Adaptive LQG/LTR Control; Discontinuity Issue

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Abstract: An adaptive LQG control with no control cost is considered. In such case the loop transfer recovery (LTR) effect can be obtained. The control problem is handled using discrete-time state-space model and the parameter estimation is performed for corresponding ARMAX model which can be represented in innovation state-space form. Thus the direct estimation of model parameters is possible by means of standard ERLS procedure and the adaptive control is implemented through *certainty equivalence principle*. In such a situation the problem of solution continuity of Riccati equation can arise for nonminimum-phase systems. Computer simulations of third-order systems modeled by a second-order minimum-phase and nonminimum-phase models are given to illustrate the robustness and performance properties of the adaptive controller, particularly with respect to the modelling error parameter η.

1 INTRODUCTION

The problem of adaptive LQG/LTR control is not much investigated in the literature. It is known that even without LTR an adaptive LQG control itself is difficult to solve in optimal manner. Adaptive LQG control has been discussed e.g. in (Kumar, 1983; Bitmead et al., 1990; Tay and Moore, 1991; Krolikowski, 1995; Mäkilä et al., 1984; Duncan and B.Pasik-Duncan, 1999), where in (Tay and Moore, 1991) an adaptive LQG/LTR problem was solved augmenting the basic estimator-based controller with a stable proper linear system feeding back the estimation residuals. This idea was also used for non-adaptive continuous-time systems in (Tay and Moore, 1989) using the H^{∞}/H^{2} optimization technique. The LQG/LTR problem for continuous-time systems was considered in (A. Saberi and Sannuti,) where the analysis of loop transfer recovery is given for not necessarily left invertible and not necessarily minimum-phase (mph) systems.

The key issue in adaptive LQG control is the closed-loop identifiability. For example in (Kumar, 1983) a discrete-time system and a cost-biased least-squares parameter estimation was used in order to achieve overall asymptotic system optimality. In (Duncan and B.Pasik-Duncan, 1999) a continuous-time system was considered and a modified weighted least-squares parameter estimation algorithm was used to obtain good properties of estimates. In both papers only a fully state observation case was considered.

In this paper, LQG adaptive control with LTR effect is considered. The adaptive discrete-time LQG control with no control cost (cheap control) is proposed when the full LTR effect, at least for mph systems can be obtained. The controller/filter parameters are tuned on-line based on parameter identification of ARMAX model. This can sometimes lead to discontinuous behaviour of solution to Riccati equation depending on system parameters (Saberi and Stoorvogel, 1996). Asymptotic performance and robustness properties are analyzed and simulations for third-order system considered as a second-order model are given. The impact of modelling error parameter η on performance, robustness and solution discontinuity of corresponding Riccati equation is underlined.

2 LQG/LTR PROBLEM FOR DISCRETE-TIME SYSTEMS

The state-space discrete-time system is given by

$$\underline{x}_{t+1} = F\underline{x}_t + G\underline{u}_t + \underline{w}_t \quad (1)$$
$$\underline{y}_t = H\underline{x}_t + \underline{v}_t \quad (2)$$

where n, m, r are dimensions of the state, control and output vectors, respectively and where $\{\underline{w}_t\}$ and $\{\underline{v}_t\}$ are sequences of independent random vector variables

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with zero mean and covariances $E\underline{w}_t\underline{w}_s^T = \Sigma_w\delta_{t,s}$, $\underline{v}_t\underline{v}_s^T = \Sigma_v\delta_{t,s}$.
The Kalman predictor for (1), (2) in steady-state is given by

$$\hat{\underline{x}}_{t+1/t} = F\hat{\underline{x}}_{t/t-1} + G\underline{u}_t + K_p\tilde{\underline{y}}_t^p \tag{3}$$

where $\tilde{\underline{y}}_t^p = \underline{y}_t - H\hat{\underline{x}}_{t/t-1}$ is an innovation of output at time t. The predictor gain is given by

