

An Information-theoretical Approach to Classify Hospitals with Respect to Their Diagnostic Diversity using Shannon's Entropy

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Abstract: In Germany hospital comparisons are part of health status reporting. This article presents the application of Shannon's entropy measure for hospital comparisons using reported diagnostic data. We used Shannon's entropy to measure the diagnostic diversity of a hospital department by means of reported ICD-9-codes. Entropy values were compared both with respect to the hospital status (i.e. primary, secondary, tertiary or specialized hospital) and specialisations (e.g. surgery, gynaecology). There were relevant differences in entropy values between the different types of hospitals. Primary hospitals differed from specialized hospitals (0.535 ± 0.09 vs. 0.504 ± 0.07). Furthermore, specialized departments like obstetrics or ophthalmology did generate lower entropy values than area-spanning departments like paediatrics or general internal medicine, having significantly higher values. In conclusion, we showed how entropy can be used as a measure for classifying hospitals. Besides of hospital comparisons, this approach can be implemented in all fields of health services research for measuring variability in nominal or ordinal data. The use of entropy as a measure for health services research and classification algorithms should be encouraged to learn more about this measure, which unreasonably has fallen into oblivion in health services research.

1 INTRODUCTION

With emerging costs in the health care sector, hospitals more and more compete for limited financial resources. Hence, not only the comparison of hospitals with respect to their outcome, but also with respect to structural qualities plays an important role for health care providers, physicians and of course for the patients (Betzler and Haun, 1998; Aiken et al., 1998). In Germany however, hospital comparisons are part of health status reporting and thus have been put on a legal basis by the Federal Government. Despite this fact, some methodological problems are still unsolved (Wegscheider, 2004; Schulz et al., 2004).

According to Frick et al. (2003) one of the main problems is the handling of differences observed in outcome and performance of hospitals, which are mainly produced by two sources. On the one hand, they emerge from the infrastructural properties of the hospitals themselves; on the other hand they emerge from the patients population treated in the hospitals and their diagnostic diversity and severity.

One methodological approach of solving this problem is to make a statistical adjustment for these

differences and then to perform a comparison on all hospitals. Another approach is to compare only those institutions, which have similar structural premises and join them into groups by means of cluster analysis.

This however requires the development of a measure, which maps these characteristics onto a numerical value. In the work of Gerste (1996) such a measure was pragmatically calculated from the relative differences of the ICD-9-codes as can be seen in Figure 1.

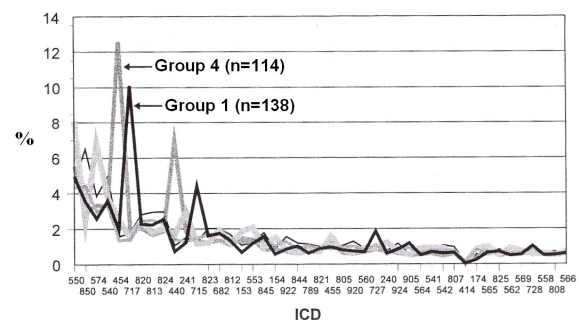


Figure 1: Hospital group-profiles "Surgery" adapted from Gerste (1996).

Most of these approaches for grouping hospitals by means of diagnostic data are empirically motivated and e.g. ask for the number n of diagnoses needed to account for, say 80% of the patient volume. Only a few systematic and intersubjectively valid approaches have been proposed for this problem so far (Izsak, 1994).

One method based on the concept of Shannon's entropy was proposed quite early by Elayat et al. (1978) to cluster hospitals in homogenous groups and has been adapted by Farley (1989) for the analysis of case-mix specialization and the concentration of diagnoses in hospitals quite early.

Although information-theoretical approaches are commonly accepted and applied as a measure for diversity in other fields of science (Nayak, 1985; Ricotta and Avena, 2003), it did not become an accepted method in Health Services Research so far.

For the special case of diagnostic diversity, a MEDLINE search only found one result dealing with the relevance of diagnostic diversity and patient volumes for quality and length of stay in pediatric intensive care units (Ruttimann et al., 2000).

This article is based on a very short conceptual paper from Erben (2000) and presents the application of Shannon's entropy measure for the calculation of diagnostic diversity on a broader dataset from hospitals and discusses the results with respect to other hospital performance measures.

