

The Fuzzy Nature of Climate Change Scenarios Maps

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Abstract: The most important uncertainties present in the global change scenarios are the climate sensibility, represented by the wide variety of GCM's available, and the uncertainty that comes from the different GHG emission scenarios. Starting from a fuzzy climate model constructed with concentrations of GHG, obtained as a result of linear emission pathways, and output temperatures obtained with a deterministic simple climate model (MAGICC) it has been determined the output fuzzy set of global delta T thresholds such as 1, 2, 3 and 4 °C for 2100 and a medium sensibility of 3.0 °C/W/m². These fuzzy sets are used for assign uncertainties to values of temperature increase and precipitation change percentage taken from a map of regional climate change and for interpret the map in a fuzzy sense. We present some maps of temperature increase and precipitation change percentage for Mexico.

1 INTRODUCTION

Many previous works have been published about the topic of climate change scenarios caused by global warming, as can be verified by reviewing the latest three assessment reports of the Working Group I of the Intergovernmental Panel on Climate Change (IPCC 2001, 2007 and 2013, all of them available at <http://www.ipcc.ch/>).

There have been various proposals for physical climate change scenarios based primarily on different ways to estimate future greenhouse gas (GHG) and, also, a collection of general circulation models with atmosphere and ocean coupled (AOGCM) or updated versions of these.

The scenarios are regularly presented as maps, grid or contours, with a certain spatial resolution, and in different spatial domains as global, regional or local. Different GHG emission scenarios, AOGCM's and time horizons for climatic variables, such as temperature and precipitation, are also considered (Conde et al., 2011).

However, the manner to interpret the scope of these projections is complicated by the fact that there are different sources of uncertainty associated with the various inputs used in the development of climate change scenarios. It is especially difficult for both, scientists and decision makers, to take into account the predominantly epistemic nature of uncertainties in climate change to design adaptation

strategies or mitigation measures, so sometimes, statistical methodologies, that may not be the appropriate, are used (Gay and Estrada, 2009).

In previous works (Gay, et al., 2012, 2013, Gay and Sánchez, 2013) it has been explored the use of fuzzy logic in the representation and interpretation of the uncertainties of climate change scenarios, because this formulation allows the natural inclusion, through linguistic rules, of the different sources of uncertainty; for fuzzy logic no concept have precise limits (Zadeh 1965).

Using models of type FIS (Fuzzy Inference System) has been achieved to relate, via linguistic rules of IF-THEN form, fuzzy sets associated to values of climate sensitivity and GHG emissions with the values of the global temperature provided as output variable from AOGCM's (Gay, et al., 2012, 2013).

It is by means of changes in temperature global, regional or local, that other climatic variables and the different climate subsystems show the effects of climate change (Gay and Sánchez, 2013).

Based on the fuzzy model presented by Gay et al., (2013) and the simple climate model contained in Magicc/Scengen (Wigley, 2008) we show how the global mean temperature increase is distributed on the globe for the significant thresholds of 1, 2, 3 and 4 °C. The linear emission pathways, used to build the fuzzy model, include all the possibilities mentioned in successive reports of IPCC.

In this work we consider the possibility of analyzing the impacts, at the regional level, of temperature increase and precipitation changes from the perspective of the year in which some temperature is reached. Two sources of uncertainty are taken into account, the emissions of GHG and the climate sensitivity.

We have learned that the larger concentration and sensitivity the sooner the successive thresholds of temperature will be reached. If the sensitivity is 6 °C/W/m² there is no way of staying at two degrees unless the concentrations of CO₂ had followed the -2CO₂ trajectory: negative emissions that means very strong subtraction of CO₂ from the atmosphere. We think that it is easier to consider a degree by degree strategy than one based on dates.

In Gay et al., (2013) were presented maps for 2 GCM's (as an example) with the necessary concentration to reach 1, 2, 3 and 4 °C limits to 2100. The maps show the spatial distribution of the temperature increase over the globe.

Emissions and sensitivity introduce uncertainties in the temperature that in turn must be reflected in the scaled temperature displayed in a map. Other source of uncertainty considered is the GCM. The maps constructed for different GCM's illustrate all possibilities for a region of the globe.

Additionally, we present maps on a regional scale for Mexico, corresponding to the global maps mentioned above. It is evident that maps were constructed from Magicc/Scengen for each GCM.

In this work we show how the GCM's introduce uncertainty in the estimates of precipitation change in global and regional scales.

2 METHOD

We use the results reported by Gay and Sanchez (2013) consisting in linear emission paths (Figure 1), and the concentrations, forcings and temperatures calculated with the use of the Magicc/Scengen up to the year 2100, to discuss the timing of reaching a warming of 1, 2, 3 and 4 degrees centigrade. To illustrate how this can be done we observe from the temperature profile that corresponds to the emission path labeled 5CO₂ (that is the linear profile whose value in 2100 is five times the emissions in 1990), when the curve crosses the 1, 2, 3 and 4 °C, thresholds and look at the time when this happens (see Figure 2). These dates depend on the sensitivity used in the model and occur sooner as the sensitivity increases.

