Maximum Message Flow and Capacity in Sensor Networks

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Abstract: The present paper considers problems for defining of the maximal messages traffic in a communication network with limited capacities of the separate sections and with arbitrary location of sensors and receivers on it. The specific requirements are described which emerge from the operation of the sensors and receivers on the communication network. Network flow methods are proposed for calculating the maximum possible messages flow, including such a flow of min cost, as well as of the set of critical sections of the network, which block the possibility of further increase of the messages flow. These methods take in account the specific features at generating and receiving of information by the sensors and the receivers respectively. Two numerical examples are given which practically illustrate the solving of the problems pointed out above, and show the effectiveness of the methods proposed for modelling and optimization.

1 PRELIMINARY

Many areas of science and technologies exist where machines and apparatuses are used, equipped with multiple sensors and receivers for the signals and messages, emitted by the former. All of them are connected in sophisticated communication networks for information transfer and distribution; as such may be considered the different centers for physical experiments, machines and equipment in the energy industry – from solar plates to heavy oil sea stations, nuclear electrical power plants, transportation systems, and so on. In fact no area – production, social, or economical – exists where the information flows are not of great importance and as so the speed and reliability of the connections should be by no means neglected. This is of course directly connected with the tremendous flourish of information technologies, which propose possibilities for information flows control.

The network flow programming methods and algorithms (Ford, Fulkerson, 1956) propose a good ground for investigation and realization of the message planning and routing. These methods and algorithms, though a particular class of mathematical programming, turn to be very effective and quickly convergent (Shakkottai, Srikant, 2007; Sgurev, 1991).

2 THE SENSOR COMMUNICATION NETWORK

It is most convenient to represent the sensors communication network as an oriented graph $G(X, U)$ (Christofides, 1986) with a set of arcs U and a set of noes X, such that:

$$X = \bigcup_{i \in I} x_i;\ U = \bigcup_{(i,j) \in G} (x_i, x_i); \tag{1}$$

$$X = (S \bigcup T) \bigcup R;\ I = (I_s \bigcup I_t) \bigcup I_r; \tag{2}$$

$$S = \bigcup_{i \in I_s} x_i;\ T = \bigcup_{i \in I_t} x_i;\ R = \bigcup_{i \in I_r} x_i; \tag{3}$$

where S is the set of sensor points; T – the set of information receiver points; R – the set of intermediate points through the information is being transported without any processing; A – the set of pairs of indices of all arcs from U such that $A = \{(i, j) / (x_i, x_j) \in U\}$; x_{ij} – brief denotation of the arc (x_i, x_j); Ø – the empty set; I – the set of indices of all nodes from X; I_s, I_t, and I_r – subsets of indices of nodes from S, T, and R respectively, for which it is supposed that:

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$$I_s \cap I_r = \varnothing;\ I_s \cap I_t = \varnothing;\ I_r \cap I_t = \varnothing \quad (4)$$

The direct and reverse mapping on the indices I on the graph $G(X, U)$ may be represented in the following way (Christofides, 1986):

$$\Gamma_i^1 = \{ j / (x_i, x_j) \in U; x_j \in X \} \quad (5)$$

$$\Gamma_i^{-1} = \{ j / (x_j, x_i) \in U; x_j \in X \} \quad (6)$$

It is expedient the discrete messages from the separate sensors and for a given time gap Δt to be averaged by number and duration. This will allow them to be considered as a continuous flow of messages with an average statistical flow density (Sgurev, 1991), from one point to another.

If a possibility exists for simultaneous transmission of messages from x_i to x_j and vice versa, then the respective section (x_i, x_j) is replaced by a pair of oppositely directed arcs and namely $\{(x_i, x_j), (x_j, x_i)\} \subset U$.

The average statistical density of the message flow being emitted from the sensor of index $i \in I$ may be defined in the following way:

$$f_i = \frac{\sum_{p \in D_i} \lambda_{ip}}{\Delta t}; i \in I_s; \quad (7)$$

where λ_{ip} – duration of the k^{th} in order message from the sensor $i \in I_s$; D_i – the set of indices of the messages received from the sensor of index $i \in I_s$ in the time gap Δt.

