Plane Wave Diffraction by a Thin Material Strip: Higher Order Asymptotics

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Abstract: The plane wave diffraction by a thin material strip is analyzed using the Wiener-Hopf technique together with approximate boundary conditions. An asymptotic solution is obtained under the condition that the thickness and the width of the strip are small and large compared with the wavelength, respectively. The scattered field is evaluated asymptotically based on the saddle point method and a far field expression is derived. Scattering characteristics of the strip are discussed via numerical results of the radar cross section.

1 INTRODUCTION

The analysis of the scattering by material strips is an important subject in electromagnetic theory and radar cross section (RCS) studies. Volakis (1988) analyzed the plane wave diffraction by a thin material strip using the dual integral equation approach (Clemmow, 1951) and the extended spectral ray method (Herman and Volakis, 1987) together with approximate boundary conditions (Senior and Volakis, 1995). In his 1988 paper, Volakis first solved rigorously the diffraction problem involving a single material half-plane, and subsequently obtained a high-frequency solution to the original strip problem by superposing the singly diffracted fields from the two independent half-planes and the doubly/triply diffracted fields from the edges of the two half-planes. Therefore his analysis is mathematically not rigorous from the viewpoint of boundary value problems, and may not be applicable unless the strip width is relatively large compared with the wavelength.

In this paper, we shall consider the same problem as in Volakis (1988), and analyze the plane wave diffraction by a thin material strip for both H and E polarizations with the aid of the Wiener-Hopf technique. Analytical details are presented only for the H-polarized case, but numerical results will be shown for both H and E polarizations.

Introducing the Fourier transform of the scattered field and applying approximate boundary conditions in the transform domain, the problem is formulated in terms of the simultaneous Wiener-Hopf equations, which are solved exactly via the factorization and decomposition procedure. However, the solution is formal since branch-cut integrals with unknown integrands are involved. We shall further employ an asymptotic method established by Kobayashi (2013) to derive a high-frequency solution to the Wiener-Hopf equations, which is expressed in terms of an infinite asymptotic series and accounts for all the higher order multiple diffraction effects rigorously. It is shown that the higher-order multiple diffraction is explicitly expressed in terms of the generalized gamma function introduced by Kobayashi (1991). Our solution is valid for large strip width and requires numerical inversion of an appropriate matrix equation. The scattered field in the real space is evaluated asymptotically by taking the Fourier inverse of the solution in the tranform domain and applying the saddle point method. It is to be noted that our final solution is uniformly valid in incidence and observation angles. Numerical examples of the RCS are presented for various physical parameters and far field scattering characteristics of the strip are discussed in detail. Some comparisons with Volakis (1988) are also given. The results presented in this paper provide an important extension of our earlier analysis of the same problem (Koshikawa and Kobayashi, 2000; Nagasaka and Kobayashi, 2013).

The time factor is assumed to be $\mathrm{e}^{-\mathrm{i}\omega t}$ and suppressed throughout this paper.

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2 FORMULATION OF THE PROBLEM

We consider the diffraction of an H-polarized plane wave by a thin material strip as shown in Fig. 1, where the relative permittivity and permeability of the strip are denoted by ε_r and μ_r, respectively. Let the total magnetic field $\phi^t(x,z)[\equiv H_y(x,z)]$ be

$$\phi^t(x,z) = \phi^i(x,z) + \phi(x,z), \tag{1}$$

where $\phi^i(x,z)$ is the incident field given by

$$\phi^i(x,z) = \mathrm{e}^{-\mathrm{i}k(x\sin\theta_0 + z\cos\theta_0)} \tag{2}$$

for $0 < \theta_0 < \pi/2$ with $k[=\omega(\varepsilon_0\mu_0)^{1/2}]$ being the free-space wavenumber. The term $\phi(x,z)$ in (1) is the unknown scattered field and satisfies the two-dimensional Helmholtz equation.

