A New Adaptive Universal Fuzzy Inference System with Application

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Keywords: Fuzzy Inference System, Adaptive Universal Fuzzy Inference System, Choquet Integral-OWA Operator based Fuzzy Inference System, Choquet Integral-OWA Operator based Adaptive Universal Fuzzy Inference System.

Abstract: Through comprehensive study on existing fuzzy inference systems, this paper presents a Choquet integral-OWA operator based fuzzy inference system (AggFIS) in order to solve the traditional FIS disadvantages and its adaptive model which is named Choquet integral-OWA operator based adaptive universal fuzzy inference system (Agg-AUFIS). By considering the universal fuzzy inference operators and importance factor during reasoning process, Agg-AUFIS tries to express the essence of fuzzy logic and simulate human thinking pattern sufficiently, which could provide a new methodology for fuzzy modeling in future.

1 INTRODUCTION

Fuzzy sets theory (linguistic variable) was introduced by Zadeh in 1965 as a new modeling technique based on human knowledge and thinking pattern (Zadeh, 1965), which has gained an increasing level of acceptance in science and engineering. Hybrid tools which are developed by using combinations of fuzzy logic and neural networks require relatively short development times and are robust. Through understanding fuzzy logic essence and various adaptive fuzzy inference systems, this paper presents a Choquet integral-OWA operator based fuzzy inference system (AggFIS) and its adaptive model named as Choquet integral-OWA operator based adaptive universal fuzzy inference system (Agg-AUFIS), which are tested by historical sample data of traffic flow.

2 FUZZY MODELING

When the application domain moves from simple systems to complex ones, the usual operation procedure becomes infeasible and we need the intelligent and adaptive methods to solve all these problems. System modeling based on conventional mathematical tools (e.g., differential equations) is not well suited for dealing with ill-defined and uncertain systems. By contrast, a fuzzy model of a system (or a fuzzy inference system) employing expert knowledge and fuzzy if-then rules can model the qualitative aspects of human reasoning processes without implementing precise quantitative analysis.

2.1 Fuzzy Model of a System

The introduction of fuzzy sets can express the human thinking pattern with relatively simple mathematics format so that it is "intelligent" and possible to handle complex nonlinear problems consistent with the human mind (Zadeh, 1973). Fuzzy logic aims at modeling the imprecise modes of reasoning that play an essential role in the remarkable human ability to make rational decisions in an environment of uncertainty and imprecision. This ability depends on our ability to infer an approximate answer to a question based on a store of knowledge that is inexact, incomplete, or not totally reliable (Zadeh, 1988).

Recent interest has developed in the use of fuzzy set theory for modeling of complex systems. We call such a representation a fuzzy model of a system (FMS). Fuzzy modeling based on fuzzy inference is used to describe the model of the object using fuzzy if-then rules and aims at constructing a mathematics paradigm of linguistic analysis for complex system or progress, which can transform nature language into algorithm language that can be handled by computer (Pedrycz *et al.*, 1995; Pedrycz and Oliveira, 1996). Once a model is identified, it can be

Chai Y., Chen J., Luo W. and Jia L..
A New Adaptive Universal Fuzzy Inference System with Application.
DOI: 10.5220/0005130501630168
In *Proceedings of the International Conference on Fuzzy Computation Theory and Applications* (FCTA-2014), pages 163-168
ISBN: 978-989-758-053-6

applied to analysis, prediction, control, diagnosis of the object.

Comparing with the traditional mathematics modeling, a fuzzy model of a system (or a fuzzy inference system), which is established according to fuzzy set theory and fuzzy logic, implements a non-linear mapping from its input space to output space by simulating human thinking mode. The Stone-Weierstrass theorem indicates that a fuzzy inference system (FIS) can approximate arbitrary nonlinear function (Zadeh, 1994), the reasoning process of which is shown in Figure 1.

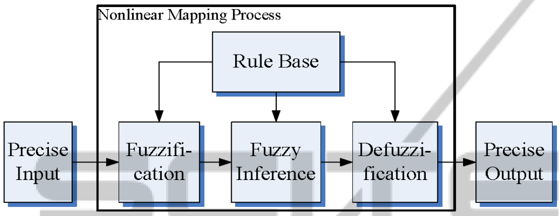

Figure 1: Reasoning process of fuzzy inference system.