$$K_p = FP_fH^T[HP_fH^T + \Sigma_v]^{-1} \tag{4}$$

where P_f is the solution of Riccati equation

$$P_f = FP_fF^T + \Sigma_w - FP_fH^T[HP_fH^T + \Sigma_v]^{-1}HP_fF^T \tag{5}$$

The filtered estimate $\hat{\underline{x}}_{t/t}$ in terms of $\hat{\underline{x}}_{t/t-1}$ is

$$\hat{\underline{x}}_{t/t} = \hat{\underline{x}}_{t/t-1} + K_f\tilde{\underline{y}}_t^p \tag{6}$$

and its recursive version is

$$\hat{\underline{x}}_{t+1/t+1} = F\hat{\underline{x}}_{t/t} + (I - K_fH)G\underline{u}_t + K_f\tilde{\underline{y}}_{t+1}^f \tag{7}$$

where $\tilde{\underline{y}}_{t+1}^f = \underline{y}_{t+1} - HF\hat{\underline{x}}_{t/t}$ and the filter gain

$$K_f = P_fH^T[HP_fH^T + \Sigma_v]^{-1}, \tag{8}$$

so $K_p = FK_f$ in view of (4). An alternative equation for (7) is

$$\hat{\underline{x}}_{t+1/t+1} = F\hat{\underline{x}}_{t/t} + G\underline{u}_t + K_f\tilde{\underline{y}}_{t+1}^p \tag{9}$$

The Kalman filter-based control law

$$\underline{u}_t = K_c\hat{\underline{x}}_{t/t} \tag{10}$$

aims to minimize the cost function

$$J = E\sum_{t=0}^{\infty}\underline{y}_t^TQ\underline{y}_t. \tag{11}$$

where the gain K_c is given by

$$K_c = -[G^TP_cG]^{-1}G^TP_cF \tag{12}$$

and P_c is the solution of Riccati equation

$$P_c = F^TP_cF - F^TP_cG[G^TP_cG]^{-1}G^TP_cF + Q \tag{13}$$

When the weighting matrix is $Q = H^TH$ and assuming that the system (1), (2) is stabilizable, detectable and mph then it can be shown (Tadjine et al., 1994), (Maciejowski, 1985) that K_c takes very simple form

$$K_c = -(HG)^{-1}HF. \tag{14}$$

and the optimal cost function is

$$J^{opt} = trH^TH[\Sigma_w + FP_fF^T]. \tag{15}$$

If $G(z) = H(zI - F)^{-1}G$ is mph and K_c takes a form (14) then the perfect recovery takes place, that is

$$\Delta(z) = G(z)G_f(z) - \Phi(z) = 0, \tag{16}$$

where the transfer function $G_f(z)$ of compensator defined by (7) and (10) can be manipulated into the form

$$\begin{aligned} G_f(z) &= -zK_c[zI - (I - K_fH)(F + GK_c)]^{-1}K_f = \\ &= -zK_c[zI - F - GK_c]^{-1}K_f, \end{aligned} \tag{17}$$

and the filter's open-loop return ratio is

$$\Phi(z) = H(zI - F)^{-1}K_p. \tag{18}$$

Putting (14) into (16) it can be seen that $\Delta(z) = 0$ so the recovery takes place. When $G(z)$ is nonminimum-phase (nmph) then the perfect recovery is in general not possible, however is recommended because the partial recovery is frequently achieved in closed-loop bandwidth.
The robustness in the presence of additive modeling error i.e. $G^* = G + \eta G_p$, can be measured by means of the $||H||_\infty$ norm of sensitivity transfer function

$$S(z) = (I + G(z)G_f(z))^{-1}, \tag{19}$$

by checking up the inequality

$$\eta||G_p(z)||_\infty||G_f(z)S(z)||_\infty \leq 1, \tag{20}$$

where $\eta||G_p(z)||_\infty$ is an upper bound of additive modeling error.
It can be noted that for the Kalman predictor-based feedback control $\underline{u}_t = K_c\hat{\underline{x}}_{t/t-1}$ the perfect recovery cannot in general be possible.

