1. All inventories are null at the beginning, even those in which acquired returned products are stored before they are disassembled.

Table 1: Example – Demand of products.

$t \rightarrow$	1	2	3	4	5	6
$D_{1,t}$	21	26	31	33	28	37
$D_{2,t}$	27	21	16	19	17	19
$D_{3,t}$	25	30	18	25	19	25

$t \rightarrow$	7	8	9	10	11	12
$D_{1,t}$	35	40	31	26	22	32
$D_{2,t}$	22	23	22	29	23	30
$D_{3,t}$	26	16	19	18	25	28

It is assumed that the quality of the returned products can be discretized in 3 levels ($V = 3$), being 1 the better level and 3 the worst one. The lead times for recovering components are 1, 1, and 2 (respectively for quality 1, 2, and 3) for products of type 1, and 0, 1, and 1 for products of type 2. Moreover, the percentage of components that can be retrieved "as new" is set to 0.8, 0.6, and 0.4 for products of type 1, and to 1, 0.75, and 0.5 for products of type 2. The number of returned products that can be acquired varies with time and with the quality level; it is here assumed that all such numbers range in the interval $[5, 30]$.

The unitary costs are not reported here due to the lack of space; it is only mentioned that the cost for not producing a (demanded) unit of product is significantly larger than the other unitary costs, in order to discourage the non-fulfilment of the external demand. Besides, in the last part of the example, an analysis of the sensitiveness of the solution with respect of the unitary costs will be carried out. Finally, all weights for the objective function components are unitary.

4.1 Solution of the Problem

The instance of the problem briefly described in the previous section has been solved by the Cplex 12.5 MIP solver. During the experimental analysis the MIP model was solved with a 120 seconds time limit, generating solutions with a 0.3-0.4% of optimality gap on a standard laptop with Intel's Core i7 processor and 8 Gb RAM.

Tables 2-7 report an example of the obtained solutions, showing in particular the values for the decision variables $x_{p,m,t}$ and $a^{\mathrm{N}}_{p,t}$ (relevant to the manufacturing operations) and $a^{\mathrm{A},v}_{r,t}$, $s^{v}_{r,t}$, and $a^{\mathrm{B}}_{p,t}$ (relevant to the remanufacturing process). In all solutions the demand for the products was satisfied within the planning horizon with a quite limited tardiness: in particular, the initial demands for products 1 and 3 were not satisfied on time due to the lead times involved in their production processes, having assumed no initial inventory for basic parts and intermediate components; in addition, only 7 units of product 3 due on bucket 5 and 1 unit of product 2 due on bucket 9 were delivered with tardiness.

4.2 Sensitivity of the Solution

A first analysis of the sensitiveness of the solution with respect of the unitary costs has been carried out. The solution of the problem has been determined as a function of the rates μ_p/ν_p, $\forall p \in B^{\mathrm{R}}$, being $\mu_p = \mu_p(u^{\mathrm{N}}_{p,t})$ and $\nu_p = \nu_p(u^{\mathrm{A},v}_{r,t}, u^{\mathrm{IR}}_{r,t}, u^{\mathrm{R},v}_{r,p,t}, u^{\mathrm{D},v}_{r,p,t})$ two values adopted to estimate the cost to buy and the cost to recover, respectively, a unit of a basic part of type p. The quantities μ_p and ν_p are computed as:

$$\mu_p = \frac{1}{T}\sum_{t=1}^{T} u^{\mathrm{N}}_{p,t} \tag{23}$$

$$\begin{aligned}\nu_p = \frac{1}{|R_p|}\sum_{r\in R_p}\Bigg[\frac{1}{\sum_p \delta_{r,p}}\Bigg(\frac{1}{VT}\sum_{v=1}^{V}\sum_{t=1}^{T} u^{\mathrm{A},v}_{r,t} + \\ + \frac{1}{T}\sum_{t=1}^{T} u^{\mathrm{IR}}_{r,t}\Bigg) + \frac{1}{VT}\sum_{v=1}^{V}\sum_{t=1}^{T}\rho^{v}_{r,p} u^{\mathrm{R},v}_{r,p,t} + \\ + \frac{1}{VT}\sum_{v=1}^{V}\sum_{t=1}^{T}\left(1-\rho^{v}_{r,p}\right) u^{\mathrm{D},v}_{r,p,t}\Bigg]\end{aligned} \tag{24}$$