If the strip thickness is small compared with the wavelength, the material strip can be replaced by a strip of zero thickness satisfying the second order impedance boundary conditions (Senior and Volakis, 1995). Then the total electromagnetic field satisfies the approximate boundary conditions as given by

$$\begin{aligned}&\left[E_z(+0,z) + E_z(-0,z)\right]\\ &\quad - 2R_e\left[H_y(+0,z) - H_y(-0,z)\right] = 0,\end{aligned} \tag{3}$$

$$\begin{aligned}&\left[\frac{1}{2R_m} + \frac{1}{2\tilde{R}_e}\left(1 + \frac{1}{k^2}\frac{\partial^2}{\partial x^2}\right)\right]\\ &\quad \cdot\left[H_y(+0,z) + H_y(-0,z)\right]\\ &\quad - \left[E_z(+0,z) - E_z(-0,z)\right] = 0,\end{aligned} \tag{4}$$

where

$$\left.\begin{aligned} R_e &= \mathrm{i}Z_0 / \left[kb(\varepsilon_r - 1)\right],\\ \tilde{R}_e &= \mathrm{i}\varepsilon_r Y_0 / \left[kb(\varepsilon_r - 1)\right],\\ R_m &= \mathrm{i}\,Y_0 / \left[kb(\mu_r - 1)\right] \end{aligned}\right\} \tag{5}$$

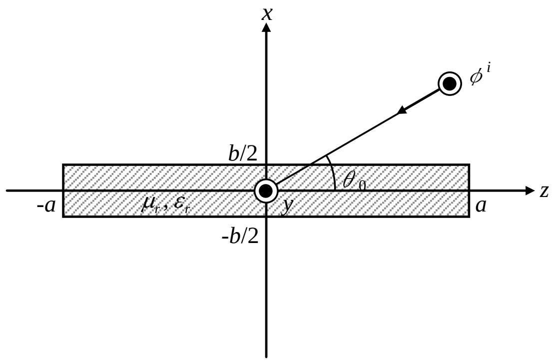


Figure 1: Geometry of the problem.

with Z_0 and Y_0 being the intrinsic impedance and admittance of free space, respectively. In the following, we shall assume that the medium is slightly lossy as in $k = k_1 + \mathrm{i}k_2$ with $0 < k_2 \ll k_1$. The solution for real k is obtained by letting $k_2 \to +0$ at the end of analysis.

In view of the radiation condition, it follows that

$$\phi(x,z) = O(\mathrm{e}^{-k_2|z|\cos\theta_0}) \tag{6}$$

as $|z| \to \infty$. We define the Fourier transform $\Phi(x,\alpha)$ of the scattered field $\phi(x,z)$ with respect to z as

$$\Phi(x,\alpha) = (2\pi)^{-1/2}\int_{-\infty}^{\infty}\phi(x,z)\,\mathrm{e}^{\mathrm{i}\alpha z}\,\mathrm{d}z, \tag{7}$$

where $\alpha(\equiv \mathrm{Re}\,\alpha + \mathrm{i}\,\mathrm{Im}\,\alpha) = \sigma + \mathrm{i}\tau$. Then we see that $\Phi(x,\alpha)$ is regular in the strip $|\tau| < k_2\cos\theta_0$ of the α-plane. Introducing the Fourier integrals as

$$\Phi_\pm(x,\alpha) = \pm(2\pi)^{-1/2}\int_{\pm a}^{\pm\infty}\phi(x,z)\,\mathrm{e}^{\mathrm{i}\alpha(z\mp a)}\,\mathrm{d}z, \tag{8}$$

$$\Phi_1(x,\alpha) = (2\pi)^{-1/2}\int_{-a}^{a}\phi(x,z)\,\mathrm{e}^{\mathrm{i}\alpha a}\,\mathrm{d}z, \tag{9}$$

it follows that $\Phi_+(x,\alpha)$ and $\Phi_-(x,\alpha)$ are regular in the half-planes $\tau > -k_2\cos\theta_0$ and $\tau < k_2\cos\theta_0$, respectively, whereas $\Phi_1(x,\alpha)$ is an entire function. In view of the notation as given by (8) and (9), $\Phi(x,\alpha)$ is expressed as follows:

$$\begin{aligned}\Phi(x,\alpha) = \mathrm{e}^{-\mathrm{i}\alpha a}\Phi_-(x,\alpha) + \Phi_1(x,\alpha)\\ + \mathrm{e}^{\mathrm{i}\alpha a}\Phi_+(x,\alpha).\end{aligned} \tag{10}$$