Conceptually a fuzzy logic controller or more generally any FMS can be also seen as a function f mapping input to output. See in Figure 2.

Figure 2: Nonlinear mapping model of FMS.

Essentially f can be seen as a mapping $f : X \rightarrow Y$, $y = f(x)$, where X and Y are the base sets of U and V respectively (U and V are fuzzy sets of X and Y). Formally we can represent this mapping as a relationship F on X*Y such that $(x, y) \in F$, if $f(x) = y$ (Yager, 1993a). It is also reported that a fuzzy inference system (FIS) is a universal approximator (Furuhashi, 2001).

Fuzzy modeling is an iterative progress and knowledge is the reasoning cores which make the output improve continually. It is concluded that FMS (or FIS) could be an intelligent modeling method because of its reasoning ability and nonlinear mapping function.

2.2 Some Shortcomings of FIS

According to fuzzy reasoning type and format of fuzzy if-then rules, most FIS can be divided into three classes: Mamdani fuzzy inference system, Sugeno fuzzy inference system and Tsukamoto fuzzy inference system.

T-S fuzzy inference system works well with linear techniques and guarantees continuity of the output surface. The consequent of each rule is a linear combination of input variables rather than membership functions and each rule has a crisp output. But T-S fuzzy inference system has difficulties in dealing with the multi-parameter synthetic evaluation and in assigning weight to each input and fuzzy rule.

Mamdani fuzzy inference system has some advantages superior to T-S system (Mamdani, 1977; Esragh and Mamdani, 1981):

- It's intuitive;
- It has widespread acceptance;
- It's well-suited to human cognition and reflects the nature of fuzzy logic.

Mamdani model can show its legibility and understandability to the laypeople and has advantages in consequent expression and intuitive reasoning, which is applied in many practical problems. M-FIS does well in solving multi-parameter evaluation problems, but it needs large calculation amount for defuzzification and it also has difficulties in weight expression.

So we conclude that there are the following two major disadvantages in the existing FIS:

(1) In the existing fuzzy inference systems, the choice of fuzzy inference operators is relatively fixed and reasoning composite method is also limited to two methods, such as Max-min and Sum-product. For example, in MATLAB fuzzy logic toolbox, the choice of AND operator is only Product or Min and the output of FIS is calculated by using the given operators. The commonly used fuzzy inference operators and reasoning composite methods may make us more and more limited in understanding nature of FIS.

The essence of fuzzy inference system is a non-linear mapping and each step of fuzzy reasoning is also a non-linear mapping process. From this point of view, other reasoning operators could be used to achieve each reasoning step of FIS in addition to traditional inference operators and we should have a comprehensive understanding of fuzzy inference system.

(2) In traditional fuzzy inference systems, importance (weight) of each input and each rule is not considered, which means their contribution to the overall output is same during the reasoning. This inference process is not consistent with human cognition. For example, when we want to buy a bus, price is most important factor to be considered first, followed by comfort, colour, etc.

Therefore, considering the importance (weight) of each criterion (input or rule) will make FIS more close to human thinking manner and reflect the nature of fuzzy logic better.

In order to solve these shortcomings and limitations, we need a "universal" fuzzy inference system, which could:

- use the universal fuzzy inference operators and replace the existing traditional operators to realize the reasoning process;
- consider the importance factor of objects (inputs and rules) and establish the inference model that could completely represent fuzzy logic essence and simulate human thinking pattern.

This paper introduces a Choquet integral-OWA operator based fuzzy inference system known as AggFIS, which is a more "universal" fuzzy inference system, and its adaptive model respectively in next chapters.

3 CHOQUET INTEGRAL-OWA OPERATOR BASED FUZZY INFERENCE SYSTEM

Fuzzy inference is a computer paradigm based on fuzzy set theory, fuzzy if-then-rules and fuzzy reasoning. The basic structure of fuzzy inference system mainly consists of three parts: rule base which contains the set of fuzzy rules; database (or dictionary) which defines the membership functions used in the fuzzy rules; a reasoning mechanism which performs the inference procedure to derive a conclusion from facts and rules.