For the receiver points with indices from It this value will look like this:

$$f_j = \frac{\sum_{k \in H_j} \varphi_{jp}}{\Delta t}; j \in I_t; \quad (8)$$

where φ_{jp} is the duration of the k^{th} in order message to the receiver of index $j \in I_t$; H_j – the set of indices of messages received by point j.

If we proceed from the assumption that no loss of messages is admissible at their transportation through the network, then equality is necessary between the sum of the densities of the messages emitted by all sensors of indices from I_s and the sum of densities of the messages, received by all receivers with indices from I_t, i.e.:

$$\sum_{i \in I_s} f_i = \sum_{j \in I_t} f_j = v; \quad (9)$$

where v is the total density of all messages being transferred from all sensors to all receivers.

In most cases the increase or decrease of the flow density from any sensor of index $i \in I_s$ and to any receiver of index $j \in I_t$ is proportional to their inherent technical characteristics defined by the parameters f_i and f_j from (7) and (8) respectively. It follows then from (9) that for each $i \in I_s$ and $j \in I_t$ the following coefficients could be calculated:

$$k_i = \frac{f_i}{\sum_{i \in I_s} f_i} = \frac{f_i}{v}; \; f_i = k_i v; \quad (10)$$

$$k'_j = \frac{f_j}{\sum_{i \in I_s} f_j} = \frac{f_j}{v}; \; f_j = k'_j v \;. \quad (11)$$

If both sides of the equalities (10) and (11) are summed on $i \in I_s$ and $j \in I_t$ respectively, then:

$$\sum_{i \in I_s} k_i = \sum_{j \in I_t} k'_j = 1. \quad (12)$$

The density of the message flow from x_i to x_j will be denoted by the arc flow function f_{ij} ; $(i, j) \in A$ and by c_{ij}; $(i, j) \in A$ will be denoted the capacity of the arc x_{ij}. Then the next requirement shows the physical impossibility the flow function density f_{ij} to exceed the capacity c_{ij} of the arc x_{ij} , i.e. for each $(i, j) \in A$:

$$0 \le f_{ij} \le c_{ij} \quad (13)$$

The value of a unit of density of the messages flow will be denoted by the non-negative arc rate $a_{ij} \ge 0$; $(i, j) \in A$ on the respective arc (section) x_{ij}.

The following two important problems may be formulated on the sensor communication networks:

A. Find the maximum possible flow v_{max} from the sensor points S to receiver points T. This may be most effectively performed through the following network programming problem:

$$L = v \rightarrow \max \quad (14)$$

subject to the following constraints, for each $i \in I$:

$$\sum_{j \in \Gamma_i^1} f_{ij} - \sum_{j \in \Gamma_i^{-1}} f_{ji} = \begin{cases} k_i v, \text{if } i \in I_s; \\ 0, \text{if } i \in I_r; \\ -k'_i, \text{if } i \in I_t; \end{cases} \quad (15)$$

$$f_{ij} \leq c_{ij}, \text{ for each } (i,j) \in A \quad (16)$$

$$f_{ij} \geq 0, \text{ for each } (i,j) \in A \quad (17)$$

Solving the problem above results in:

$$L = v_{max} \quad (18)$$

Let cuts $(X_0, \overline{X}_0)$ be defined between S and T as sets of arcs, such that:

$$X_0 \subset X; \quad (19)$$

$$\overline{X}_0 = X \setminus X_0;\ X_0 \cap \overline{X}_0 = \varnothing;$$
$$(X_0, \overline{X}_0) = \{x_{ij} / x_i \in X_0; X_j \in \overline{X}_0; x_{ij} \in U\} \quad (20)$$

Then, according to the well-known min-cut max-flow theorem of Ford-Fulkerson (Ford, Fulkerson, 1956) a minimal cut $(X_0^*, \overline{X}_0^*)$ is the one for which:

$$f(X_0^*, \overline{X}_0^*) = c(X_0^*, \overline{X}_0^*);\ f(\overline{X}_0^*, X_0^*) = 0 \quad (21)$$

It follows then that the max flow value may be increased only if the capacity of some arcs of the minimal cut $x_{ij} \in (X_0^*, \overline{X}_0^*)$ is increased. Further on the arcs with equality between the capacity and the arc flow function will be called saturated and otherwise – unsaturated.