Taking the Fourier transform of the two-dimensional Helmholtz equation, we find that

$$\left(\mathrm{d}^2/\mathrm{d}x^2 - \gamma^2\right)\Phi(x,\alpha) = 0 \tag{11}$$

for any α in $|\tau| < k_2\cos\theta_0$, where $\gamma = (\alpha^2 - k^2)^{1/2}$. Since γ is a double-valued function of α, we choose a proper branch of γ such that γ reduces to $-\mathrm{i}k$ when $\alpha = 0$. According to the choice of this branch, we can show that $\mathrm{Re}\,\gamma > 0$ for any α in the strip $|\tau| < k_2$. Equation (11) is the transformed wave equation. Solving (11) and applying the boundary conditions, we derive, after some manipulations, that

$$\begin{aligned}&-M(\alpha)J_m(\alpha)\\ &\quad = \mathrm{e}^{-\mathrm{i}\alpha a}U_-(\alpha) + \mathrm{e}^{\mathrm{i}\alpha a}U_{(+)}(\alpha),\end{aligned} \tag{12}$$

$$\begin{aligned}&-K(\alpha)J_e(\alpha)\\ &\quad = 2\left[\mathrm{e}^{-\mathrm{i}\alpha a}V_-(\alpha) + \mathrm{e}^{\mathrm{i}\alpha a}V_{(+)}(\alpha)\right],\end{aligned} \tag{13}$$

where

$$M(\alpha) = 1 - \frac{\mathrm{i}kY_0}{2\gamma}\left[\frac{1}{R_m} + \frac{1}{\tilde{R}_e}\left(1 + \frac{\gamma^2}{k^2}\right)\right], \quad (14)$$

$$K(\alpha) = \gamma - 2\mathrm{i}kY_0R_e, \quad (15)$$

$$U_{(\pm)}(\alpha) = \tilde{\Phi}_\pm(\alpha) \mp \frac{A_{1,2}}{\alpha - k\cos\theta_0}, \quad (16)$$

$$V_{(\pm)}(\alpha) = \Phi'_\pm(\alpha) \mp \frac{B_{1,2}}{\alpha - k\cos\theta_0}, \quad (17)$$

$$J_m(\alpha) = \frac{\mathrm{i}}{\omega\varepsilon_0}\left[\frac{\mathrm{d}\Phi_1(+0,\alpha)}{\mathrm{d}x} - \frac{\mathrm{d}\Phi_1(-0,\alpha)}{\mathrm{d}x}\right], \quad (18)$$

$$J_e(\alpha) = \Phi_1(+0,\alpha) - \Phi_1(-0,\alpha) \quad (19)$$

with

$$\tilde{\Phi}_\pm(\alpha) = \left(\frac{1}{R_m} + \frac{1}{\tilde{R}_e}\right)\Phi_\pm(0,\alpha) + \frac{1}{\tilde{R}_e k^2}\frac{\mathrm{d}^2\Phi_\pm(0,\alpha)}{\mathrm{d}x^2}, \quad (20)$$

$$\Phi'_\pm(\alpha) = \frac{\mathrm{d}\Phi_\pm(0,\alpha)}{\mathrm{d}x}, \quad (21)$$

$$A_{1,2} = \frac{1}{(2\pi)^{1/2}\mathrm{i}}\left(\frac{1}{R_e} + \frac{\cos^2\theta_0}{\tilde{R}_m}\right)\mathrm{e}^{\mp\mathrm{i}ka\cos\theta_0}, \quad (22)$$

$$B_{1,2} = -\frac{k\sin\theta_0}{(2\pi)^{1/2}}\mathrm{e}^{\mp\mathrm{i}ka\cos\theta_0}. \quad (23)$$

Equations (12) and (13) are the Wiener-Hopf equations satisfied by unknown spectral functions, where $U_{(+)}(\alpha)$ and $V_{(+)}(\alpha)$ are regular in the upper half-plane $\tau > -k_2\cos\theta_0$ except for a simple pole at $\alpha = k\cos\theta_0$.