3.1 Fuzzy Inference Operator

To completely specify the operation of a fuzzy inference system, we need to assign a function for each of the following operators:

- AND operator (usually T-norm) for calculating the rule firing strength with AND'ed antecedents
- OR operator (usually T-conorm) for calculating the rule firing strength with OR'ed antecedents
- Implication operator (usually T-norm) for calculating qualified consequent MFs based on given firing strength
- Aggregate operator (usually T-conorm) for aggregating qualified consequent MFs to generate an overall output MF
- Defuzzification operator for transforming an output MF to a crisp single output value

Once the above five kinds of operators are determined, we have identified one fuzzy inference system (FIS). According to the fuzzy inference operators, FIS can be generally divided into five layers:

The first layer is fuzzification layer, which is used to make crisp input fuzzification and the degree of compatibility is based on antecedent MF; the fifth layer is defuzzification layer, which extracts a crisp value that best represents a fuzzy set from overall MFs. There are five methods for defuzzifying a fuzzy set A of a universe of discourse Z and the centroid of area (COA) is the most popular one (Yager and Filev, 1992).

The second layer is the inference layer or rule layer, which is used to calculate degree of the fuzzy rule fulfilment (firing strength) corresponding to the above AND or OR operator;

The third layer is the implication layer, which is used to represent how the firing strength gets propagated or used in a fuzzy implication statement corresponding to the above implication operator;

The fourth layer is the aggregation layer, which is used to aggregate all the qualified consequent MFs to obtain an overall output MF corresponding to the above aggregate operator.

In the case of crisp inputs and outputs, FIS implements a nonlinear mapping from its input space to output space. This mapping is completed by a group of fuzzy if-then rules. We can assume that each rule describes the local mapping behaviour. Fuzzy rules are used to obtain inaccurate information and expressed as inaccurate (natural language) reasoning model, which makes FIS have reasoning abilities similar to human mind under the imprecise and uncertain environment.

3.2 AggFIS

This paper presents a Choquet integral-OWA operator based fuzzy inference system named as AggFIS, which uses the Choquet integral-OWA operator composite method instead of T-conorm-T-norm composite and COA defuzzification method in reasoning process.

In the inference layer, we apply OWA operator which replaces AND (OR) operator to calculate firing strength (Yager, 1988, 1993b); in the aggregation layer, we use Choquet integral instead of the traditional T-conorm operator (Sum or Max) to aggregate the qualified MFs and generate an overall output MF (Grabisch, 2000); and the defuzzification operator is centroid of area (COA). Furthermore, in the reasoning process we assign weight to each input, expressed by μ_i; we also assign weight to each rule, expressed by τ_i.

If we regard these FIS steps as aggregation process, the replaced operators can be expressed as "Agg" and we call this model "AggFIS". Corresponding to different choice of each "Agg" operator, we can get the different special FIS cases (Kelman and Yager, 1995; Yager, 1996).

The reasoning process of AggFIS is shown in Figure 3.

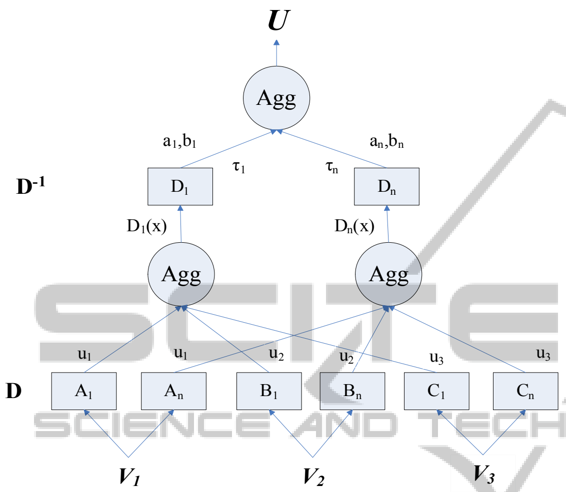

Figure 3: Choquet integral-OWA operator based FIS.