B. As it is possible several minimal cuts to exist the problem arises to find the one of them which is of minimal value of the parameter $\sum_{(i,j)\in A} a_{ij} f_{ij}$. For solving this problem it is necessary problem A. to be first solved, i.e. the max flow v_{max} from (18) to be found through relations (14) to (17) and then with fixed max flow the minimal cut of minimal cost to be defined. For this purpose the values of $\{k_i\, v / i \in I_s\}$ and $\{k'_j\, v / j \in I_t\}$ are calculated with known $v = v_{max}$ and the latter to be put down as fixed values in the right hand side of (15). Then finding of the minimal cut of minimal cost may be carried out by solving the following network flow programming problem:

$$L = \sum_{(i,j)\in A} a_{ij} f_{ij} \rightarrow \min \quad (22)$$

observing constraints (14) to (17).

This method provides a possibility for optimal distribution (max flow and min cost) of the messages traffic between the sensors and the receivers in the sensor communication network.

3 EXEMPLARY PROBLEM AND NUMERICAL SOLUTIONS

The numerical examples which follow demonstrate the abilities of the method proposed for finding the maximal flow from the sensors to receivers (Problem A.) and the minimal cut with minimal cost (Problem B.).

EXAMPLE: A sensor communication network with 9 nodes and 17 arcs (sections) is conditionally shown in Figure 1.

Three nodes are sensors, 3 – receivers, and 3 – intermediate, and namely:

$$S = \{x_1, x_2, x_3\};\ T = \{x_7, x_8, x_9\};\ R = \{x_4, x_5, x_6\}.$$

The oriented arcs in Figure 1 show from which initial node to which final node messages are being transmitted. The capacities $\{c_{ij}\}$ and the rates $\{a_{ij}\}$ for each arc of the network are shown in Table 1. The messages densities from sensors S to receiver points T are put down in Table 2. In the same table the values of coefficients $\{k_i\}$ and $\{k'_j\}$ are given, calculated according to formulae (10) and (11).

Table 1: Capacities and Rates

A	(1,2)	(1,4)	(1,5)	(1,7)	(2,3)	(2,4)	(3,4)	(3,6)	(3,9)	(4,5)	(4,6)	(5,7)	(5,8)	(6,8)	(6,9)	(8,7)	(9,8)
c_{ij}	5	3	7	6	7	6	6	9	4	8	5	7	8	6	11	5	6
a_{ij}	10	5	5	10	11	6	6	5	10	5	5	3	4	7	4	6	10

Table 2: Coefficients $\{k_i\}$ and $\{k'_j\}$

Nodes X	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
$\{f_i\}$	14	10	6	0	0	0	9	9	12
$\{k_i\} \cup \{k'_j\}$	0,47	0,33	0,2	-	-	-	0,3	0,3	0,4
Node type	Sensor			Intermediate			Receiver		

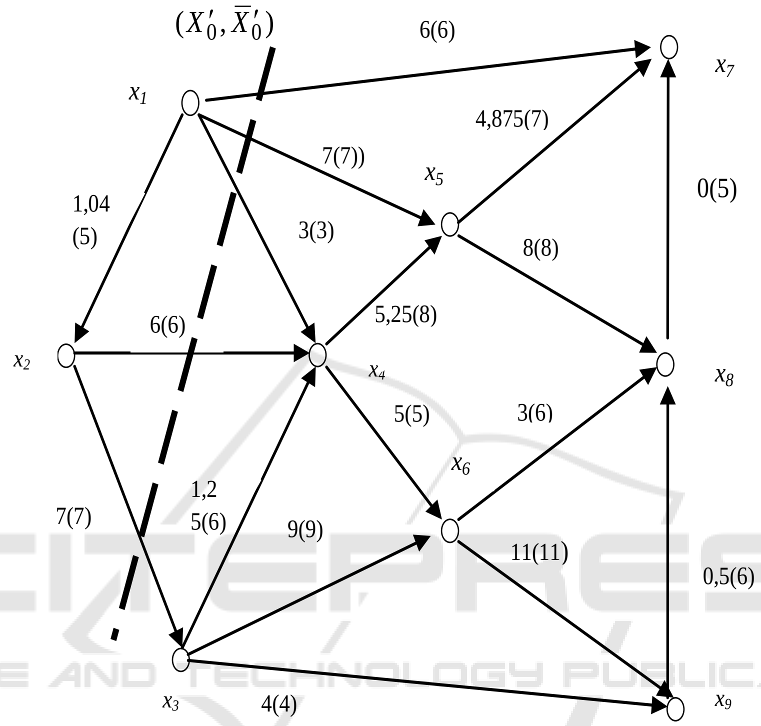

Figure 1: A sensor communication network with 8 nodes and 17 arcs

A. On the base of the data from Tables 1 and 2 the problem for finding the maximal flow v_{max} may be reduced to the following problem of network flow programming. Maximization of v from the linear form (14) observing the following equalities and inequalities:

$z_1)$ $f_{1,2} + f_{1,4} + f_{1,5} + f_{1,7} = 0,47\, v$;
$z_2)$ $f_{2,3} + f_{2,4} - f_{1,2} = 0,33\, v$;
$z_3)$ $f_{3,4} + f_{3,6} + f_{3,9} + f_{2,3} = 0,2\, v$;
$z_4)$ $f_{4,5} + f_{4,6} - f_{1,4} - f_{2,4} - f_{3,4} = 0$;
$z_5)$ $f_{5,7} + f_{5,8} - f_{1,5} - f_{4,5} = 0$;
$z_6)$ $f_{6,8} + f_{6,9} - f_{3,6} - f_{4,6} = 0$;
$z_7)$ $f_{1,7} + f_{5,7} +- f_{8,7} = 0,3\, v$;
$z_8)$ $f_{5,8} + f_{6,8} + f_{9,8} - f_{8,7} = 0,3\, v$;
$z_9)$ $f_{3,9} + f_{6,9} - f_{9,8} = 0,4\, v$;

$z_{10})$ $f_{1,2} \le 5$;	$z_{11})$ $f_{1,4} \le 3$	$z_{12})$ $f_{1,5} \le 7$;
$z_{13})$ $f_{1,7} \le 6$;	$z_{14})$ $f_{2,3} \le 7$	$z_{15})$ $f_{2,4} \le 6$;
$z_{16})$ $f_{3,4} \le 6$;	$z_{17})$ $f_{3,6} \le 9$	$z_{18})$ $f_{3,9} \le 4$;
$z_{19})$ $f_{4,5} \le 8$;	$z_{20})$ $f_{4,6} \le 5$	$z_{21})$ $f_{5,7} \le 7$;
$z_{22})$ $f_{5,8} \le 8$;	$z_{23})$ $f_{6,8} \le 6$	$z_{24})$ $f_{6,9} \le 11$;
$z_{25})$ $f_{8,7} \le 5$;	$z_{26})$ $f_{9,8} \le 6$	$z_{27})$ $f_{i,j} \ge 0$ for each $(i, j) \in A$.

Table 3: Arc flow function density

Arc flow density $f_{i,j}$	$f_{1,2}$	$f_{1,4}$	$f_{1,5}$	$f_{1,7}$	$f_{2,3}$	$f_{2,4}$	$f_{3,4}$	$f_{3,6}$	$f_{3,9}$	$f_{4,5}$	$f_{4,6}$	$f_{5,7}$	$f_{5,8}$	$f_{6,8}$	$f_{6,9}$	$f_{8,7}$	$f_{9,8}$
Value	1,04	3	7	6	7	6	1,25	9	4	5,25	5	4,875	7,375	3	11	0	0,5

The problem described above was solved by the software product WebOptim (Genova et al., 2011). The results obtained are summarized in the next Table 3 with value of $\boldsymbol{v_{max}}$ **= 36,25**.

If data above for $\{f_{ij}\}$ are used and also the arc rates $\{a_{ij}\}$ from Table 1, then the costs for messages transportation, corresponding to the maximal flow defined above, and namely:

$$\sum_{(i,j)\in A} a_{ij} f_{ij} = 491{,}25 \qquad (23)$$

On the base of the coefficients $\{k_i\}$ and $\{k'_j\}$ from Table 2 and the maximal flow achieved $\boldsymbol{v_{max}}$ **= 36,25** the maximum admissible flow densities of messages may be calculated from the sensors S to the receiver points T, i.e.:

$$k_1 v = 17{,}04;\ k_2 v = 11{,}96;\ k_3 v = 7{,}25 \qquad (24)$$

$$k'_7 v = 10{,}875;\ k'_8 v = 10{,}875;\ k'_9 v = 14{,}50 \qquad (25)$$

On each arc in Figure 1 its main parameters are shown – the arc flow function, and in brackets the arc capacity. On the same figure the cut is shown by thick dotted line $(X'_0, \bar{X}'_0)$ = $\{x_{14}, x_{15}, x_{17}, x_{23}, x_{24}\}$ for which there is equality between the maximal possible flow and the minimal cut, i.e. for which requirements (21) are observed. Node x_3 cannot be added to the nodes $X'_0 = \{x_1, x_2\}$ of this cut $(X'_0, \bar{X}'_0)$ because its parameter $k_3 v$ is linearly related to $k_1 v$ and $k_2 v$ which are blocked by the minimal cut $(X'_0, \bar{X}'_0)$. Therefore $k_3 v$ cannot be increased although that a path exists from it $\{x_{34}, x_{45}, x_{57}\}$ to the receiver point x_7 with unsaturated arcs. This is a specific feature of the sensor communication networks reflected in (10) and (11) which does not allow Ford-Fulkeson theorem to be directly applied, but in an oblique way only. In case that increase of the flow v is needed from S to T this should be performed by increasing the capacity of an arc from the cut:

$$(X'_0, \bar{X}'_0) = \{x_{14}, x_{15}, x_{17}, x_{23}, x_{24}\} \qquad (26)$$

B. For calculating the maximal flow of minimal cost relations z_1 to z_{27} with the following changes:

- the right hand sides of equations z_1 to z_3 are replaced by the respective right hand parts of the three relations from (25);
- the right hand sides of equations z_7 to z_9 are replaced by the respective right hand parts of the three relations from (26). In this way the maximal possible flow v_{max} is fixed both in the sensors S and in the receivers T.

For finding the minimal value of this flow the following linear relation is used in thich the rates $\{a_{ij}\}$ are taken from Table 1:

$$L_1 = 10 f_{1,2} + 5 f_{1,4} + 5 f_{1,5} + 10 f_{1,7} + 11 f_{2,3} + 6 f_{2,4} + 6 f_{3,4} + 5 f_{3,6} + 10 f_{3,9} + 5 f_{4,5} + 5 f_{4,6} + 3 f_{5,7} + 4 f_{5,8} + 7 f_{6,8} + 4 f_{6,9} + 6 f_{8,7} + 10 f_{9,8} \rightarrow \min \qquad (27)$$

The problem (27) with the modified relations z_1 to z_{27} was solved by the software product mentioned above. The values of the arc flow functions and of the linear form (27) are summarized in the Table 4:

$$L_1 = 485{,}53 \qquad (28)$$

Table 4: Arc Flow Function

Arc flow function $f_{i,j}$	$f_{1,2}$	$f_{1,4}$	$f_{1,5}$	$f_{1,7}$	$f_{2,3}$	$f_{2,4}$	$f_{3,4}$	$f_{3,6}$	$f_{3,9}$	$f_{4,5}$	$f_{4,6}$	$f_{5,7}$	$f_{5,8}$	$f_{6,8}$	$f_{6,9}$	$f_{8,7}$	$f_{9,8}$
Value	1,03	3	7	6	7	6	1,24	9	4	5,87	4,37	4,87	8	2,87	10,5	0	0

These data are put down in the Figure 2 like in Figure 1. In both numerical examples – in case A (Figure 1) and in case B (Figure 2) the configuration of the graph $G(X,U)$, capacities $\{c_{ij}\}$, coefficients $\{k_i\}$ and $\{k'_j\}$, arc rates $\{a_{ij}\}$ and the max flow v_{max} are identical but there is a difference in the flow realization of $\{f_{ij}\}$. The flow value on the arc $x_{4,5}$ in case A is 5,25 and in case B – 5,87. There are changes and on the arcs $\{x_{4,6}, x_{5,7}, x_{5,8}, x_{6,8}, x_{6,9}, x_{8,9}\}$. Some of them ($x_{4,6}$, $x_{6,9}$) has turned from saturated into unsaturated ones, another one ($x_{5,8}$) – from unsaturated into saturated, and third – ($x_{4,6}$, $x_{5,7}$, $x_{6,8}$, $x_{8,9}$), has only changed the flow function values.