3 FACTORIZATION OF THE KERNEL FUNCTIONS

The solutions of (12) and (13) require factorization of the kernel functions defined by (14) and (15) in the form

$$M(\alpha) = M_+(\alpha)M_-(\alpha) = M_+(\alpha)M_+(-\alpha), \quad (24)$$

$$K(\alpha) = K_+(\alpha)K_-(\alpha) = K_+(\alpha)K_+(-\alpha). \quad (25)$$

In order to factorize (14) and (15), let us introduce the auxiliary functions $N_n(\alpha)$ for $n = 1,2,3$ as

$$N_n(\alpha) = N_{n+}(\alpha)N_{n-}(\alpha) = 1 + \frac{\mathrm{i}}{k\delta_n}\gamma, \quad (26)$$

where

$$\delta_{1,2} = -\frac{\tilde{R}_e}{Y_0}\left[1 \pm \left(1 + Y_0^2\frac{\tilde{R}_e + R_m}{R_m\tilde{R}_e^2}\right)^{1/2}\right], \quad (27)$$

$$\delta_3 = 2Y_0R_e. \quad (28)$$

Substituting (26) into (14) and (15), it follows that

$$M(\alpha) = \frac{kY_0}{\mathrm{i}\gamma}\left(\frac{\tilde{R}_e + R_m}{2\tilde{R}_eR_m}\right)N_1(\alpha)N_2(\alpha), \quad (29)$$

$$K(\alpha) = -2\mathrm{i}kY_0R_eN_3(\alpha). \quad (30)$$

Applying the method developed by Noble (1958), $N_n(\alpha)$ for $n = 1,2,3$ are factorized as

$$\begin{aligned} N_{n\pm}(\alpha) &= \left(1 + \delta_n^{-1}\right)^{1/2} \\ &\cdot \exp\left\{-\frac{\delta_n}{\pi}\int_{\pi/2}^{\arccos(\pm\alpha/k)}\frac{t\cos t}{\sin^2 t - \delta_n^2}\mathrm{d}t\right. \\ &\pm \frac{\mathrm{i}}{2\pi}\ln\left[\delta_n + (\delta_n^2 - 1)^{1/2}\right] \\ &\cdot \ln\left[\frac{\mathrm{i}k(\delta_n^2 - 1)^{1/2} + \alpha}{\mathrm{i}k(\delta_n^2 - 1)^{1/2} - \alpha}\right] \\ &\left. + \frac{1}{4}\ln\left[1 + \frac{\alpha^2}{k^2(\delta_n^2 - 1)}\right]\right\}. \end{aligned} \quad (31)$$

From (29)-(31), we find that the split functions $M_\pm(\alpha)$ and $K_\pm(\alpha)$ are expressed as follows:

$$M_\pm(\alpha) = \left[\frac{kY_0(\tilde{R}_e + R_m)}{2\tilde{R}_eR_m}\right]^{1/2}\frac{N_{1\pm}(\alpha)N_{2\pm}(\alpha)}{(k \pm \alpha)^{1/2}}, \quad (32)$$

$$K_\pm(\alpha) = (2kY_0R_e)^{1/2}\mathrm{e}^{-\mathrm{i}\pi/4}N_{3\pm}(\alpha). \quad (33)$$

4 FORMAL SOLUTION

Multiplying both sides of (13) by $\mathrm{e}^{\pm\mathrm{i}\alpha a}/K_\mp(\alpha)$ and applying the decomposition procedure with the aid of the edge condition, we derive, after some manipulations, that

$$\frac{V_{(+)}^{s,d}(\alpha)}{K_+(\alpha)} + \frac{B_1}{K_+(k\cos\theta_0)(\alpha - k\cos\theta_0)} \mp \frac{1}{2\pi\mathrm{i}}\int_{-\infty+\mathrm{i}c}^{\infty+\mathrm{i}c}\frac{\mathrm{e}^{2\mathrm{i}\beta a}V_{(+)}^{s,d}(\beta)}{K_-(\beta)(\beta + \alpha)}\mathrm{d}\beta = 0, \quad (34)$$

