

# Consensus Coordination in the Network of Autonomous Intersection Management

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**Keywords:** Autonomous Intersection Management, Intelligent Transportation System, Autonomous Vehicle, Vehicle to Infrastructure Communication, Infrastructure to Infrastructure Communication, Consensus Algorithm.

**Abstract:** The Autonomous Intersection Management (AIM) will be a future method for the Intelligent Transportation System. It combines wireless communication and the autonomous vehicle in order to create the new concept for managing road traffic more safely and efficiently. The distributed control principle is applied to the intersection network to control the traffic in the macroscopic level. The Vehicle to Infrastructure (V2I) and Infrastructure to Infrastructure (I2I) communication are used to exchange the traffic information between a single autonomous vehicle to the network of autonomous intersections. The discrete time consensus algorithm is implemented to coordinate the gross traffic density of an intersection and its neighborhoods in the network. The boundary condition for the uncongested flow is created by using the Greenshield's traffic model. The proposed method represents the ability to maintain the traffic flow rate of each intersection and operates with the uncongested flow condition. The simulation results of the network of a multiple autonomous intersection are provided.

## 1 INTRODUCTION

The traffic congestion problem is increasingly becoming a severe problem in the road transportation. The research in the Intelligent Transportation System tries to find a solution to improve the traffic safety and efficiency. There were several researches in controlling the traffic signal due to the fixed timing traffic signal, indicating a poor performance in managing traffic. One of the active solutions is using the technique of the adaptive traffic signalling. The traffic signal can be adjusted adaptively based on the current traffic situation. There are many methods to adjust the traffic signal. The commercial solution called SCOOT (Robertson, 1991) determines the period of green and red light by using the queue length of each street. In (Chiu, 1993), Fuzzy logic was applied to update the signal, based on the constructing rules.

The Autonomous Intersection Management (AIM) concept is a totally autonomous system that combines the technology of the autonomous vehicle and the wireless communication. According to the intelligence of an autonomous vehicle (Wuthishuwong, 2008), the road accidents that are

caused by human driver errors can be reduced. The objectives of creating a full autonomous system are to improve the traffic safety and traffic efficiency by using autonomous vehicles and an autonomous intersection manager. The AIM (Dresner, 2008) was studied based on the multi-agents technique. Vehicle agents communicate to an intersection agent to reserve the area. The successful reservation will have no conflict with the others. Otherwise, the reservation will be rejected. In (Naumann, 1998), (Zou, 2003) used the same concept but without the intersection agent. Vehicle agent negotiates with each other in order to cross an intersection. In (Wuthishuwong, 2013) used the V2I communication to plan the safe trajectory for each vehicle whilst crossing an intersection. The extend version from a single AIM to the multiple AIM in (Wuthishuwong, 2013) was studied the technique for maintaining the traffic flow in the network by coordinating the local traffic information between its neighbourhood.

In this paper, the authors propose the consensus algorithm in order to coordinate the traffic information between each autonomous intersection in the network. The multiple intersections scenario is modelled. As well, the communication topology

between Vehicle to Infrastructure (V2I) and Infrastructure to Infrastructure (I2I) are designed. Maintaining the continuity of the traffic flow in the network, the boundary condition of the uncongested flow is derived based on the Greenshield's traffic model. The simulation of a multiple Autonomous Intersection Management is presented. The results are plotted and evaluated with the Greenshield's model

## 2 INTERSECTION NETWORK

The intersection network is modelled by connecting 9 single intersections, where each intersection has 4 ways. It is based on the distributed control structure. Then, a single intersection is considered as an autonomous agent that has ability to control itself, whilst the control strategy is dependent on the information between its neighborhoods.

The graph theory (Murray, 2009) is used to visualize and interpret the interaction of a network. Technically, each intersection manager is assigned by a node and the connection between each node is represented by an edge. However, there are 2 classes of a node relationship.