2 MATERIAL AND METHODS

Shannon's entropy is based on a system of mutually exclusive and exhaustive events A_1, A_2, \dots, A_n and a set of probabilities $p_1 := p(A_1), p_2 := p(A_2), \dots, p_n := p(A_n)$. Then, the entropy is given by

$$E(p_1, \dots, p_n) = - \sum_{k=1}^n p_k \log p_k \quad (1)$$

where $0 \cdot \log 0 = 0$ is assumed. The largest value of E is given for the equal distribution of the events A_i with $p_k = 1/n; k=1, 2, \dots, n$, which is easy to proof.

Thus, to standardize E on the interval $[0, 1]$, it has to be divided by $E_{\max} = \log(1/n)$.

In the following, we use the term entropy for this kind of standardised entropy value. Except from a scaling factor λ which depends on which logarithm is used, there is only one such function E , which quantifies the content of information in the above defined way.

To illustrate our approach, we will give the original example from Erben (2000): In his work he

analysed the L4-hospital diagnosis statistics and thus, the events A_i are given by aggregated three-digit-ICD-9 codes (e.g. 820 = "Fracture of the neck of the femur").

Suppose now, we have two hospitals A and B. Hospital A is highly specialized in orthopaedic surgery. In this case, 75% of all cases are covered by one two three-digit-ICD-9 codes and four three-digit-ICD-9 codes include about 90% of all cases treated in this hospital. In total, 71 three-digit-ICD-9 cover the complete spectrum of this hospital. This leads to a value of $E=0,368$.

Hospital B is an orthopaedic clinic and additionally serves as a university clinic. In this case, 17 three-digit-ICD-9 codes are needed to cover 75% of all cases. In total 113 three-digit-ICD-9 codes are needed to cover the complete range of cases. This leads to an entropy-value of $E = 0,729$. With a low value for the specialized clinic A and a value double that size for the university clinic B, entropy thus might serve as a good indicator for diagnostic diversity.

To see if the results of this example are generalizable to a greater variety of hospital departments, we analysed the complete spectrum of hospital departments in Schleswig- Holstein. For further analysis these departments are subdivided with respect to their area (e.g. internal medicine, surgery, gynaecology) but also with respect to the hospital status they are affiliated with. This status is defined as follows:

Primary hospitals include at least the fields of Internal Medicine and Surgery, and according to requirements may include the fields of Obstetrics and/or Gynaecology, Otorhinolaryngology and Ophthalmology and, in special cases Urology and Orthopaedics.

Secondary hospitals: Additionally to the fields covered by primary hospitals, secondary hospitals include paediatrics, neurology and dental surgery.

Tertiary hospitals: The range of services of tertiary hospitals significantly goes far beyond those of secondary and primary hospitals. This includes the provision of a highly differentiated range of technical equipment e.g. medical devices like a positron emission tomograph.

Specialized hospitals offer the best medical therapy and care for a limited range of diagnoses including the referral of complicated cases.

Just like in the original work of Erben (2000), the statistics are based on the data set of the aggregated three-digit ICD-9-codes from the L4-hospital statistics of 1998. We excluded hospitals with incomplete diagnostic data, which led to a sample of

977 hospitals and hospital departments that were included in our analysis.

Descriptive statistics and the calculation of the entropy were performed with SPSS for Windows Version 20.

3 RESULTS

Figure 2 shows the distribution of entropy values for all 977 institutions. The distribution is quite symmetric with a minimum entropy value of 0.12, a maximum of 0.95 and a median of 0.516, which is near the mean of 0.520.

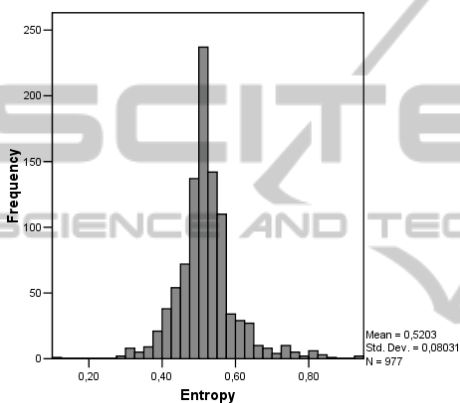


Figure 2: Entropy-distribution of all hospitals and departments (N=977).