The minimal cut $(X'_0, \bar{X}'_0)$; $X_0 = \{x_1, x_2\}$ remains the same as in Figure 1 and due to the same reasons it blocks the maximal flow increase. If the total value of the maximum possible traffic in both cases – A and B, then as expected from (23) and (29) for the max flow of min cost the total value L_1 is less by about 1,2% less than the analogical value L corresponding to the first case, i.e.:

$$\Delta L = L - L_1 = 491{,}25 - 485{,}53 = 5{,}72 \qquad (29)$$

The two examples given in the cases A and B demonstrate the effectiveness of the method proposed for finding of the maximum messages flow from sensor to receiver points on an arbitrary sensor communication network, and of max flow of min cost.

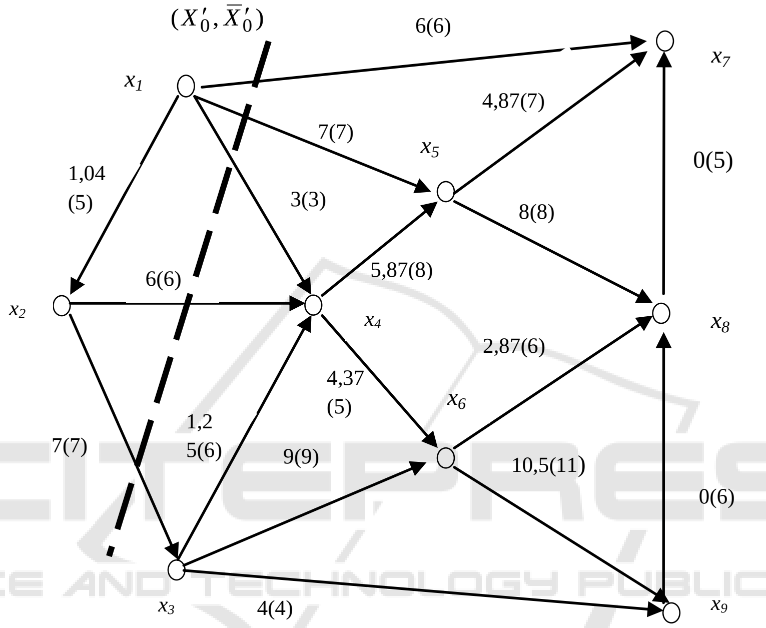

Figure 2: The same network with optimal values

4 SUMMARY

Here we show that the graph theory and network flow methods and algorithms are still up-to-date for control and optimization of the ‘commodity’ traffic in our case – messages from sensors to receivers, ensuring max flow at min cost of the traffic across the network. Two approaches are proposed for sensor networks, which maximize the flow from sensors to receivers and minimize the cost of this flow. In the first one the max flow is found and in the second one – alternative paths of min cost are found. The advantage of the network flow optimization is that it is independent on the nature and the physical characteristics of the network and operates with abstract and relative quantities, which when scaled in appropriate way are applicable to any type of real networks.

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REFERENCES

Sgurev V., 1991. Network Flows with General Constraints, Publishing House of the Bulgarian Academy of Sciences, Sofia (in Bulgarian).

Christofides N., 1986. Graph Theory: An Algorithmic Approach, London: Academic Press.

Shakkottai S., R. Srikant, 2007. Network Optimization and Control. Foundations and Trends in Networking, Vol. 2, № 3, pp. 271–379.

Ford L. R., D. R. Fulkerson, 1956. Maximal flow through a network. Canadian Journal of Mathematics, Vol. 8, pp. 399–404.

Bertsecas D., 1991. Linear Network Optimization, Massachusetts Institute of Technology.

Jensen P., J. Barnes, 1980. Network Flow Programming, John Wiley & Sons, NY, USA.

Genova K., L. Kirilov, V. Guliashki, B. Staykov, D. Vatov, 2011. A Prototype of a Web-based Decision Support, System for Building Models and Solving Optimization and Decision Making Problems, In: Proceedings of XII International Conference on Computer Systems and Technologies (Eds. B. Rachev, A. Smrikarov), Wien, Austria, 16-17 June 2011, ACM ICPS, Vol. 578, pp. 167–172.