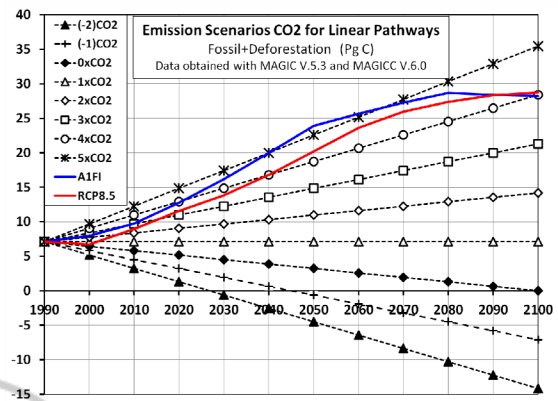


Figure 1: Linear emission pathways as proposed by Gay et al., (2012). As noted there, (-2) CO₂ means -2 times the emission (fossil + deforestation) of CO₂ of 1990 by 2100 and so for -1, 0, 1, to 5 CO₂. All the linear pathways contain the emission of non CO₂ GHG as those of the A1FI and were inserted in MAGICC V.5.3 (Wigley, 2008). Here we include the emission scenario corresponding to RCP 8.5 obtained from Magicc V.6.0 (Meinshausen, et al., 2011).

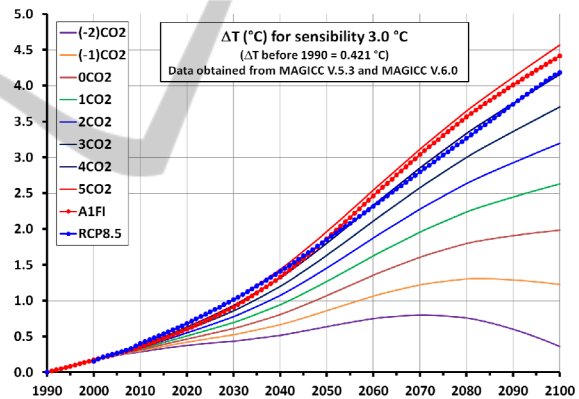


Figure 2: The corresponding global temperature increments for emission pathways of Figure 1, calculated from MAGICC V.5.3 and 6.0. Note the similarity between curves A1FI and 5CO₂, and also between curves RCP8.5 and 4CO₂.

For example, one degree may happen as soon as 2021, 2 °C as soon 2039 as it is shown in Tables 1 and 2 of Gay and Sanchez (2013) reproduced here for clarity. If the emissions followed more moderate paths the dates of crossing the thresholds would be delayed. The lesson from this is obvious: the lower the emissions the later the thresholds would be crossed and the more time we would have to adapt to increased temperatures. Then again using the Magicc/Scengen, maps were found for 1 to 4 °C. These maps may serve as planning tools for adaptation studies.

Since the temperature and concentration of the A1FI and RCP8.5, of the IPCC are very close to our profile corresponding to large emissions then the timing can be applied to those profiles as well (as we can see in Figure 2).

Table 1: Dates to achieve the 1 °C threshold following linear emission trajectories from -2CO2 to 5CO2.

Emission Trajectory	Sensitivity (deg C/W/m2)		
	1.5	3.0	6.0
-2CO2			2049
-1CO2		2057	2039
0CO2	2079	2048	2033
1CO2	2063	2042	2029
2CO2	2056	2038	2027
3CO2	2051	2035	2024
4CO2	2047	2032	2023
5CO2	2044	2030	2021
B1-IMA	2090	2043	2027
A1FI-MI	2046	2033	2024

Table 2: Dates to achieve the 2 °C threshold following linear emission trajectories from -2CO2 to 5CO2.

Emission Trajectory	Sensitivity (deg C/W/m2)		
	1.5	3.0	6.0
-2CO2			
-1CO2			2073
0CO2		2100 (1.98°C)	2059
1CO2		2072	2052
2CO2		2064	2048
3CO2	2093	2058	2045
4CO2	2081	2054	2042
5CO2	2053	2051	2039
B1-IMA			2057
A1FI-MI	2076	2053	2042

In this work we emphasize the possibility of analyzing the impacts of temperature increase from the perspective of the year in which some temperature is reached at the regional level. We think that it is easier to consider a degree by degree strategy than one based on dates. In other words having information of the approximate dates in which different thresholds, for example, one degree would be crossed, would enable a policy maker to

act on those sectors or activities that would be affected by one degree leaving for later those that would be affected by larger temperature changes. Knowing the timing can help the planning process.

Postponing the action complicates matters because the uncertainties become larger. For example the projections of temperatures and precipitation in 2100 depend on two sources of uncertainty, the emissions path of GHG and the climate sensitivity. For example, in 2100 the uncertainty associated to a one degree of global increase extends to more than two degrees, then for a 1 °C global increase, maps for one and two degrees are to be considered. For 4 °C and sensitivity 3, uncertainty can expand the temperature range to 6.41 °C.

Emissions and sensitivity introduce uncertainties in the temperature that in turn must be reflected in the scaled temperature and precipitation displayed in a map. Other source of uncertainty considered is the GCM itself. What we mean here is that the map for a one degree increase given for the GFDL 2.0 (Geophysical Fluid Dynamics Laboratory Coupled Model, version 2.0) is going to be slightly (or seriously when talking of precipitation) different that the one coming from the HADGEM1 (Hadley Centre Global Environmental Model version 1). The choice of AOGCM's has been somewhat arbitrary.