where c is a constant such that $0 < |\tau| < c < k_2\cos\theta_0$, and

$$V_{(+)}^{s,d}(\alpha) = V_{(+)}(\alpha) \pm V_-(-\alpha). \quad (35)$$

It is verified from (17), (33), and (35) that the singularities associated with the integral in (34) for $\mathrm{Im}\,\beta > c$ are a simple pole at $\beta = k\cos\theta_0$ and a branch point at $\beta = k$. We now choose a branch cut emanating from $\beta = k$ as a straight line that is parallel to the imaginary axis and goes to infinity in the upper half-plane. Evaluating the integral by enclosing the contour into the upper half-plane, we derive that

$$V_{(+)}^{s,d}(\alpha) = K_+(\alpha)\left[-\frac{B_1}{K_+(k\cos\theta_0)(\alpha - k\cos\theta_0)} \mp \frac{B_2}{K_-(k\cos\theta_0)(\alpha + k\cos\theta_0)} \pm v_{s,d}(\alpha)\right], \quad (36)$$

where

$$v_{s,d}(\alpha) = \frac{1}{\pi \mathrm{i}}\int_k^{k+\mathrm{i}\infty} \frac{\mathrm{e}^{2\mathrm{i}\beta a}(\beta - k)^{1/2}}{\beta + \alpha} \cdot V_{(+)}^{s,d}(\beta) T_+(\beta)\mathrm{d}\beta, \quad (37)$$

$$T_+(\beta) = \frac{(\beta + k)^{1/2} K_+(\beta)}{\beta^2 - k^2 + 4k^2 Y_0^2 R_e^2}. \quad (38)$$

Equation (36) provides the exact solution to the Wiener-Hopf equation (13), but it is formal in the sense that the branch-cut integrals $v_{s,d}(\alpha)$ with unknown integrands are involved.

Equation (12) can be solved in a similar manner, but the solution will not be discussed here. In the next section, we shall derive explicit high-frequency solutions to the Wiener-Hopf equations.

5 HIGH-FREQUENCY ASYMPTOTIC SOLUTION

In order to eliminate the singularities of $V_{(+)}^{s,d}(\alpha)$ at $\alpha = k\cos\theta_0$, we introduce

$$\Phi_+'^{s,d}(\alpha) = \Phi_+'(\alpha) \pm \Phi_-'(-\alpha). \quad (39)$$

Then (36) can be written in the following form:

$$\Phi_+'^{s,d}(\alpha) = K_+(\alpha)\left[\chi_{s,d}^v(\alpha) + \frac{C_{s,d}}{\pi \mathrm{i}} \cdot \int_k^{k+\mathrm{i}\infty} \frac{\mathrm{e}^{2\mathrm{i}\beta a}(\beta - k)^{1/2}}{\beta + \alpha}\Phi_+'^{s,d}(\beta) T_+(\beta)\mathrm{d}\beta\right]. \quad (40)$$

In (40), several quantities are defined by

$$\chi_{s,d}^v(\alpha) = B_1\left[Q_1(\alpha) \pm \sum_{n=0}^{\infty} T_n \eta_{2n}(\alpha)\right] \pm B_2\left[Q_2(\alpha) \pm \sum_{n=0}^{\infty} T_n \eta_{1n}(\alpha)\right], \quad (41)$$

$$C_{s,d} = \pm 1 \quad (42)$$

with

$$T_n = \left.\frac{T_+^{(n)}(\beta)}{n!}\right|_{\beta=k}, \quad (43)$$

$$Q_{1,2}(\alpha) = \frac{K_+^{-1}(\alpha) - K_\pm^{-1}(k\cos\theta_0)}{\alpha \mp k\cos\theta_0}, \quad (44)$$

$$\eta_{1n,2n}(\alpha) = \frac{\xi_{0n}^{1/2}(\alpha) - \xi_{0n}^{1/2}(\pm k\cos\theta_0)}{\alpha \mp k\cos\theta_0}, \quad (45)$$

$$\xi_{pn}^{1/2}(\alpha) = \frac{\mathrm{e}^{\mathrm{i}[2ka + (\pi/2)(n-p-1/2)]}}{\pi (2a)^{1/2+n-p}}(-1)^p p! \cdot \Gamma_{p+1}\left(3/2 + n, -2\mathrm{i}(\alpha + k)a\right). \quad (46)$$