3 ADAPTIVE CONTROL

The SISO ARMAX model is given by

$$A(q^{-1})y_t = B(q^{-1})u_t + C(q^{-1})e_t \tag{21}$$

where $A(q^{-1}), B(q^{-1})$ and $C(q^{-1})$ are polynomials in the backward shift operator q^{-1}, i.e. $A(q^{-1}) = 1 + a_1q^{-1} + ... + a_nq^{-n}, B(q^{-1}) = b_1q^{-1} + ... + b_nq^{-n}, C(q^{-1}) = 1 + c_1q^{-1} + ... + c_nq^{-n}$ and y_t is the output , u_t is the control input, and $\{e_t\}$ is assumed to be a sequence of independent variables with zero mean and variance σ_e^2. Unknown system parameters $\underline{\theta} = (a_1, ..., a_n, b_1, ..., b_n, c_1, ..., c_n)^T$ are estimated on-line to obtain an updated model at time t, i.e. $\hat{\underline{\theta}}_t$.
ARMAX model (21) has an equivalent innovation state-space representation

$$\underline{x}_{t+1} = F\underline{x}_t + \underline{g}u_t + \underline{k}_e e_t \tag{22}$$

$$y_t = \underline{h}^T\underline{x}_t + e_t \tag{23}$$

where $\underline{g} = (b_1, ..., b_n)^T$, $\underline{k}_e = (c_1 - a_1, ..., c_n - a_n)^T$, $\underline{h}^T = (1, 0, ..., 0)$

$$F = \begin{bmatrix} -a_1 & 1 & \ldots & 0 \\ . & . & \ldots & 0 \\ -a_{n-1} & . & \ldots & 1 \\ -a_n & . & \ldots & 0 \end{bmatrix}.$$

For ARMAX system different forms of Kalman filter can be proposed. One of them takes the Kalman predictor (3) associated with eq.(22) as

$$\hat{\underline{x}}_{t+1/t} = F\hat{\underline{x}}_{t/t-1} + \underline{g}u_t + \underline{k}_p\tilde{y}_t^p \quad (24)$$

where $\tilde{y}_t^p = y_t - \underline{h}^T\hat{\underline{x}}_{t/t-1}$ and $\sigma^2_{\tilde{y},p}$ is the variance of $\tilde{y}_t^p$ for which it holds $\sigma^2_{\tilde{y},p} = \sigma^2_e$. The predictor gain is now given by

$$\underline{k}_p = (FP_f\underline{h} + \sigma_e^2\underline{k}_e)[\underline{h}^TP_f\underline{h} + \sigma_e^2]^{-1} \quad (25)$$

where P_f is the solution of Riccati equation

$$\begin{aligned} P_f &= FP_fF^T + \underline{k}_e\underline{k}_e^T\sigma_e^2 - (FP_f\underline{h} + \underline{k}_e\sigma_e^2)\times \\ &\times[\underline{h}^TP_f\underline{h} + \sigma_e^2]^{-1}(FP_f\underline{h} + \underline{k}_e\sigma_e^2)^T \end{aligned} \quad (26)$$

The actual model used for control is obtained for current parameter estimates $\hat{\underline{\theta}}_t$. The LTR control law (14) is especially useful for adaptive control because there is no need solving the Riccati equation (13) for every model $\hat{\underline{\theta}}_t$, and the feedback gain K_c can be tuned directly, however this holds only for mph systems. The investigated problem is to check up how the modeling error influences the performance, robustness and parameter estimation quality of adaptive control.

4 THE DISCONTINUITY ISSUE

The stability of the proposed adaptive control system depends not only on the magnitude of modeling error but also on the asymptotic convergence of parameter estimates. The parameter estimation in LQG adaptive control does not in general assure the convergence to the true parameters, even in the lack of modelling error. Moreover, it is known that the solutions of Riccati equation may have discontinuities as a function of system parameters. This can occur for non left-invertible and nmph systems (Saberi and Stoorvogel, 1996). Obviously, possible discontinuities have impact on *certainty equivalence*-based control and the closed-loop global stability cannot be guaranteed. The concept of left-invertibility plays here a crucial role. System (1) with $r \geq m$ is said to be left-invertible if it satisfies

$$\max_z rankP_l(z) = n + m \quad (27)$$

where

$$P_l(z) = \begin{bmatrix} zI - F & -G \\ H & 0 \end{bmatrix}.$$

If the system is not left-invertible then it is almost always possible to find parameter perturbations that cause discontinuity (Saberi and Stoorvogel, 1996). Even if the system is left-invertible discontinuity problems can occur when the system is nmph.