3 REGIONAL VIEW OF GLOBAL ΔT OVER TEMPERATURE AND PRECIPITATION IN MEXICO

Maps on a regional scale for Mexico, corresponding to the global maps mentioned above are constructed by interpolation methods for temperature and precipitation; we show some of them (Figures 3 to 10). Arguments mentioned above for global maps apply as well to these.

The size of the grid in the data obtained from Magicc/Scengen, is relatively big (2.5° x 2.5°) and, for purposes of regionalization, we reduced it to 0.5° x 0.5° (about 10 Km x 10 Km) applying the method of splines as presented in Conde et al., (2011).

The maps of regional climate change scenarios for Mexico, over temperature and precipitation, are presented according to each ΔT threshold.

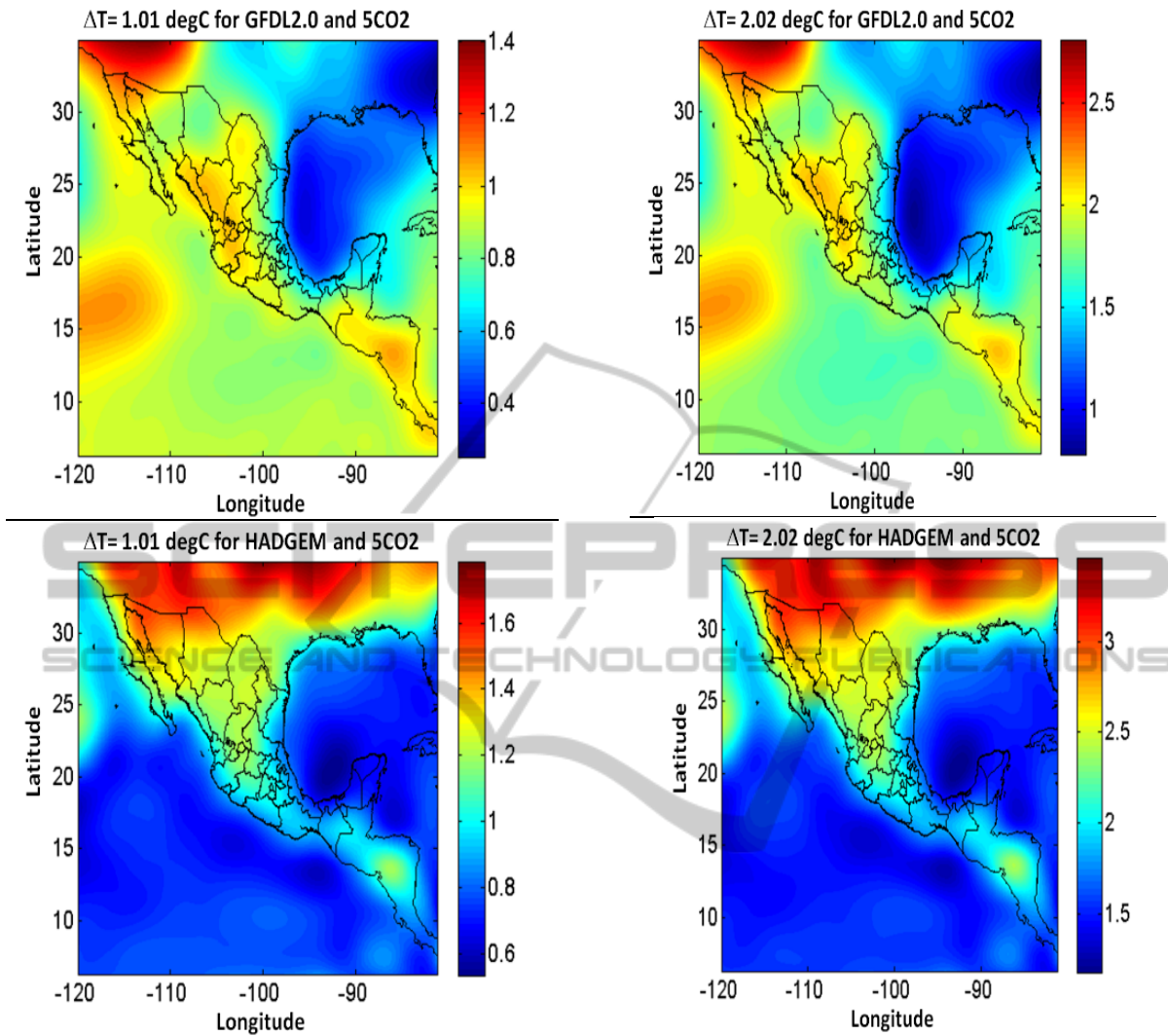


Figure 3: Regional scenario for temperature at $\Delta T_{\text{global}} = 1.01 \text{ }^\circ\text{C}$ threshold, according to GFDL 2.0 (upper panel) and HADGEM1 (lower panel) for 5CO2 emission trajectory. Maps were obtained using Magicc/Scengen V. 5.3 data and MATLAB script.

Figure 4: Regional scenario for temperature at $\Delta T_{\text{global}} = 2.02 \text{ }^\circ\text{C}$ threshold, according to GFDL 2.0 (upper panel) and HADGEM1 (lower panel) for 5CO2 emission trajectory. Maps were obtained using Magicc/Scengen V. 5.3 data and MATLAB script.