### 2.1 Street Network

The street network is modeled based on the real physical connection of each intersection. Typically, an intersection is connected through an incoming and outgoing street. Hence, the street network is a set of street that connects a group of neighbored intersections as illustrated in Fig.1. At the street network, a single intersection acts as a central manager. Each single intersection collects the local traffic density on the connected street by counting

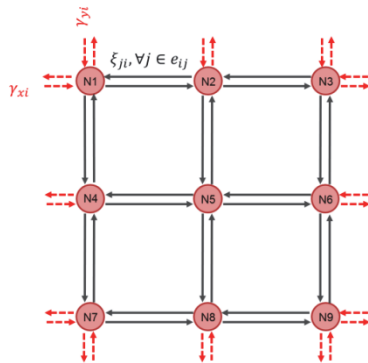


Figure 1: Network of streets with the direction flow of all intersection.

the requested messages that are transmitted from the incoming vehicles over the V2I communication. Therefore, each intersection manager in the network is identical and it responds to manage only its own local intersection. The collected traffic density information of each intersection can be determined by summing the traffic density of all incoming streets to intersection.

$$\rho_i = \sum_{j \in e_{ij}} \xi_{ji} + \sum_{x,y \in N_i} \gamma_{(x,y)i} \quad (1)$$

Where,  $\rho_i$  is the gross incoming traffic density of the intersection  $i$ ,  $\xi_{ji}$  is the incoming traffic density of a street has traveled from intersection  $j$  to intersection  $i$ , and  $\gamma_{(x,y)i}$  is the external incoming traffic from the sources  $x, y$  connect to the intersection  $i$ .

### 2.2 Communication Network

The communication topology of the intersection network is illustrated in Fig. 2. The connection between couple of nodes uses the bi-directional communication. Each node, which represents an intersection manager, can either receive or transmit the data package to their destination node

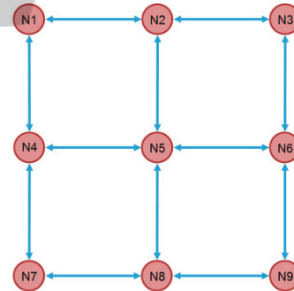


Figure 2: Intersection communication network topology.

The properties of a graph theory are used to represent the relationship of the intersection network. The adjacency element  $a_{ij}$ , will have value 1 when there is an edge between each node, otherwise the value is equal to 0.

$$a_{ij} = \begin{cases} 1, & (n_i, n_j) \in E \\ 0, & \text{Otherwise} \end{cases} \quad (2)$$

$$\underline{\mathcal{A}} = [a_{ij}]; i, j \in N_i$$

The degree matrix describes the number of connections at each intersection.

$$d_{ij} = \begin{cases} \sum_{i=1}^n a(i,j), & i = j \\ 0, & \text{Otherwise} \end{cases} \quad (3)$$

$$\underline{\mathcal{D}} = [d_{ij}]; i, j \in N_i$$

The Laplacian matrix describes the complete relationship of the intersection network. The simple way to determine the Laplacian matrix is subtracting the degree matrix with the adjacency matrix.

$$\underline{\mathcal{L}} = \underline{\mathcal{D}} - \underline{\mathcal{A}} \quad (4)$$

Where;  $i$  is the row element of the matrix,  $j$  is the column element of the matrix,  $\underline{\mathcal{A}}$  is the adjacency matrix,  $\underline{\mathcal{D}}$  is the degree matrix, and  $\underline{\mathcal{L}}$  is the Laplacian matrix.

### 3 CONSENSUS COORDINATION OF AIM

In this work, the discrete consensus algorithm is implemented for coordinating the traffic information in AIM in order to balance the overall traffic flow in the network. The consensus algorithm has been recently studied in robot application such as robot formation in (Ren, 2007; Olfati-Saber, 2003; Olfati-Saber, 2007). Naturally, it is the distributed control that gives the convergence property, which fits for the large scale system. The system architecture of the multiple, autonomous intersections management is illustrated in Fig. 3.