The format of fuzzy rule is as follows:

If V_1 is A_1 and V_2 is B_1 and V_3 is C_1, Then U is D_1;

V_1, V_2, V_3 are the crisp inputs, U is the single crisp output; A_1, B_1, C_1 represents the membership function of each input variable; D_1 represents the membership function of output variable. Where,

- D: membership function for the antecedents (membership neural module);
- D^{-1}: membership function for the consequents (inverse membership neural module);
- $D_i(x)$: firing strength of each rule;
- μ_i: importance (weight) of each input;
- τ_i: importance (weight) of each rule;
- $[a_i, b_i]$: range value for each rule's output.

AggFIS has advantages in universal expression of fuzzy inference operators and importance factor of each criterion, which is trying to reflect fuzzy logic essence and simulate human thinking pattern.

4 CHOQUET INTEGRAL-OWA OPERATOR BASED ADAPTIVE UNIVERSAL FUZZY INFERENCE SYSTEM

A "universal" fuzzy inference system, which has the universal fuzzy inference operators and considers the importance factor during reasoning process, can be transformed into the general structure of fuzzy neural network by combining with feedforward neural network. Such model has the capability of learning and we call it as "adaptive universal fuzzy inference system (AUFIS)". Through training, the parameters in this model can be adjusted and it is easy to extract fuzzy rules that described I/O relationships of a nonlinear system from the trained AUFIS.

If Choquet integral-OWA operator based fuzzy inference system (AggFIS) is incorporated into a feedforward neural network according to the above theory, we obtain the adaptive model for AggFIS, which is known as Choquet integral-OWA operator based adaptive universal fuzzy inference system (Agg-AUFIS).

In AggFIS, we choose the differentiable fuzzy inference operators which ensure the reasoning process continuity so that Agg-AUFIS has the ability of learning and the adaptability to the data.

In this section, we first put forward the model structure of Choquet integral-OWA operator based adaptive universal fuzzy inference system (Agg-AUFIS) and then discuss the learning rules for it.

4.1 Model Description

We assume Agg-AUFIS under consideration has two inputs x and y and one output f. Suppose that the rule base contains two fuzzy if-then rules:

Rule 1: *If x is A_1 and y is B_1 , Then f is C_1;*

Rule 2: *If x is A_2 and y is B_2 , Then f is C_2.*

Agg-AUFIS model consists of five layers which is shown in Figure 4, output of each layer is as following.

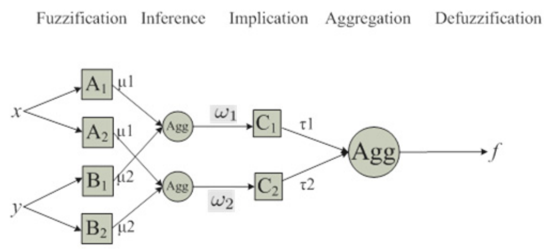

Figure 4: Agg-AUFIS model.

Layer 1: fuzzification layer:

Generate the membership grades μ_a, μ_b

$$O_{1,i} = \mu_{A_i}(x) \quad i=1,2 \quad \text{or}$$
$$O_{1,i} = \mu_{B_{i-2}}(y) \quad i=3,4 \qquad (1)$$

The membership function is the generalized bell function:

$$\mu_{A_i}(x) = \frac{1}{1 + [((x - c_i)/a_i)^2]^{b_i}} \quad (2)$$

Where $\{a_i, b_i, c_i\}$ is the parameter set which refers to premise parameters.

Layer 2: inference layer or rule layer:

$$O_{2,i} = \omega_i = [\overline{\mu_1} + \mu_1 \times \mu_{A_i}(x)] \times [\overline{\mu_2} + \mu_2 \times \mu_{B_i}(y)] \quad i=1,2 \quad (3)$$

Generate ω_1, ω_2 by OWA operator, where μ_i indicates the importance (weight) of each input.

Layer 3: implication layer:

$$O_{3,i} = \omega_i \circ C_i \quad i=1,2 \quad (4)$$

Implication operator is product. The consequent parameters are determined by Ci.

Layer 4: aggregation layer:

$$O_4 = \sum(\omega_i \circ C_i - \omega_{i-1} \circ C_{i-1}) \times \tau_i \quad i=1,2 \quad (5)$$

Aggregation operator is Choquet integral, where τ_i indicates the importance (weight) of each rule.

Layer 5: defuzzification layer

$$O_5 = f = D \circ O_4 \quad (6)$$

Compute the crisp output f. The defuzzification method is COA (center of area).