In this work we show how the GCM's introduce uncertainty in the estimates of precipitation change in global and regional scales. This is simply due to the fact that modeling strategies and parameterizations differ from model to model. For example Table 3 and 4 illustrate for 3 points in the map the differences in the values of the temperature and precipitation (for the same global temperatures) at the same geographical position produced by different models.

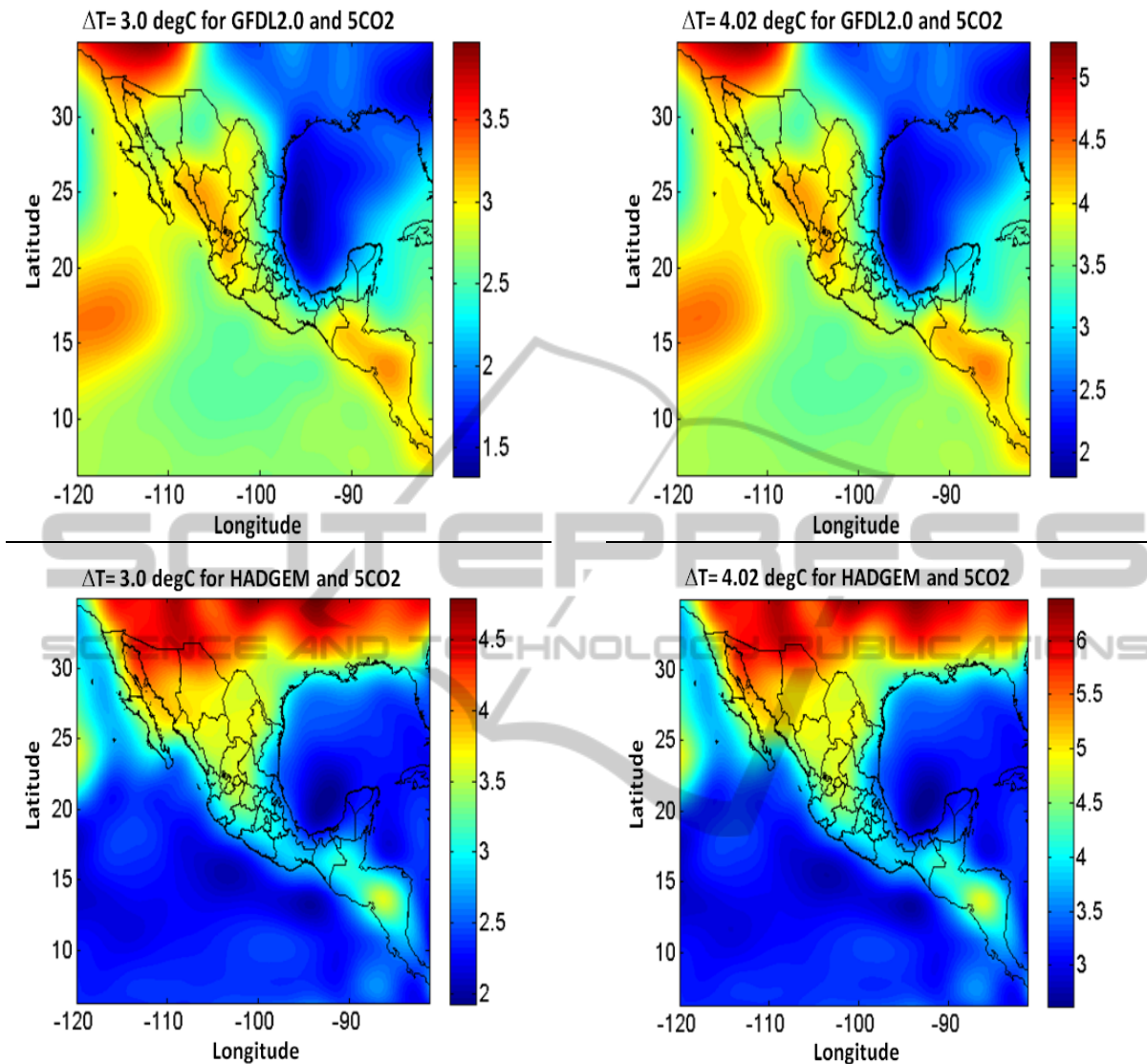


Figure 5: Regional scenario for temperature at $\Delta T_{\text{global}} = 3.0 \text{ }^\circ\text{C}$ threshold, according to GFDL 2.0 (upper panel) and HADGEM1 (lower panel) for 5CO2 emission trajectory. Maps were obtained using Magicc/Scengen V. 5.3 data and MATLAB script.

Figure 6: Regional scenario for temperature at $\Delta T_{\text{global}} = 4.02 \text{ }^\circ\text{C}$ threshold, according to GFDL 2.0 (upper panel) and HADGEM1 (lower panel) for 5CO2 emission trajectory. Maps were obtained using Magicc/Scengen V. 5.3 data and MATLAB script.