In (46), $\Gamma_{p+1}(\cdot,\cdot)$ is the generalized gamma function (Kobayashi , 1991) defined by

$$\Gamma_m(u,v) = \int_0^\infty \frac{t^{u-1}\mathrm{e}^{-t}}{(t+v)^m}\mathrm{d}t \quad (47)$$

for $\mathrm{Re}\, u > 0, |v| > 0, |\arg v| < \pi$, and positive integer m. Applying the method established by Kobayashi (2013) to the integral in (36), we can obtain a high-frequency asymptotic expansion of (36) with the result that

$$T_+(\alpha)\Phi_+'^{s,d}(\alpha) \sim T_+(\alpha)K_+(\alpha) \cdot \left[\chi_{s,d}^v(\alpha) + C_{s,d}\sum_{n=0}^{\infty} f_n^{vs,vd}\xi_{0n}^{1/2}(\alpha)\right] \quad (48)$$

for $ka \to \infty$, where

$$f_n^{vs,vd} = \frac{1}{n!}\left.\frac{\mathrm{d}^n\left[T_+(\alpha)\Phi_+'^{s,d}(\alpha)\right]}{\mathrm{d}\alpha^n}\right|_{\alpha=k}. \quad (49)$$

We can show that the unknowns $f_n^{vs,vd}$ for $n = 0, 1, 2, \cdots$ in (48) satisfy the system of linear algebraic equations as in

$$f_m^{vs,vd} - C_{s,d}\sum_{n=0}^{\infty} A_{mn}^v f_n^{vs,vd} \sim B_m^{vs,vd} \quad (50)$$

for $m = 0, 1, 2, \cdots$, where

$$A_{mn}^v = \sum_{p=0}^{m} \frac{h^{(m-p)}(k)\xi_{pn}^{1/2}(k)}{p!(m-p)!}, \quad (51)$$

$$B_m^{vs,vd} = \sum_{p=0}^{m} \frac{h^{(m-p)}(k) g_{vs,vd}^{(p)}(k)}{p!(m-p)!}, \quad (52)$$

$$h^{(m-p)}(k) = \left.\frac{\mathrm{d}^{m-p}\left[T_+(\alpha)K_+(\alpha)\right]}{\mathrm{d}\alpha^{m-p}}\right|_{\alpha=k}, \quad (53)$$

$$g_{vs,vd}^{(p)}(k) = \left.\frac{\mathrm{d}^p \chi_{s,d}^v(\alpha)}{\mathrm{d}\alpha^p}\right|_{\alpha=k}. \quad (54)$$

Equation (48) together with the matrix equations (50) provides a high-frequency asymptotic solution of (40) for the strip width large compared with the wavelength. Making use of the above results and carrying out further manipulations, we finally arrive at an explicit asymptotic solution to the Wiener-Hopf equation with the result that

$$V_{(+)}^{s,d}(\alpha) \sim K_{+}(\alpha)\Bigg\{-\frac{B_1}{K_{+}(k\cos\theta_0)(\alpha - k\cos\theta_0)} \mp \frac{B_2}{K_{-}(k\cos\theta_0)(\alpha + k\cos\theta_0)} \pm \sum_{n=0}^{\infty} T_n\left[B_1\eta_{2n}(\alpha) \pm B_2\eta_{1n}(\alpha)\right] \pm \sum_{n=0}^{\infty} f_n^{vs,vd}\xi_{0n}^{1/2}(\alpha)\Bigg\} \quad (55)$$

as $ka \to \infty$. It is to be noted that this solution rigorously takes into account the multiple diffraction between the edges of the strip. A similar procedure may also be applied to (12) for a high-frequency solution but the details will not be discussed here.