In Agg-AUFIS, the parameters that need to be adjusted include:

(1) Premise parameters: A_1, A_2, B_1, B_2 represent the premise parameters. The type of the inputs membership functions (MF) is generalized bell functions, each MF has 3 nonlinear parameters. The total number of premise parameters is 12.

(2) Consequent parameters: C_1, C_2 represent the consequent parameters. If the consequent MF is trapezoidal membership function, each MF has 4 nonlinear parameters to be adjusted. The total number is 8.

(3) μ_i: importance (weight) of each input. Total number is 2.

(4) τ_i: importance (weight) of each rule. Total number is 2.

Total nonlinear parameters in this example are 24 which can be adjusted according to the parameter updating formula discussed in the following.

4.2 Learning Rules for Agg-AUFIS

In this section, we conclude the learning rules for Agg-AUFIS based on main idea of back propagation (BP) in NN through discussing the general weight updating formula in detail, which could provide a theoretical reference for further study of learning rules in adaptive fuzzy inference system (Rumelhart *et al*., 1986; Rumelhart, 1994).

As the name implied, an adaptive FIS is a network structure, the overall input-output behaviour of which is determined by a collection of modifiable parameters. A feedforward adaptive FIS is a mapping between its inputs and output spaces. Our goal is to construct a network for achieving a desired nonlinear mapping which is regulated by a data set consisting of desired input-output pairs of a target system to be modeled: this data set is called training data set. The procedures that adjust the parameters to improve the network's performance are called the learning rules that explain how these parameters (or weights) should be updated to minimize a predefined error measure.

The basic idea of BP is to define a measure of the overall performance of the system and then to find a way to optimize that performance. The error measure computes the discrepancy between the network's actual output and a desired output. We then obtain the learning rules by putting a specific optimization technique on the error measure. The steepest descent method is used as a basic learning rule. It is also called back propagation because the gradient vector is calculated in the direction opposite to the flow of the each node output.

We change the parameters of the system in proportion to the derivative of the error with respect to the weights. This simple procedure works remarkably well on a wide variety of problems. A key advantage of neural network systems is that these simple, yet powerful learning procedures can be defined, allowing the systems to adapt to their environments.

In Agg-AUFIS model, we use back propagation as the basic learning rule which means using gradient vector in steepest descent method to update all the nonlinear parameters. Once the gradient is computed, regression techniques are used to update parameters in the model and we conclude the parameters updating formula for Agg-AUFIS as follows:

$$\omega_{next} = \omega_{now} + \Delta\omega_{ij}$$
$$\Delta\omega_{ij} = -\eta \cdot \frac{\partial E}{\partial \omega_{ij}} = -\eta \cdot \frac{\partial E}{\partial x_i} \cdot \frac{\partial f_i}{\partial \omega_{ij}} = -\eta\varepsilon_i \cdot \frac{\partial f_i}{\partial \omega_{ij}} \quad (7)$$

$j<i$, that is $x_i = f_i(\sum \omega_{ij} . x_j + \theta)$, f_i and x_i means the activation function and output of node i. Error signal ε_i, which can be derived by the previous layer nodes, starts from the output layer and goes backward until the input layer is attained.

The general parameters updating formula is:

$$\omega_{next} = \omega_{now} - \eta \cdot (d_i - x_i) \cdot x_j \cdot X \quad (8)$$

Where, η is the learning step, d_i is the desired output for node i, x_i is the real output for node i, x_j is the input for node i, X is a Polynomial, which is:

$$X = x_i \times (1 - x_i) \quad (9)$$

5 EXPERIMENTS

In order to verify the validity of Choquet integral-OWA operator based Adaptive Universal Fuzzy Inference System (Agg-AUFIS) presented in this paper, we established Agg-AUFIS for evaluation of traffic level of service, which are trained and tested by historical sample data (1429 pairs for training and 640 pairs for testing).

Testing errors are shown in Figure 5. Average test error is 0.057391. The worst test error is 0.4154 while the best test error is 1.6785e-005. The results indicated that Agg-AUFIS could be well adapted to sample data and it is a kind of universal approximator.