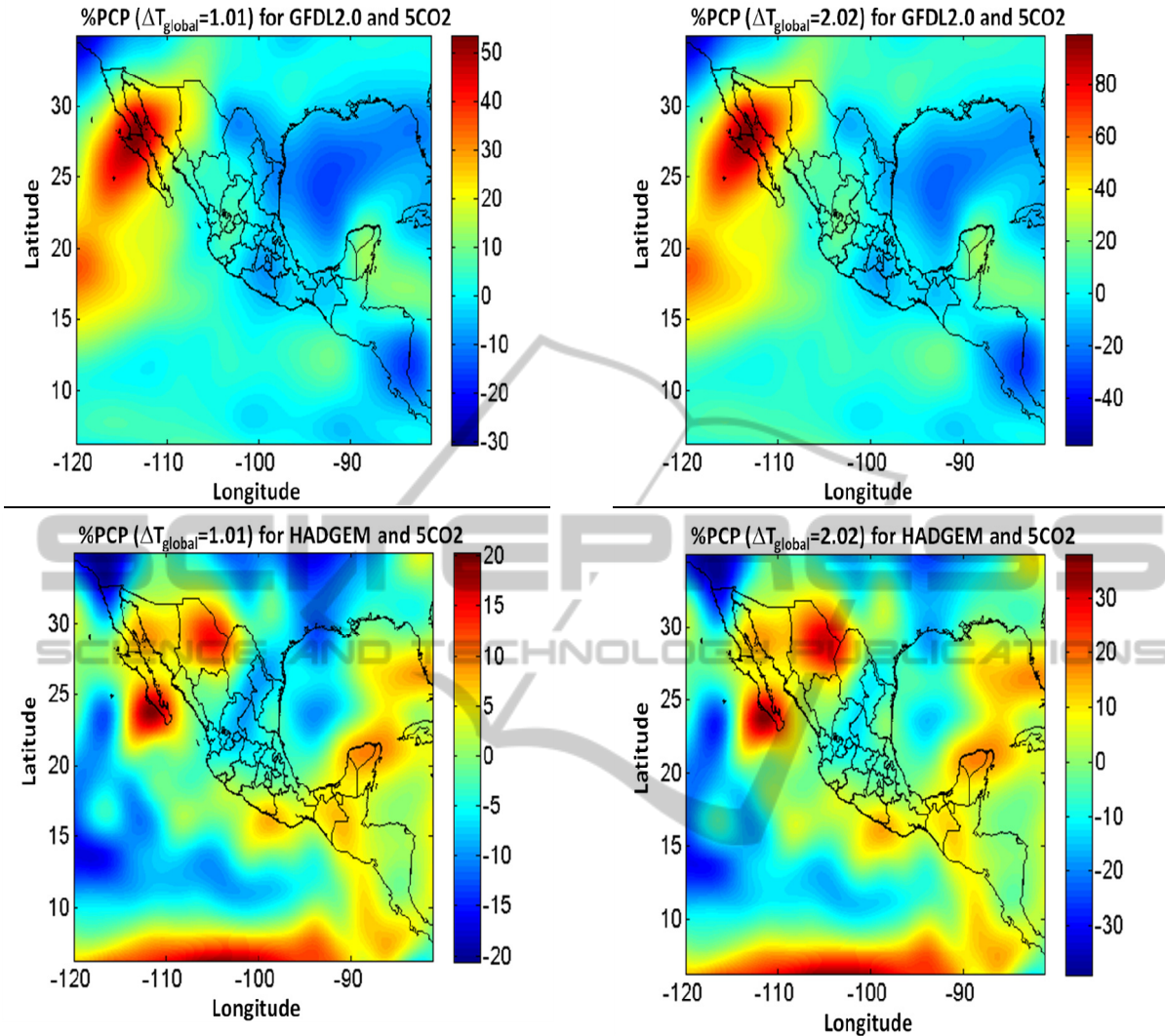


Figure 7: Regional scenario for precipitation at $\Delta T_{\text{global}} = 1.01$ °C threshold, according to GFDL 2.0 (upper panel) and HADGEM1 (lower panel) for 5CO2 emission trajectory. Maps were obtained using Magicc/Scengen V. 5.3 data and MATLAB script.

Figure 8: Regional scenario for precipitation at $\Delta T_{\text{global}} = 2.02$ °C threshold, according to GFDL 2.0 (upper panel) and HADGEM1 (lower panel) for 5CO2 emission trajectory. Maps were obtained using Magicc/Scengen V. 5.3 data and MATLAB script.