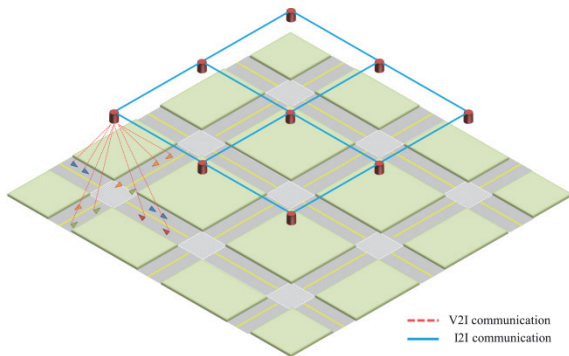


Figure 3: The system architecture of the multiple autonomous intersections management.

Each intersection acts as the centralized controller. The traffic density of each intersection is collected in the street network layer by using V2I communication and this information is distributed to

its neighborhoods in the intersection network layer over I2I communication. The AIM will compute the control command, based on the traffic density of itself and its neighborhoods.

The consensus algorithm is applied to coordinate the traffic information among the intersections in the group. The traffic density is used as the coordinated information, as well as, representing the state of an intersection. The dynamics of each local intersection and a global network can be expressed as the following equation.

$$\dot{\rho}_i = \sum_{j \in N_i} a_{ij}(\rho_j - \rho_i) \quad (5)$$

$$\dot{\rho} = -\mathcal{L}\rho \quad (6)$$

Substituting the gross traffic density of each intersection, which is defined in Eq.1, the consensus of a collective AIM can be derived as:

$$\begin{bmatrix} \dot{\rho}_1 \\ \vdots \\ \dot{\rho}_{N_i} \end{bmatrix} = -\mathcal{L} \cdot \begin{bmatrix} \rho_1 \\ \vdots \\ \rho_{N_i} \end{bmatrix} = -\mathcal{L} \cdot \begin{bmatrix} \sum_i \xi_{ji} + \sum_i \gamma_{ji} \\ \vdots \\ \sum_{N_i} \xi_{jN_i} + \sum_i \gamma_{jN_i} \end{bmatrix} \quad (7)$$

The discrete time consensus is derived by applying the difference equation. Then, the discrete time consensus for a local intersection and a global network can be expressed as the following equation.

$$\rho_i(k+1) = \rho_i(k) + \varepsilon \sum_{j \in N_i} a_{ij}(\rho_j(k) - \rho_i(k)) \quad (8)$$

$$\rho(k+1) = \mathcal{P}\rho(k) \quad (9)$$

Where,  $\mathcal{P}$  is a Perron matrix  $\mathcal{P} = I - \varepsilon\mathcal{L}$  and  $\varepsilon$  is the step size  $\varepsilon > 0$ . The sufficient conditions for the stability of a consensus in the network are provided in [9].

The control system of a multiple autonomous intersections is composed of nine units of AIM, which is the distributed control schema. Thus, each intersection control strategy is identical. The closed loop control block diagram of the autonomous traffic control of a single intersection is illustrated in Fig.4.

The autonomous vehicle is used in AIM system and practically AIM can only prioritize the timing of crossing an intersection. Thus, the control variable is the incoming time which can be transformed to the average velocity when the distance between a vehicle and intersection is known. Basically, every vehicle has to send the requesting message to AIM before crossing an intersection. With this point, the

traffic density and number of vehicles approaching an intersection, is measured through the V2I communication. It counts the number of messages of the incoming vehicles and subtracts the number of outgoing vehicles. However, the information is in a discrete time domain after sampling.

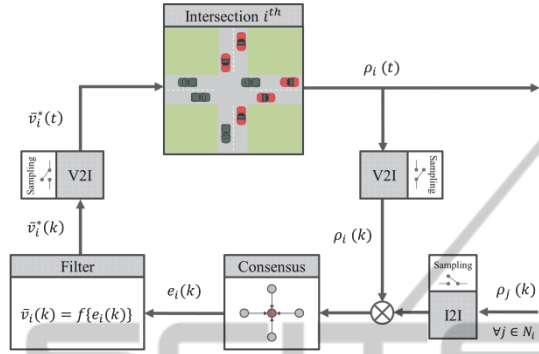


Figure 4: Closed loop control block diagram of a single intersection.