6 SCATTERED FAR FIELD

Using the boundary condition, the scattered field in the Fourier transform domain is expressed as

$$\Phi(x,\alpha) = \tilde{\Phi}(\alpha)\,\mathrm{e}^{-\gamma|x|}, \quad (56)$$

where

$$\tilde{\Phi}(\alpha) = -\frac{\mathrm{i}kY_0\left[\mathrm{e}^{-\mathrm{i}\alpha a}\,U_{-}(\alpha) + \mathrm{e}^{\mathrm{i}\alpha a}\,U_{(+)}(\alpha)\right]}{2\gamma M(\alpha)} \mp \frac{\mathrm{e}^{-\mathrm{i}\alpha a}\,V_{-}(\alpha) + \mathrm{e}^{\mathrm{i}\alpha a}\,V_{(+)}(\alpha)}{K(\alpha)}, \quad x \gtrless 0. \quad (57)$$

The scattered field in the real space is obtained by taking the inverse Fourier transform of (56) according to the formula

$$\phi(x,z) = (2\pi)^{-1/2}\int_{-\infty+\mathrm{i}c}^{\infty+\mathrm{i}c} \tilde{\Phi}(\alpha)\,\mathrm{e}^{-\gamma|x|-\mathrm{i}\alpha z}\,\mathrm{d}\alpha, \quad (58)$$

where c is constant such that $\left|c\right| < k_2\cos\theta_0$. We introduce the cylindrical coordinate (ρ,θ) centered at the origin as

$$x = \rho\sin\theta, \quad z = \rho\cos\theta \quad (59)$$

for $0 < \left|\theta\right| < \pi$. Then a far field expression of (58) can be derived with the aid of the saddle point method, leading to

$$\phi(\rho,\theta) \sim \tilde{\Phi}(-k\cos\theta)k\sin\left|\theta\right| \cdot \frac{\mathrm{e}^{\mathrm{i}(k\rho-\pi/4)}}{(k\rho)^{1/2}}, \quad x \gtrless 0 \quad (60)$$

as $k\rho \to \infty$. Equation (60) is uniformly valid for arbitrary incidence and observation angles.

7 NUMERICAL RESULTS AND DISCUSSION

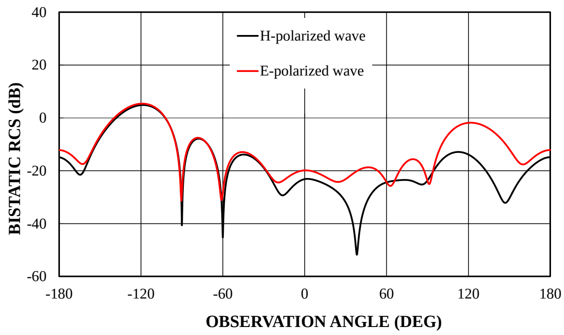
We shall now present numerical results on the RCS for both H and E polarizations, and discuss far field scattering characteristics of the strip in detail. The normalized RCS per unit length is defined by

$$\sigma / \lambda = \lim_{\rho\to\infty}\left(k\rho\left|\phi / \phi^i\right|^2\right) \quad (61)$$

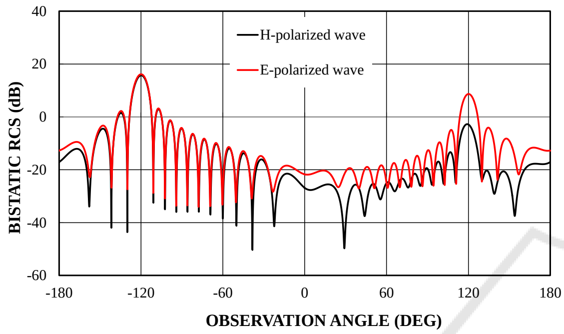
with λ being the free-space wavelength.

Figures 2 shows the bistatic RCS as a function of observation angle θ, where the width and the thickness of the strip are taken as $2a = 2\lambda, 7\lambda$ and $b = 0.05\lambda$, respectively. In numerical computation, we have chosen the ferrite with $\varepsilon_r = 2.5 + \mathrm{i}1.25$, $\mu_r = 1.6 + \mathrm{i}0.8$ as an example of existing lossy materials. The incident angle θ_0 is fixed as 60°. It is seen from the figure that the RCS shows noticeable peaks along the reflected $(\theta = 120^\circ)$ and incident $(\theta = -120^\circ)$ shadow boundaries. We also notice that the RCS exhibits sharp oscillation with an increase of the strip width as can be expected. Comparing the RCS characteristics between H and E polarizations, we observe that the RCS level for H polarization is lower than that for E polarization in the reflection region $(0^\circ < \theta < 180^\circ)$ but the results for both polarizations show close features in the shadow region $(-180^\circ < \theta < 0^\circ)$.