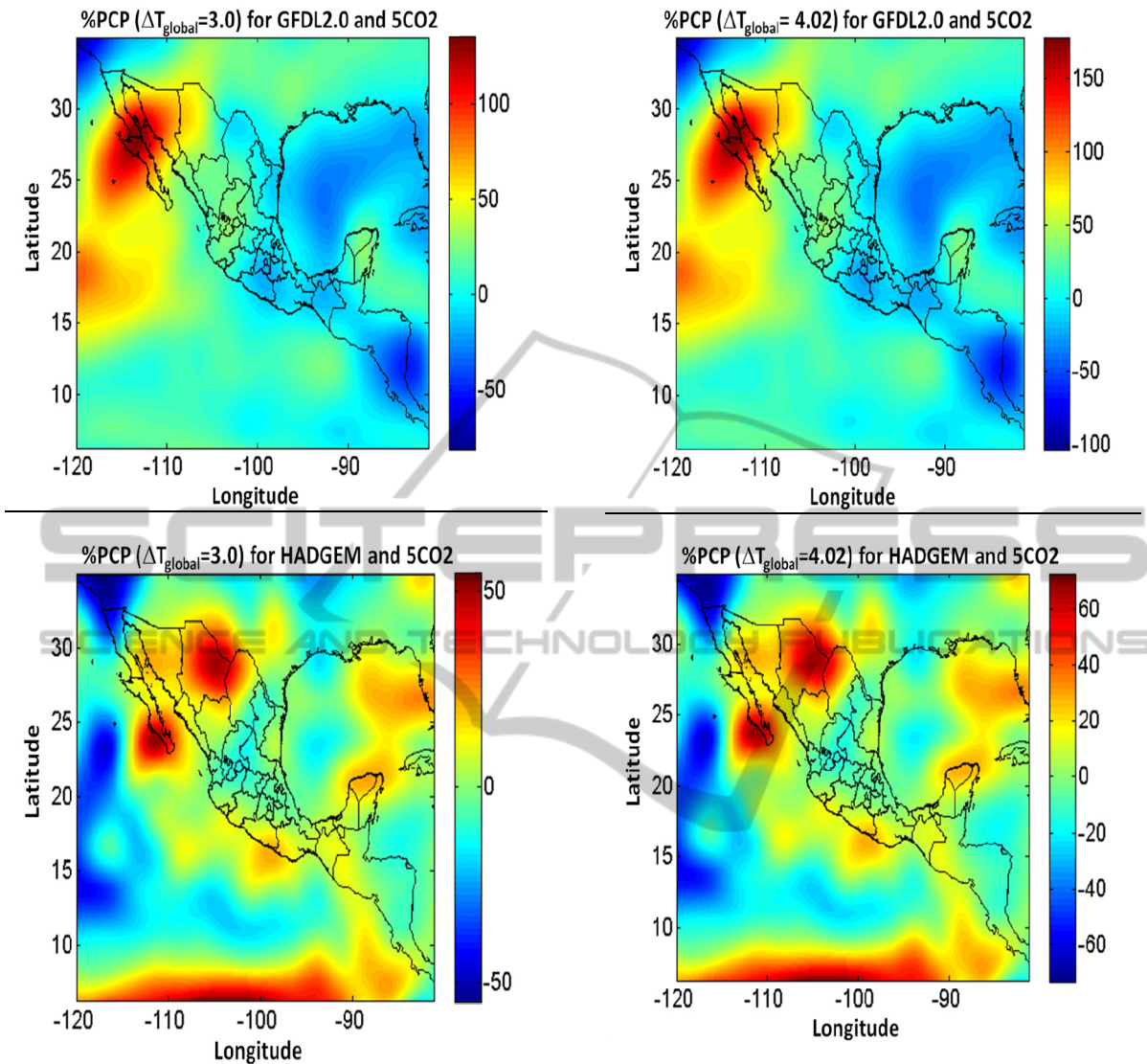


Figure 9: Regional scenario for precipitation at $\Delta T_{\text{global}} = 3.0$ °C threshold, according to GFDL 2.0 (upper panel) and HADGEM1 (lower panel) for 5CO2 emission trajectory. Maps were obtained using Magicc/Scengen V. 5.3 data and MATLAB script.

Figure 10: Regional scenario for precipitation at $\Delta T_{\text{global}} = 4.02$ °C threshold, according to GFDL 2.0 (upper panel) and HADGEM1 (lower panel) for 5CO2 emission trajectory. Maps were obtained using Magicc/Scengen V. 5.3 data and MATLAB script.

Table 3: Uncertainty produced by using model GFDL 2.0, projected to 2100 with emission pathway of 5CO2. Temperature and precipitation increments calculated for 3 points located NW (-108.75, 31.25), SW (-101.25, 21.25) and Central (-88.75, 21.25) Mexico. Data obtained from Magicc/Scengen V.5.3.

Delta T threshold (°C)			
1	2	3	4
Temperature (°C)			
0.90	1.92	2.80	3.75
0.92	1.87	2.80	3.77
0.61	1.32	2.12	2.87
Precipitation (%)			
19.96	38.54	54.8	72.96
1.33	5.8	11.88	16.71
9.69	18.25	22.31	28.97

Table 4: Uncertainty produced by using model HADGEM1, projected to 2100 with emission pathway of 5CO2, Temperature and precipitation increments calculated for 3 points located NW (-108.75, 31.25), SW (-101.25, 21.25) and Central (-88.75, 21.25) Mexico. Data obtained from Magicc/Scengen V.5.3.

Delta T threshold (°C)			
1	2	3	4
Temperature (°C)			
1.54	3.12	4.45	5.95
1.16	2.33	3.44	4.61
0.68	1.45	2.29	3.1
Precipitation (%)			
4.23	9.17	13.99	18.86
-6.76	-9.29	-9.09	-11.09
9.61	18.1	22.11	28.7

For example for the Northwest point (-108.75, 31.25) the local temperature depends on the global one and is not necessarily the same: If the global T is 1°C the local is 0.9 for the GFDL model and is different for the other model, 1.54 for the HADGEM. The contrasts for the precipitation are very large.