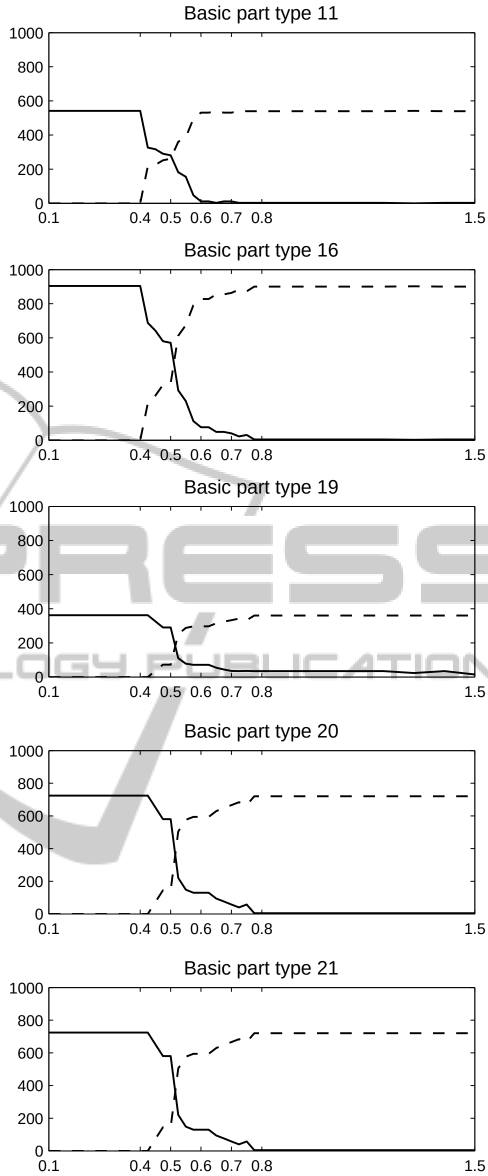

Figure 4: Overall number of basic parts acquired new (straight line) and recovered (dash line) as function of the rate μ_p/ν_p ranging from 0.1 to 1.5.

The analysis has been carried out by considering a fixed ν_p and a variable μ_p. In particular, for any $p \in B^{\mathrm{R}}$, μ_p has been set in accordance with ν_p in order to obtain, for the rate μ_p/ν_p, the values in the interval $[0.1, 1.5]$, with step size 0.025 from 0.4 to 0.8 and step size 0.1 elsewhere in the interval. The results of such analysis are in Figure 4, which shows patterns that are coherent with the considered class of system. Besides, such a first analysis can be considered as a basic validation of the integration of manufacturing and remanufacturing activities. In this connection, unlimited capacity of the resources (both of the

Table 2: Example – Solution variable $x_{p,m,t}$ (production plan on the four machines).

$t \rightarrow$	1	2	3	4	5	6	7	8	9	10	11	12
$x_{3,1,t}$	73	25	12	32	42	0	19	18	25	28	0	0
$x_{4,1,t}$	0	111	28	37	35	40	31	26	25	29	0	0
$x_{6,1,t}$	100	46	28	51	59	19	43	41	44	58	43	10
$x_{8,1,t}$	0	0	0	35	0	0	0	0	0	0	0	0
$x_{9,1,t}$	0	0	0	37	0	40	0	26	0	29	0	0
$x_{12,1,t}$	73	25	12	0	0	0	19	18	0	28	0	0
$x_{15,1,t}$	0	222	56	0	70	0	62	0	0	0	0	0

$t \rightarrow$	1	2	3	4	5	6	7	8	9	10	11	12
$x_{6,2,t}$	0	0	0	0	0	0	0	0	0	0	0	0
$x_{8,2,t}$	111	49	16	0	40	53	4	25	29	0	0	0
$x_{9,2,t}$	0	111	28	0	35	0	31	0	25	0	0	0
$x_{12,2,t}$	0	0	0	32	42	0	0	0	25	0	0	0
$x_{15,2,t}$	0	0	0	74	0	80	0	52	50	58	0	0

$t \rightarrow$	1	2	3	4	5	6	7	8	9	10	11	12
$x_{1,3,t}$	0	0	0	111	28	37	35	40	31	26	25	29
$x_{2,3,t}$	27	21	16	19	17	19	22	23	21	30	23	30
$x_{5,3,t}$	27	21	0	0	0	0	0	23	0	30	23	30
$x_{7,3,t}$	73	25	0	32	42	0	19	18	0	28	0	0

$t \rightarrow$	1	2	3	4	5	6	7	8	9	10	11	12
$x_{5,4,t}$	0	0	16	19	17	19	22	0	21	0	0	0
$x_{7,4,t}$	0	0	12	0	0	0	0	0	25	0	0	0
$x_{10,4,t}$	100	157	56	88	94	59	74	67	69	87	43	10
$x_{14,4,t}$	111	49	16	35	40	53	4	25	29	0	0	0

Table 3: Example – Solution variable $z_{p,t}$ (demand satisfied on time).