Figure 3 shows the monostatic RCS versus incidence angle θ_0, where the same parameters as in Fig. 2 have been chosen for computation. We see from the figure that the RCS level for H polarization is lower than that for E polarization except in the neighbourhood of the specular reflection direction at $\theta_0 = 90^\circ$. Figure 4 shows comparison with the results obtained by Volakis (1988), where the strip dimension is $2a = 2\lambda$, $b = 0.05\lambda$, and the material parameters are $\varepsilon_r = 1.5 + \mathrm{i}0.1$, $\mu_r = 4.0 + \mathrm{i}0.4$. It is seen from the figure that our results agree well with Volakis's results over $45^\circ < \theta_0 < 90^\circ$, but there are some discrepancies for $0^\circ < \theta_0 < 45^\circ$. These discrepancies are perhaps due to the fact that Volakis's solution is constructed based on the solutions for the two independent half-planes and becomes less accurate at relatively low frequencies $(2a = 2\lambda)$.

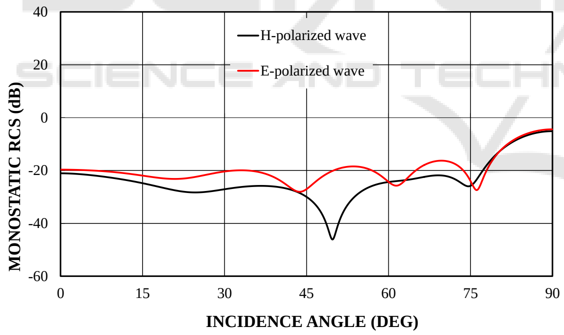


(a) $2a = 2\lambda$.



(b) $2a = 7\lambda$.

Figure 2: Bistatic RCS versus observation angle for $\theta_0 = 60^\circ, b = 0.05\lambda, \varepsilon_r = 2.5 + i1.25, \mu_r = 1.6 + i0.8$.



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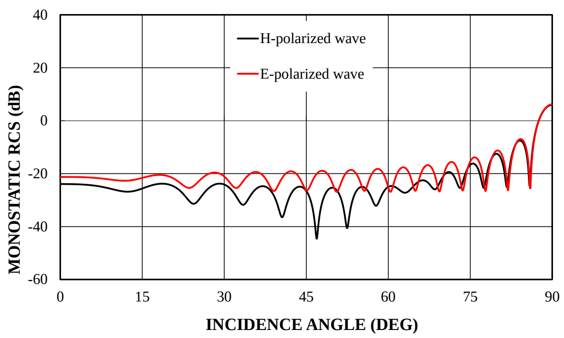

(b) $2a = 7\lambda$.

Figure 3: Monostatic RCS versus incidence angle for $b = 0.05\lambda, \varepsilon_r = 2.5 + i1.25, \mu_r = 1.6 + i0.8$.

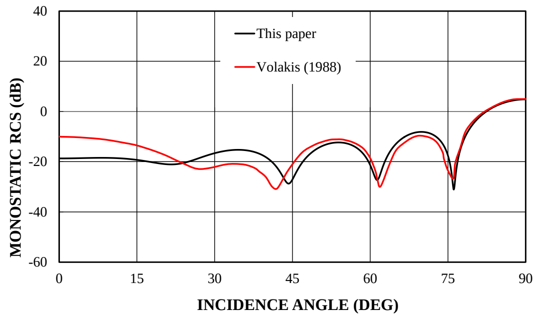

Figure 4: Monostatic RCS versus incidence angle for H polarization, $2a = 2\lambda$, $b = 0.05\lambda$, $\varepsilon_r = 1.5 + i0.1$, $\mu_r = 4.0 + i0.4$ and its comparison with Volakis (1988).

8 CONCLUSIONS

In this paper, we have analyzed the plane wave diffraction by a thin material strip for both H and E polarizations using the Wiener-Hopf technique and approximate boundary conditions. Employing a rigorous asymptotics, a high-frequency solution for large strip width has been obtained. Illustrative numerical examples on the RCS are presented, and far field scattering characteristics of the strip have been discussed in detail. Some comparisons with the other existing method have also been provided.

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