In order to control the traffic flow of an intersection, the traffic density of the neighborhoods is inputted through I2I communication. The consensus algorithm coordinates the information from itself with its neighborhood in order to determine the desired value of the traffic density following the Eq. 9. Therefore, the error term of each intersection is the difference between the desired traffic density and the current traffic density value. It can be expressed as the following equation.

$$e_i(k) = \rho_i(k + 1) - \rho_i(k) \quad (10)$$

Technically, the consensus algorithm try to balance the traffic density between the local intersections. This means it will maintain the level of traffic density closed to its neighborhoods, to keep the low variation between them. Theoretically, the error term must be minimized and approach zero in the finite time in order to make the current traffic density equal to the desired traffic density.

Referring to the field of transportation engineer, the traffic model is composed of three corresponding parameters: traffic density, traffic flow rate and average velocity. These relationships are used to represent the macroscopic traffic. In this work, the Greenshield's model is used as the reference traffic model. For controlling the traffic, the condition of the congested and uncongested traffic are defined by using the empirical data of the traffic density, average velocity and traffic flow rate. the relationship between the average velocity and the

traffic density with the boundary of congested and uncongested traffic is illustrated in Fig.5.

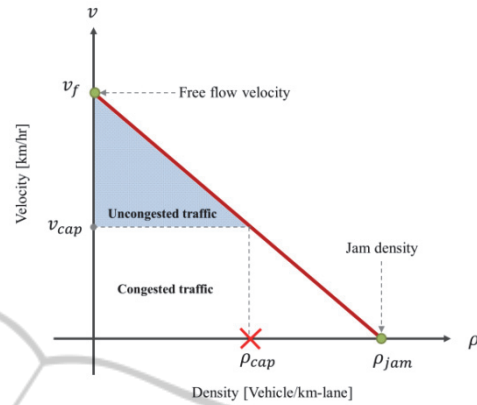


Figure 5: Greenshield's traffic model: the relationship between the average velocity and the traffic density.

According to the parameters of the Greenshield's model (Hall, 1996), the free flow velocity ( $v_f$ ) is given at 91 km/hr and the jamming density ( $\rho_{jam}$ ) is given at 78 vehicles/km/lane. The velocity at capacity ( $v_{cap}$ ) is given at 46 km/hr. The velocity at capacity is the lower boundary of the average velocity that vehicles can drive under the uncongested traffic. The traffic will begin to congest after this boundary, if the vehicles cannot keep the driving velocity at least at this level. Consequently, the traffic density at capacity ( $\rho_{cap}$ ) is the maximum number of vehicles on the street that still keeps the average velocity within the boundary. It can be determined as:

$$\rho_{cap} = \rho_{jam} \left( 1 - \frac{v_{cap}}{v_f} \right) \quad (11)$$

The traffic density at capacity is round up to 38 vehicles/km/lane. The boundary condition of the uncongested traffic can be summarized into the following equations.

$$\begin{aligned} v_{cap} &\leq v \leq v_f \\ 0 &\leq \rho \leq \rho_{cap} \end{aligned} \quad (12)$$

The uncongested traffic is satisfied when the average velocity is higher than the velocity at capacity and less than the free flow velocity, as well as, the traffic density being greater than zero and less than the traffic density at capacity. On the other hand, the congested traffic conditions are vice versa.

The second relationship represents the relationship between the traffic flow rate and the traffic density.