5 SIMULATIONS

Consider an example of a third-order actual system

$$G^*(s) = \frac{s+2}{(s+1)(s+3)} + \eta\frac{1}{s+2}$$

whose nominal model $G(s)$ is mph, so the case $\eta = 0$ corresponds to the lack of modeling error and $\eta = 1$ is the case of undermodeling. Discretizing the continuous-time system with ZOH and sampling period $T_s = 0.5s$ yields the following transfer function in q^{-1} operator

$$G^*(q^{-1}) = \frac{-0.3262q^{-1} - 0.1224q^{-2}}{1 - 0.8297q^{-1} + 0.1535^{-2}} + \eta\frac{0.3101q^{-1}}{1 - 0.3679q^{-1}}. \quad (28)$$

As already mentioned, a second-order model was taken for identification and *certainty equivalence principle* was used to implement the adaptive control system to demonstrate the robustness of adaptive controller with respect to undermodeling.

The corresponding RLS parameter estimates of the nominal model are shown in Fig.1, for $\eta = 0$ and in Fig.2 for $\eta = 1$ where the noise variance σ_e^2 was set at 0.1. From Fig.1 one can see that for $\eta = 0$ parameter estimates converge to the true parameters of second-order nominal model. It is to observe that when $\eta = 1$ as in Fig.2 the parameter estimates converge to some stationary points, however different from the true ones.

As a second example consider a third-order actual system

$$G^*(s) = \frac{-s+1}{(s+1)(s+2)} + \eta\frac{1}{s+3}$$

whose nominal model is nmph. Discretizing the continuous-time system with ZOH and sampling period $T_s = 0.5s$ yields the following transfer function operator

$$G^*(q^{-1}) = \frac{-0.1612q^{-1} + 0.2856q^{-2}}{1 - 0.9744q^{-1} + 0.223q^{-2}} + \eta\frac{0.259q^{-1}}{1 - 0.223q^{-1}}. \quad (29)$$

From Fig.3 one can see that for $\eta = 0$ parameter estimates again converge to the true parameters of second-order nominal model. The case with $\eta = 0.5$ is shown in Fig.4. Again the parameter estimates converge to some stationary points, however different from true values.

Fig.5 shows the gain $\underline{k}_c$ versus η obtained in adaptive control for mph and nmph nominal systems where both the formula (12) based on the Riccati equation (13) and the formula (14) are used for calculation of $\underline{k}_c$. This is illustrated by left and right column, respectively. In the mph case the result is the

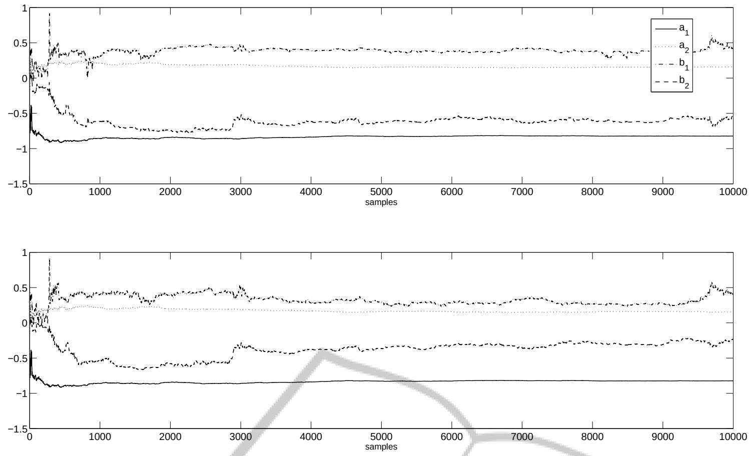

Figure 1: MPH: estimates for $\eta = 0$.