4 UNCERTAINTY OF ΔT_{GLOBAL} PROJECTED OVER THE MAPS

As mentioned before, (Gay and Sanchez, 2013) considered two sources of uncertainty contributing to the temperatures in 2100, the first coming from the emissions: large emissions mean large temperature changes, and the second due to our

imprecise knowledge of the climate sensitivity of the models. The first uncertainty is for the politicians to resolve because emissions depend on policy and if this is oriented towards lowering them, the temperatures could be kept within certain limits determined in part by the uncertainty in the sensitivity of the models. Therefore the second source is for the scientists who need to narrow the interval of climate sensitivity which is still too large as it is shown in our discussion.

Once we have the global mean temperatures and an idea of the associated uncertainty due to different emission paths and sensitivities, using the same idea for scaling employed in the Magicc/Scengen system (Wigley, 2008), we convert this information to a two dimensional maps of temperatures and precipitations.

If we denote the uncertainty by a Δ then we propose the following equations:

$$\Delta T_{new} = T_{grid}/T_{map} \times \Delta T_{magicc} \quad (1)$$

$$\Delta P_{new} = P_{grid}/T_{map} \times \Delta T_{magicc} \quad (2)$$

where ΔT_{magicc} , is the temperature produced by the fuzzy model of Gay and Sanchez (2013) which is in fact a fuzzy number and consequently ΔT_{new} and ΔP_{new} also are. T_{grid}/T_{map} (or P_{grid}/T_{map}) represent the normalized pattern of change for T (or P), i.e., the change of the variable per each degree centigrade of global warming.

The fuzzy model mentioned above, consisting of 18 fuzzy rules (Gay et al. 2013), is run to obtain global temperatures increases in year 2100 and their corresponding uncertainty intervals. This information is then used to produce two-dimensional maps depicting physically consistent geographical distributions of temperatures which in turn are consistent with global temperatures obtained from our fuzzy model.

In the fuzzy model we use the value of the sensitivity is fixed at the best estimate of 3 °C/W/m2 and varying the concentration we try to get 1, 2, 3 and 4 degrees centigrade. The temperature is a function of the concentration, the T7, T8, ... , T12 shown in the Figures 11 to 14 are the output temperature increase fuzzy sets whose characteristics were calculated via MAGICC data (Gay et al., 2013). In this way we obtain the following fuzzy values for the global temperatures:

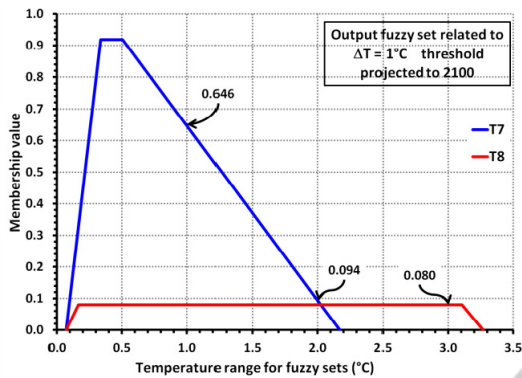


Figure 11: Fuzzy output temperature for 1 °C, its membership value is 0.646. The membership of 2 and 3 °C is marginal. Data obtained from the 18 rules FIS (Gay and Sanchez, 2013).

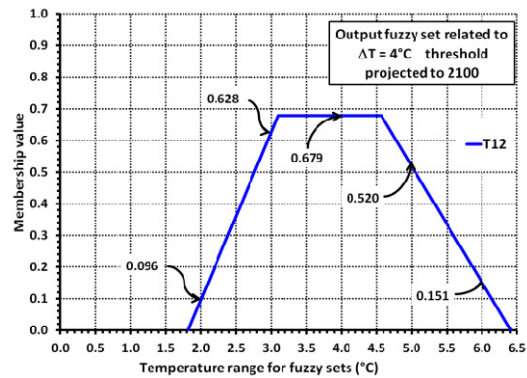


Figure 14: Fuzzy output temperature for 4 °C, its membership value is 0.679. The figure also shows the membership values for 2, 3, 5 and 6 °C. Data obtained from the 18 rules FIS (Gay and Sanchez, 2013).

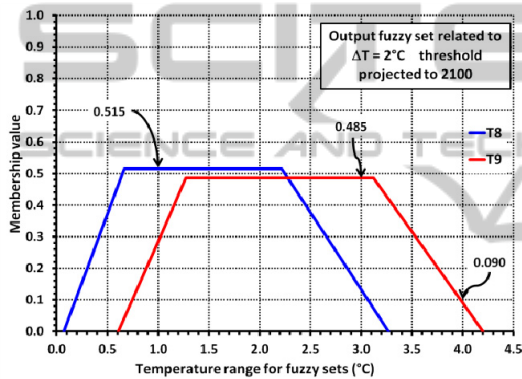


Figure 12: Fuzzy output temperature for 2 °C, its membership value is 0.596. The figure also shows the membership values for 1, 3 and 4 °C. Data obtained from the 18 rules FIS (Gay and Sanchez, 2013).

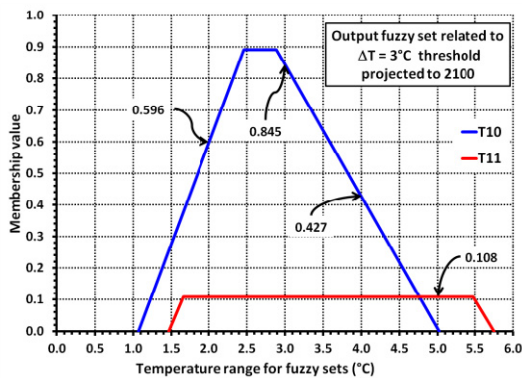


Figure 13: Fuzzy output temperature for 3 °C, its membership value is 0.845. The figure also shows the membership values for 2, 4 and 5 °C. Data obtained from the 18 rules FIS (Gay and Sanchez, 2013).