The relationship between these two parameters is defined by a parabolic function. With the provided parameters, the traffic flow at capacity ( $q_{cap}$ ) can be determined as:

$$q_{cap} = v_f \left( \rho_{cap} - \frac{\rho_{cap}^2}{\rho_{jam}} \right) \quad (13)$$

The traffic flow rate at the capacity will be approximately 1,800 vehicles/hr. It can be said that the boundary of the uncongested traffic is  $0 \leq q \leq q_{cap}$ . However, this boundary condition cannot be alone used to indicate the traffic situation. The uncongested traffic and congested traffic condition share the same boundary since the relationship is the parabolic function. The traffic flow of the uncongested traffic is in the left region of the graph and the derivative gives the positive value. That means the average velocity is increasing from zero until it reaches the boundary of the traffic flow at capacity. Meanwhile, the traffic flow under the congested traffic is on the other side with the negative slope. The flow rate is gradually decreased to zero after the point of traffic flow at capacity is reached. The Greenshield's model of the relationship between the traffic flow rate and the traffic density with the boundary of congested and uncongested traffic is illustrated in Fig.6.

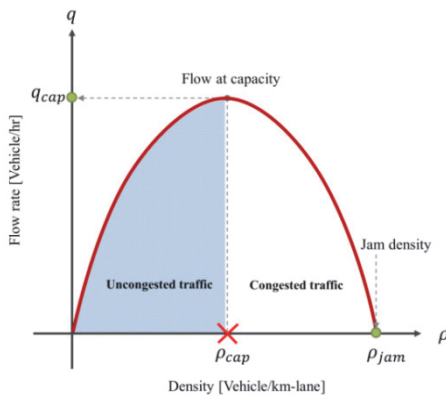


Figure 6: Greenshield's traffic model: the relationship between the traffic flow rate and the traffic density.

The Greenshield's model of the relationship between the average velocity and the traffic flow rate with the boundary of congested and uncongested traffic is illustrated in the Fig. 7. The uncongested traffic is represented in the upper part of the graph. On the contrary, the lower part of the graph represents the congested traffic condition. In addition, the graph shows that at the equilibrium point, the average

velocity at capacity and the traffic flow rate at capacity provides the value of the traffic density at capacity.

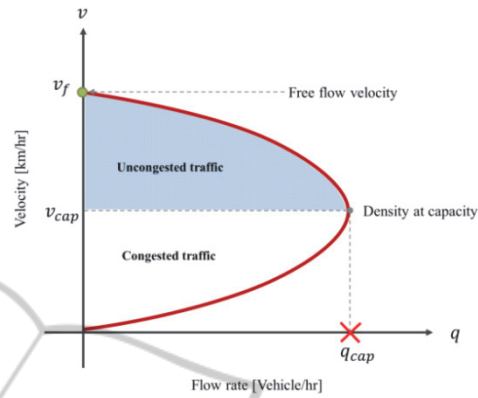


Figure 7: Greenshield's traffic model: the relationship between the average velocity and the traffic flow rate.

With the Greenshield's traffic model, the traffic of an intersection is controllable. In order to manage the current traffic density to meet the desired traffic density, the Greenshield's relationship of an average velocity and a traffic density is implemented. Since the model gives the direct relationship between them, it is obvious that changing an average velocity is the way to minimize the traffic density error of an intersection. The average velocity in the discrete time can be derived as:

$$\bar{v}_i(k) = \bar{v}_i(k-1) - \frac{v_f}{\rho_{jam}} e_i(k) \quad (14)$$

In the control block diagram, the filter is implemented for smoothing the output response in order to remove the short term fluctuation. The technique of the moving average is applied by weighting the value between the current computed value with the previous desired value. The weighting coefficient is called the degree of filtering and the summation of them will be unity. It is called the exponential moving average filter. Technically, the function of this filter is identical to the first order low pass filter in the electronics circuit, suppressing the amplitude of a signal so that the frequency is higher than the cut-off frequency. The exponential moving average filter for the desired average velocity can be expressed as:

$$\bar{v}_i^*(k) = \alpha \bar{v}_i(k) + (1 - \alpha) \bar{v}_i^*(k-1) \quad (15)$$

Where,  $\bar{v}_i^*(k)$  is the desired average velocity for an intersection at time step  $k$ ,  $\bar{v}_i(k)$  is the computed



average velocity from the Greenshield’s model at time step  $k$ ,  $\bar{v}_i^*(k-1)$  is the previous time step  $k-1$  of a desired average velocity and  $\alpha$  is the weight coefficient,  $\alpha \in [0,1]$ .