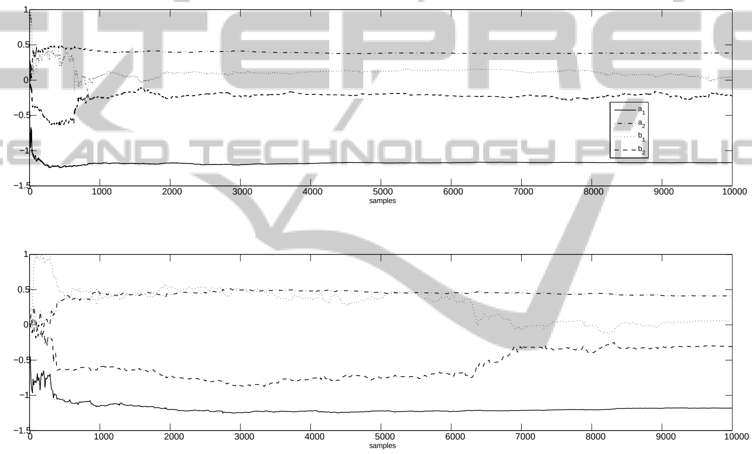

Figure 2: MPH: estimates for $\eta = 1$.

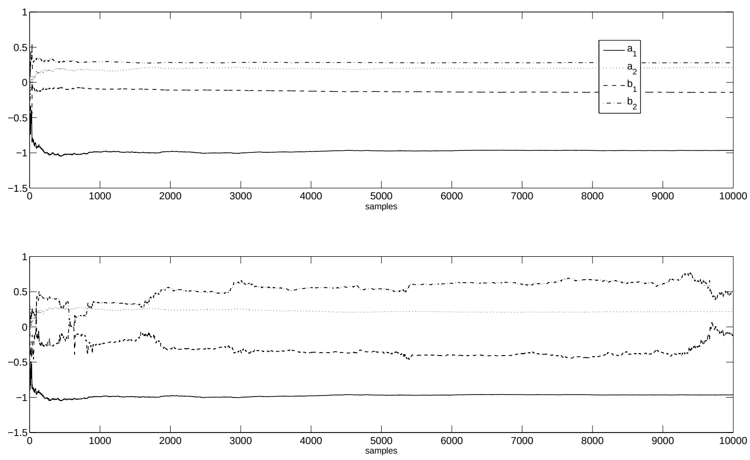

Figure 3: NMPH: estimates for $\eta = 0$.

same for both formulae while for nmph nominal system the gain $\underline{k}_c$ calculated according to (12) and (14) gives instability. The discontinuous jumps in nmph case can be caused by discontinuity in solutions of the Riccati equation (13) that can follow from parameter estimates convergence to wrong values when $\eta > 0$.

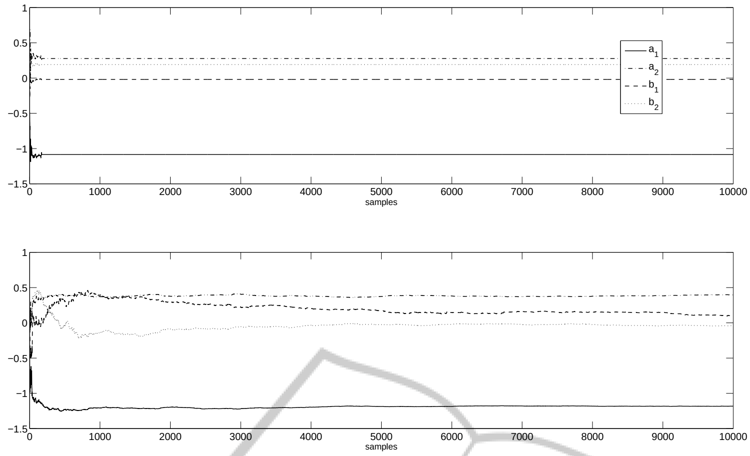

Figure 4: NMPH: estimates for $\eta = 0.5$.