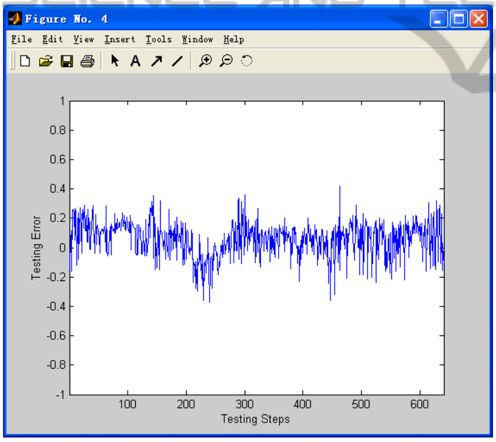

Figure 5: Testing error.

6 CONCLUSIONS

Based on FIS capability of simulating human reasoning process and dealing with nonlinear system problems, this paper presents a Choquet integral-OWA operator based fuzzy inference system (AggFIS) that is universal in reasoning operator selection, inference model structure and importance factor expression, and its adaptive model known as Agg-AUFIS. The experiments results showed that Agg-AUFIS has great non-linear mapping function and complex system modeling capacity. The comparative experiments will be made between Agg-AUFIS and existing similar systems, which could verify the advantages and effectiveness of proposed model in future work.

REFERENCES

Esragh, F., Mamdani, E. H., 1981. *A general approach to linguistic approximation. Fuzzy Reasoning and Its Applications*, Academic Press. London.

Furuhashi, T., 2001. Fusion of fuzzy/neuro/evolutionary computing for knowledge acquisition. *Proceedings of IEEE*, 89(9), 1266-1274.

Grabisch, M., 2000. A Graphical Interpretation of the Choquet Integral. *IEEE transactions on Fuzzy Systems*, 8(5), 627-631.

Kelman, A., Yager, R. R., 1995. On the Application of a Class of MICA Operators. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, 3,113-125.

Mamdani, E. H., 1977. Application of Fuzzy Logic to Approximate Reasoning Using Linguistic Synthesis. *IEEE transactions on computer*, 26(12), 1182-1191.

Pedrycz, W., Lam, P., Rocha, A. F., 1995. Distributed fuzzy system modeling. *IEEE transactions on fuzzy systems, man, and cybernetics*, 25(5), 769-780.

Pedrycz, W., Oliveira, J., 1996. Optimization of fuzzy models. *IEEE transactions on fuzzy systems, man, and cybernetics -PART B: cybernetics*, 26(4), 627-636.

Rumelhart, D. E., Hinton, G. E., Williams, R. J., 1986. *Learning internal representations by error propagation. Parallel distributed processing: explorations in the microstructure of cognition*, MIT Press. Cambridge.

Rumelhart, D. E., 1994. The Basic Ideas in Neural Networks. *Communications of the ACM*, 37(3), 87-92.

Yager, R. R., 1988. On Ordered Weighted Averaging Aggregation Operators in Multicriteria Decisionmaking. *IEEE transactions on fuzzy systems, man, and cybernetics*, 18(1),183-190.

Yager, R. R., Filev, D. P., 1992. SLIDE:A simple adaptive defuzzification method. *IEEE transactions on Fuzzy Systems*, 1(1), 69-78.

Yager, R. R., 1993a. On a Hierarchical Structure for Fuzzy Modeling and Control. *IEEE transactions on fuzzy systems, man, and cybernetics*, 23(4), 1189-1197.

Yager, R. R., 1993b. Families of OWA operators. *Fuzzy Sets and Systems*, 59, 125-148.

Yager, R. R., 1996. On Mean Type Aggregation. *IEEE transactions on fuzzy systems, man, and cybernetics - PART B: cybernetics*, 26(2), 209-221.

Zadeh, L. A., 1965. Fuzzy Set. *Information and Control*, 8, 338-353.

Zadeh, L. A., 1973. Outline of a New Approach to the Analysis of Complex Systems and Decision Processes. *IEEE Transactions on Systems, Man, and Cybernetics*, 3(1), 28-44.

Zadeh, L. A., 1988. Fuzzy Logic. *Computer*, 21(4), 83-93.

Zadeh, L. A., 1994. Soft Computing and Fuzzy Logic. *IEEE Software*, 11(6), 48-56.