Since, the intersection network is designed, based on the distributed control, every intersection control structure is identical as presented in Fig. 4. For this reason, the collective of all sub systems, intersection manager, represents the characteristic of the intersection network. The closed loop control block diagram of the intersection network can be illustrated in Fig.8.

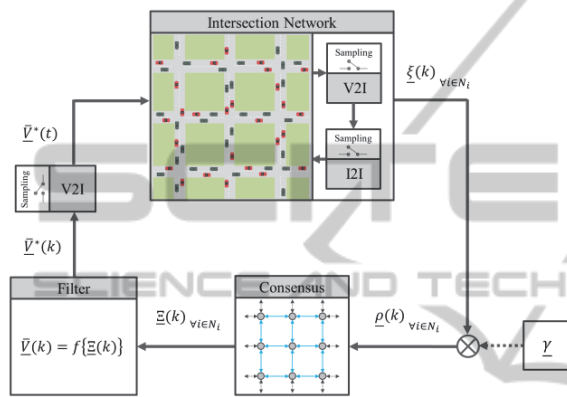


Figure 8: Closed loop control block diagram of the intersection network.

### 4 SIMULATION RESULTS

The simulation results of the multiple autonomous intersection management, which were implemented, based on the consensus algorithm with the Greenshield’s traffic model, is presented. The inputted traffic flow rate of all 12 sources is assigned randomly. The range of the traffic flow rate is set between 1,000-2,000 vehicles/hr.

All vehicles generate their own route randomly. The results of the relationship between 3 traffic parameters of each intersection are plotted. The paired relationship between average velocity, traffic density and traffic flow rate is shown in Fig. 9, 10, and 11 respectively. In addition, the collecting plots of all intersections, compared to the Greenshield’s model are shown in Fig12, 13 and 14 respectively. The corresponding plot of all traffic parameters of 9 intersections is shown in Fig.15.

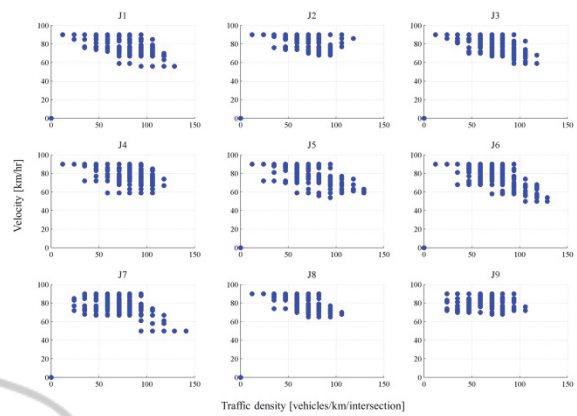


Figure 9: The average velocity and the traffic density relationship of each intersection in the network.

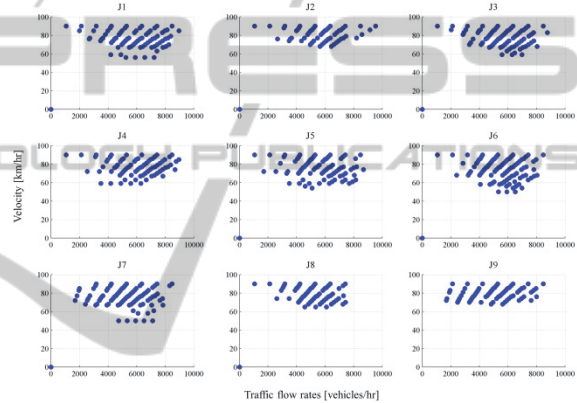


Figure 10: The average velocity and the traffic flow rate relationship of each intersection in the network.