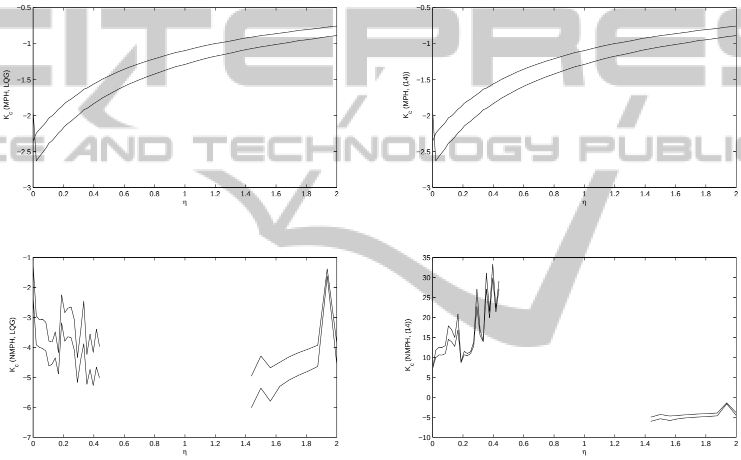

Figure 5: Plot of $\underline{k}_c$ versus η.

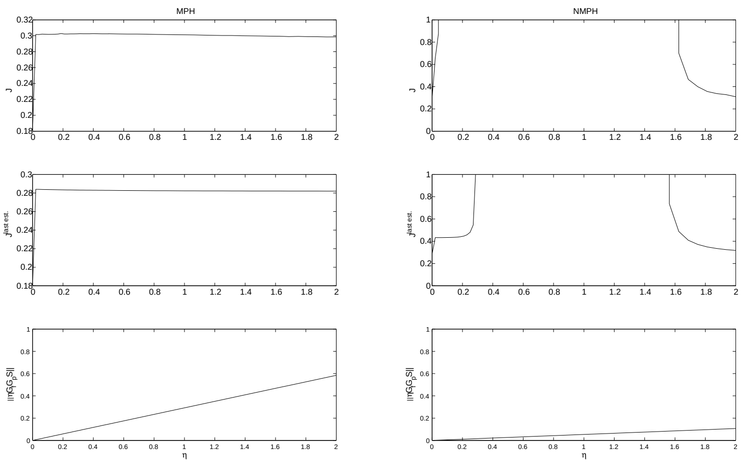

Figure 6: Plot of J and norms versus η.

This happens only when the system is nmph, moreover the results obtained on the base of (14) give unstable gains $\underline{k}_c$.

In the case of $\eta > 0$ a deterioration of cost function is to be expected. This is illustrated in Fig.6 where the simulated cost function (11) is plotted versus η for adaptive control and for nonadaptive control with system parameters taken as their final estimates. The cost function values for $\eta = 0$ are $J_0 = 0.1848, 0.1828$, and $J_0 = 0.2998, 0.2665$ for mph and nmph nominal systems, respectively. Again, some jumps of cost function are observed in adaptive case for $\eta > 0$.

The norm condition of robust stability (20), for both mph and nmph systems with known parameters is also illustrated in Fig.6 for $\eta = [0, 2]$. It can be observed that robust stability margin for nmph system is even larger than for mph system, i.e. the value of η at which the norm attains 1 is greater in the case of nmph system.

The RLS algorithm was applied for parameter identification of a considered ARX model and as already has been shown in Figures 1, 3 the estimates converge to the true nominal values for both mph and nmph systems.

For general ARMAX models, the recursive pseudolinear regression (RPLR) or recursive prediction error (RPEM) algorithms should be applied. The results shown in (Nilsson and Egardt, 2010), confirm that RPEM is then more suitable in the considered undermodelled situation taking into account the asymptotic properties of the algorithms.

6 CONCLUSIONS

Simple adaptive discrete-time LQG control in the context of LTR is presented. Parameter estimation of ARMAX model is used for tuning the discrete-time compensator. The interplay between robustness, performance and estimation convergence with respect to the modeling error is underlined. Examples of third-order actual systems described by a second-order mph and nmph nominal models are taken for simulation. Simulation results show an effectivness of the adaptive LQG control with possible LTR effect as a way for robustifying the adaptive control especially for mph systems. On the other hand, problems with discontinuous solution of Riccati equation may occur for nmph systems.

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