For an increase of one degree the concentration of CO2 required is 220 ppmv and the uncertainty interval is from 0.08 to 2.17 degrees, based on the fuzzy sets characteristics reproduced here as a graph (see Figure 11) consequently for a 1 °C global increase, maps for one and two degrees (see Figures 3 to 10) are to be considered with their respective membership value $\mu(1)=0.64$ and $\mu(2)=0.094$.

If ΔT is 2 degrees the interval is from 0.08 to 3.27 °C; for 3 and 4 degrees the uncertainty intervals are from 1.07 to 5.02 °C and from 1.82 to 6.41 °C respectively (Figure 12). Therefore for a 3 °C global increase the uncertainty extends to 5 °C so, maps corresponding to 3, 4 and 5 degrees should be considered.

Now we can interpret the maps obtained in the previous section (Figures 3 to 10) in a different way.

For 2100 the simple fuzzy model of Gay and Sanchez (2013) based on emission trajectories that span from -2CO2 to 3 times the emissions in 1990 (1CO2) (that means that the paths corresponding to 4 and 5 times the 1990 emissions have been left out) and the uncertainty of the climate sensitivity of the models produce temperatures that span from 1 to more than 5 degrees centigrade but with different membership values. This information is then used to assign a membership value to a whole map. This is done in the following way.

Let us assume that the temperature in 2100 is 2 °C but this is a fuzzy number with a membership value $\mu(2)=0.51$ but the fuzzy number also contains 1, 3 and 4 degrees with membership values of 0.51, 0.48, and 0.09 respectively. Therefore a warming of two degrees would mean that we have to take into consideration not only the map corresponding to 2 °C but also the corresponding to 1, 3, and 4 degrees except that with different weights provided by the

membership values, as it is shown in Table 5. The same assignment should be done to the precipitation maps. We have produced maps for 1, 2, 3 and 4 °C, that have been associated with different dates with the purpose of helping in the planning process. The same maps can be used to produce fuzzy results for 2100 and somehow demonstrate that the longer we wait the fuzzier the future becomes.

Table 5: Membership values for the delta T thresholds projected to 2100, corresponding to fuzzy output sets obtained from 18 rules fuzzy model. The CO2 concentrations listed are the values needed to achieve each threshold.

CO2 Concentration (ppmv)	Delta T thresholds (°C)			
	1	2	3	4
220	0.646	0.094	0.080	0
349	0.515	0.515	0.485	0.090
526	0	0.596	0.845	0.427
806	0	0.096	0.628	0.679

5 CONCLUSIONS

We have shown that, for decision-making purposes, it is easier and more convenient to consider a strategy based on degree by degree than one based on dates. The different climate subsystems are impacted by the global temperature increase, no matter what date it occurs. The only determinant parameter of the magnitude of impact is its climate sensibility. Having information of the approximate dates in which different thresholds, for example, one degree would be crossed, would enable a policy maker to act on those sectors or activities that would be affected by one degree leaving for later those that would be affected by larger temperature changes.

It has been depicted that linear emission pathways, proposed in early works, include all the possibilities mentioned in successive reports of IPCC, including the RCP ones.

We have considered the uncertainty of GHG concentrations and the uncertainty of climate system sensibility as the more important. We know now that the larger concentration and sensitivity the sooner the successive thresholds of temperature will be reached and the wider uncertainty intervals, too.

The uncertainty related to the process of selecting AOGCM's has its origin in the uncertainty of climate sensibility.

The uncertainty generated from the AOGCM's results, available in the literature, has been estimated from the construction of ensembles of model projections for different dates, the range of uncertainty in these projections is statistically assigned by means of simple standard deviation (Wehner, 1998) or, with a more complex procedure, using criteria such as the performance of the actual climatic conditions and convergence of projections, for each AOGCM over a determined geographical region (Giorgi and Mearns, 2002). The meaning of averaging the results of different AOGCM's, each one with a different physical vision of the climate, is not clear.

Several maps, representing climate change scenarios, have been presented and a discussion about them has been done. For that purpose we select a couple of AOGCM's and an emission pathway of 5CO2, the more "pessimist", but it covered till 4 degrees of temperature increase by the 2100.

Starting from a fuzzy model of type FIS and a simple climate model (MAGICC) we have obtained a method to interpret the uncertainties involved in the construction of global change scenarios of temperature and precipitation. The key has been to consider the uncertainty interval determined by the fuzzy model outputs for each one of the global temperature thresholds, and extend it to the other variable.

As a result of the above, it is possible to see the fuzzy perspective of the scenarios described in the maps. It can be stated that the climate scenario for a given global ΔT map contains, in some degree, the maps corresponding to other (adjacent) thresholds. As an example, we found that the map for a ΔT of 2 °C contains the map for 1, 3 until 4 °C, but the maps for 1, 2 and 3 °C are almost indistinguishable from each other.

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