Figures 3 and 4 show the subgroup analysis of entropy with respect to the hospital classification and the medical fields of the hospitals.

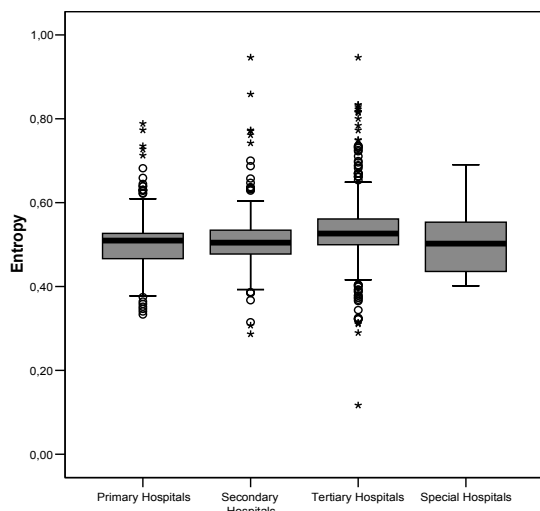


Figure 3: Boxplot of Entropy-values for all hospitals and departments subdivided for hospital status.

With median-values from 0.50 to 0.52 and a similar interquartile ranges (IQR), primary, secondary and tertiary hospitals do not extremely differ in their entropy-values. Nevertheless there is a difference with respect to the range. As can also be seen from the distribution of entropy parameters in the Table 1, there is an evidence of outliers in the group of primary to tertiary hospitals, which are marked in the boxplot-figure with stars and circles. This effect is not observed in the group of specialized hospitals.

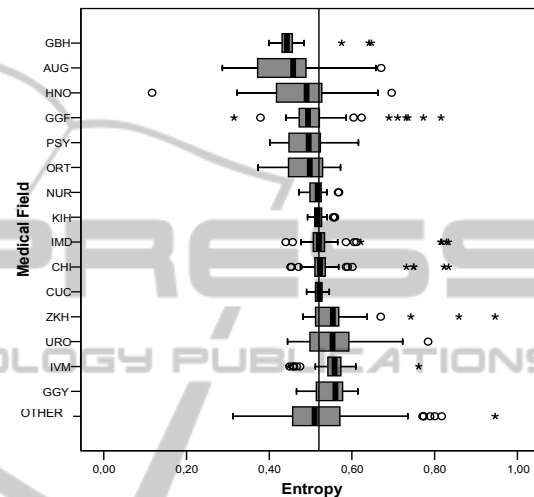


Figure 4: Boxplot of Entropy-values for all hospitals and departments subdivided into medical fields (CHI= Surgery; IMD= Internal Medicine; HNO= Otorhinolaryngology; GGF= Gynaecology and Obstetrics; URO= Urology; PSY= Psychiatry; ORT= Orthopaedics; IVM= Intensive Care; KIH= Paediatric Intensive Care; NUR= Nuclear Medicine and Radiology; ZKH= Dental Surgery; GGY= General Gynaecology; GBH= Obstetrics; AUG= Ophthalmology; CUC= Traumatology).

Table 1: Statistical parameters for the distribution of entropy subdivided by hospital classification.

	Primary	Secondary	Tertiary	Specialized
Mean ± SD	0.535±0.09	0.510±0.07	0.505 ± 0,07	0.504 ± 0.07
Median ± IQR	0.506 ± 0.06	0.505 ± 0.06	0.510 ± 0,06	0.503 ± 0.12
Minimum	0.33	0.29	0.12	0.40
Maximum	0.79	0.95	0.95	0.69
Range	0.46	0.66	0.83	0.29

Whilst the subgroup analysis of entropy by means of hospital categories did not yield to promising conclusions, the subdivision with respect to medical fields does generate some interesting findings. On first sight, highly specialized departments like obstetrics (0.44) or ophthalmology (0.46) generate lower entropy values than area-spanning departments like Nuclear Medicine and

Radiology (0.52) or general gynaecology (0.56), which have significantly higher values ($p < 0.05$).