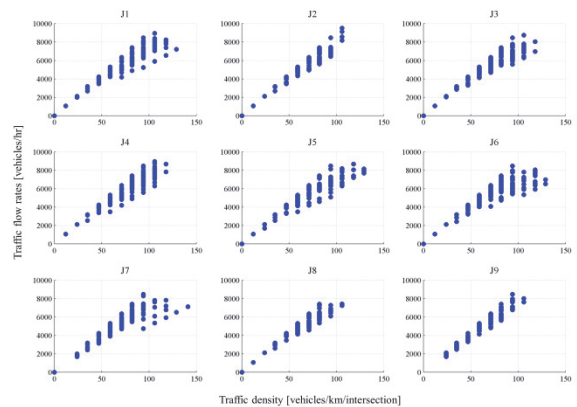


Figure 11: The traffic flow rate and the traffic density relationship of each intersection in the network.

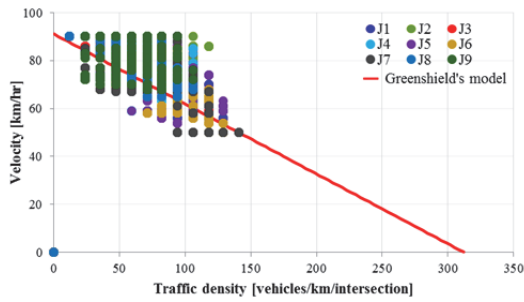


Figure 12: Collecting plot of the average velocity and the traffic density, compared with the Greenshield's model.

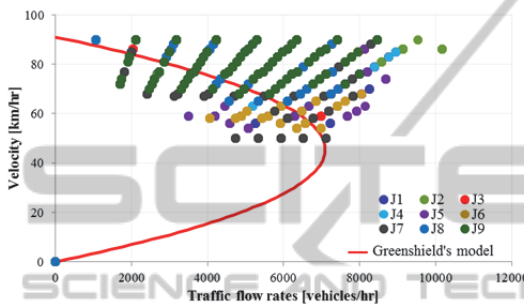


Figure 13: Collecting plot of the average velocity and the traffic flow rate, compared with the Greenshield's model.

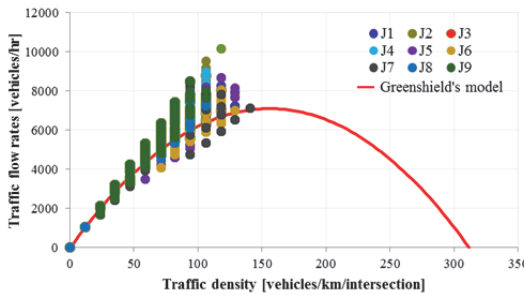


Figure 14: Collecting plot of the traffic flow rate and the traffic density, compared with the Greenshield's model.

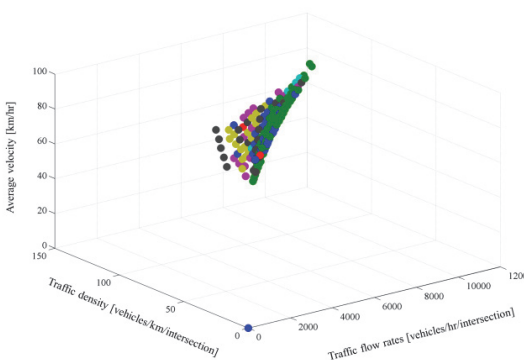


Figure 15: Summary plot of all traffic parameters, traffic flow rate, traffic density and average velocity of 9 intersections.

The results show all intersections can maintain the level of traffic density, average velocity and the traffic flow rate, within the uncongested condition. As well, AIM provides better efficiency in traffic flow rates, compared to the theoretical value that given by Greenshield's model.

## 5 CONCLUSIONS

This work introduces the coordination method for multiple, autonomous intersections by using discrete consensus algorithm with the Greenshield's model. In this paper, the proposed method presents the success performance in managing the traffic in the network of multiple autonomous intersections. The simulation results show every intersection in the network can operate under the uncongested flow condition and provides a contribution in traffic flow rate capability. The attached video presents the success driving under the green wave concept that all vehicles can maintain continuous driving and crossing multiple intersections without stop.

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