$t \rightarrow$	1	2	3	4	5	6	7	8	9	10	11	12
$z_{1,t}$	0	0	0	33	28	37	35	40	31	26	22	32
$z_{2,t}$	27	21	16	19	17	19	22	23	21	29	23	30
$z_{3,t}$	0	0	18	25	12	25	26	16	19	18	25	28

Table 4: Example – Solution variable $a^{\mathrm{N}}_{p,t}$ (acquisition plan of new basic parts).

$t \rightarrow$	1	2	3	4	5	6	7	8	9	10	11	12
$a^{\mathrm{N}}_{11,t}$	68	6	0	0	0	0	0	0	0	0	0	0
$a^{\mathrm{N}}_{13,t}$	333	147	48	105	120	171	0	75	87	0	0	0
$a^{\mathrm{N}}_{16,t}$	68	68	0	21	40	9	26	10	0	23	0	0
$a^{\mathrm{N}}_{17,t}$	146	50	24	64	84	0	38	36	50	56	0	0
$a^{\mathrm{N}}_{18,t}$	146	50	24	64	84	0	38	36	50	56	0	0
$a^{\mathrm{N}}_{19,t}$	111	0	0	19	33	30	0	0	4	0	0	0
$a^{\mathrm{N}}_{20,t}$	0	124	0	66	56	34	36	20	0	46	0	0
$a^{\mathrm{N}}_{21,t}$	0	124	0	66	56	34	36	20	0	46	0	0

Table 5: Example – Solution variable $a^{\mathrm{A},v}_{r,t}$ (acquisition plan of returned products for remanufacturing).

$t \rightarrow$	1	2	3	4	5	6	7	8	9	10	11	12
$a^{\mathrm{A},1}_{1,t}$	0	0	0	0	0	2	0	0	0	0	0	0
$a^{\mathrm{A},2}_{1,t}$	22	0	0	5	15	3	8	12	0	0	0	0
$a^{\mathrm{A},3}_{1,t}$	13	10	0	0	0	0	2	3	0	0	0	0
$a^{\mathrm{A},1}_{2,t}$	15	3	15	15	6	15	10	0	0	0	0	0
$a^{\mathrm{A},2}_{2,t}$	7	0	1	0	0	0	0	0	4	13	8	0
$a^{\mathrm{A},3}_{2,t}$	10	1	10	11	7	6	1	15	20	15	0	0

Table 6: Example – Solution variable $s^v_{r,t}$ (remanufacturing process).

$t \rightarrow$	1	2	3	4	5	6	7	8	9	10	11	12
$s^1_{1,t}$	0	0	0	0	0	2	0	0	0	0	0	0
$s^2_{1,t}$	22	0	0	5	15	3	8	12	0	0	0	0
$s^3_{1,t}$	13	10	0	0	0	0	2	3	0	0	0	0
$s^1_{2,t}$	15	0	16	17	5	6	20	0	0	0	0	0
$s^2_{2,t}$	7	0	1	0	0	0	0	0	4	13	8	0
$s^3_{2,t}$	10	0	11	11	7	6	0	16	20	15	0	0

Table 7: Example – Solution variable $a^{\mathrm{B}}_{p,t}$ (basic parts recovered in the remanufacturing process).

$t \rightarrow$	1	2	3	4	5	6	7	8	9	10	11	12
$a^{\mathrm{B}}_{11,t}$	32	40	28	63	47	27	35	41	44	58	43	10
$a^{\mathrm{B}}_{16,t}$	32	89	56	67	54	50	48	57	69	64	43	10
$a^{\mathrm{B}}_{19,t}$	0	49	28	4	7	23	13	16	25	6	0	0
$a^{\mathrm{B}}_{20,t}$	0	98	56	8	14	46	26	32	50	12	0	0
$a^{\mathrm{B}}_{21,t}$	0	98	56	8	14	46	26	32	50	12	0	0

manufacturing part and of the remanufacturing one) and unlimited availability of returned products have been assumed in this analysis, in order to make the sensitiveness independent from the availability of machines and of products to be disassembled in order to recover basic parts.

5 CONCLUSIONS

This work proposes a MIP model for planning manufacturing activities in a multi-product, multi-stage production plant which includes a remanufacturing facility. The considered model assumes a simplified aggregate production characterized by deterministic information on demand and availability of returned products. The presented experimental analysis points out the applicability of the model at least for small-medium size instances (the problem in the considered example has about 1150 variables and 560 constraints), as well as the coherence of the model behaviour with respect to the variations of part acquisition costs. In any case, more extensive testing with larger instances is on its way. Besides, future improvements of this model will focus on the consideration of explicit setups and on the relaxation of some of the deterministic assumptions.

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