4 CONCLUSIONS

This article examines the application of Shannon's entropy measure to hospital diagnostic admission data. Although Shannon's entropy has been widely used as a measure for diversity in various scientific fields, it has only sparsely been applied for classification purposes in health services research.

Based on the frequencies of three-digit-ICD-9 codes at hospital admission, we showed the possibilities of Shannon's measure of entropy as a possibility for analysing nominal scaled data of health status reporting of hospitals. Based on the L4-statistics, we exemplified how entropy can be used for clustering hospitals by using their routine diagnostic data.

Although the ICD-9 data from our example dates back almost 10 years, our approach can easily be adapted to hospital data based on ICD-10 or DRGs. Especially in DRG-data a clearer differentiation of specialized hospitals might be possible.

With increasingly limited financial resources in the health care sector, hospitals as well as networks of general practitioners are seeking for markers which distinguish them from competitors in their fields (Sabatino et al., 1992). The assessment of diversity therefore might be one promising approach, which especially in the life sciences is already a highly important issue. From the level of molecular biology i.e., the diversity of gene expressions is actually discussed, whereas on the level of evolutionary biology the diversity of species in the animal and plant kingdom is of relevance. In both situations entropy has been applied as a measure to assess the diversity or complexity (Pueyo et al., 2007).

Using entropy as a diversity marker can basically be implemented in all fields of health services research, where categorical data emerges. One actual example is the application of entropy as a measure to assess the diversity of medical devices in large inventories of medical equipment (Brindle et al., 2008). For diagnostic or therapeutic data, entropy might be useful e.g. for measuring the variability of diagnostic data.

One particular application might be the comparison of suspected diagnosis at referral of hospitalized patients versus the proven admission diagnosis at intake. Another example is the use of entropy for the analysis of clinical pathways.

Especially in integrated care, the question arises, whether entropy is created by transferring patients from a primary hospital to a specialized clinic or vice versa. This might lead to a sequential calculation of entropy by dividing diagnostic data with respect to the pathways patients were admitted.

Hence, one might wonder why outcomes research has not used this measure e.g. for the diversity classification in health outcomes. Especially for the task of measuring variability in nominal or ordinal data parameters common parameters like standard deviation or the variation coefficient are not applicable and thus, entropy can be used for such purposes.

Although our examples are quite comprehensible, one has to be aware that entropy is just a marker for variety and does not measure a difference in the distribution of categorical data. For example, groups one and four in Fig. 1 of the hospital group-profiles "Surgery" from Gerste (1996) show a similar distribution, where group one can be created by shifting group four to the right. Although their profile is completely different, their entropy, if it had been calculated, would be quite similar. Hence, entropy only classifies with respect to similar structures (e.g. centroids in special diagnostic groups), but does not give a clue whether these structures are similar to each other in respect of contents.

Thus, according to Jost (2006) it is important to distinguish between entropy and true diversities when interpreting such indices as it is not obvious on what basis these indices were computed.

Moreover a variety of entropy measures does exist summarized in the generalized diversity indices δ proposed by Patil and Taillie (1982)

$$\delta = \sum_{j=1}^S p_j \rho(p_j)$$

with

$$\rho(p_j) = \begin{cases} 1 - p_j^{\alpha-1} / \alpha - 1 & \text{if } \alpha \neq 1 \\ -\ln(p_j) & \text{if } \alpha = 1 \end{cases}$$

As a special case ($\alpha=1$) δ does also include Shannon's entropy measure.

A detailed mathematical analysis of Leinster and Cobbold (2012) found that Shannon's entropy might be more sensitive to rare events, while others like Simpson's measure of diversity ($\alpha=2$) is not that much influenced by such events. This property still makes Shannon's entropy to be one of the most reliable diversity indices.

Nevertheless different sensitivities to the occurrence of rare events have to be taken into account very carefully, when deriving conclusions from entropy measures (Ricotta and Seidl, 2006).

In our case of diagnostic diversity of hospitals we therefore believe that Shannon's entropy is a proper choice.

In conclusion the use of entropy as a measure for health services research and classification algorithms based on entropy have to be encouraged to learn more about this measure, which unreasonably has fallen into oblivion in health services research.

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