Relative Position Descriptors
A Review

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Abstract: A relative position descriptor is a quantitative representation of the relative position of two spatial objects. It is a low-level image descriptor, like colour, texture, and shape descriptors. A good amount of work has been carried out on relative position description. Application areas include content-based image retrieval, remote sensing, medical imaging, robot navigation, and geographic information systems. This paper reviews the existing work. It identifies the approaches that have been used as well as the properties that can be expected from relative position descriptors. It compares and provides a brief overview of various descriptors, including their main properties, strengths and limitations, and it suggests areas for future work.

1 INTRODUCTION

Relative position refers to the arrangement of objects in space relative to each other. In daily life, knowledge about relative positions is conveyed through linguistic expressions like, "object *A* is mostly *above* object *B*," "object *A* is quite *far* from object *B*," or "object *A* almost *touches* object *B*." Such qualitative statements use terms that denote *spatial relationships*, which are often categorized into directional (e.g., *above*), distance (e.g., *far*) and topological (e.g., *touches*) relationships. Relative position information is important in various areas of image processing and computer vision. However, many practical image processing and computer vision tasks call for quantitative measurements. Quantitative models of relative position have therefore been proposed. We call such models *relative position descriptors.* They are visual descriptors, like color, texture, and shape descriptors, but they also intend to serve as containers for spatial relationships. They therefore provide a link between low-level spatial data features and high-level concepts. The ideal relative position descriptor gives a snapshot of the arrangement of objects in space relative to each other, it encapsulates rich information about all kinds of spatial relationships between the objects, and it allows this information to be easily extracted.

Relative position descriptors have received good attention in image processing research in recent years, as they have applications in object recognition, image retrieval and indexing, map-to-image conflation, linguistic scene description, etc. Most of the attention has been focused on finding effective approaches to the modeling of relative positions and techniques for extracting spatial relationship information from relative position descriptors. Other topics that have received some attention include the design of efficient algorithms; the handling of fuzzy objects, objects in vector form, and 3D objects; similarity and affine invariance.

There are several review papers on models of spatial relationships (and these models may or may not be based on relative position descriptors). See, e.g., (Bloch, 2005) (Jaworski and Kucharski, 2010). However, to our knowledge, this is the first review paper on relative position descriptors. Section 2 identifies the approaches that have been used as well as the properties that can be expected from these descriptors. It also provides a summary of the properties of various descriptors. Section 3 briefly presents each descriptor mentioned in Section 2. Conclusions are drawn in Section 4.

2 APPROACHES & PROPERTIES

2.1 Approaches

Table 1 indicates the approaches used to define the

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relative position descriptors considered in this paper. More on the definition of each descriptor can be found in Section 3. The information necessary to fully understand these table and section is presented below.

An *object* is a nonempty, regular closed subset of the Cartesian plane. A *pixel* is a unit square whose sides are parallel to the x- and y-axes and whose center has integer coordinates. A *raster* object is the union of a finite number of pixels. A *vector* object is an object whose boundary is the union of a finite number of line segments. Consider two objects *A* and *B*. The position of *A* relative to *B* is usually represented by a histogram H^{AB}, or by a real function improperly called a histogram, or by a tuple of such functions. A working assumption is that the objects may be too close to each other to be approximated by their centroids or minimum bounding boxes.

In the *pixel-pair-based* approach, the relative position descriptor is designed with raster objects in mind. A histogram value depends on pairs of pixels, where each pair is composed of a pixel of *A* and a pixel of *B*. In the *point-pair*-based approach, the descriptor is designed with both raster and vector objects in mind. A histogram value depends on pairs of points, where each pair is composed of a point of *A* and a point of *B*. In the *segment-pair-based* approach, a histogram value depends on pairs of aligned segments, where each pair is composed of a segment of *A* and a segment of *B*. In the *core-pair-based* approach, a histogram value depends on pairs of aligned cores, where each pair is composed of a core of *A* and a core of *B*. A *core* of an object is the intersection of that object with a line.

The φ-*histogram* (Matsakis, Wendling and Ni, 2010) is a generic relative position descriptor defined using the point-pair-based approach. The symbol φ denotes a function that maps any triple like (θ,p,q) to a real number, where p and q are points and θ is a direction in the plane, i.e., θ is an element of the interval $(-\pi,\pi]$. The histogram value $\varphi^{AB}(\theta)$ is the sum (integral) of all the φ(θ,p,q) values, where p belongs to *A* and q to *B*. The F-*histogram* is defined using the core-pair-based approach. F denotes a function that maps any triple like (θ,I,J) to a real number, where θ is a direction in the plane and I and J are unions of aligned segments. The histogram value $F^{AB}(\theta)$ is the sum of all the F(θ,I,J) values, where I is a core of *A* and J a core of *B* aligned with I. Likewise, the f-*histogram* is defined using the segment-pair-based approach. f denotes a function that maps any triple like (θ,I,J) to a real number, where I and J are aligned segments.

Table 1: Approaches.

DESCRIPTOR		**APPROACH** pixel/point/segment/core-pair-based	**APPROACH** boundary/region-based
generic	φ-histogram	point-pair-based	region-based
	f-histogram	segment-pair-based	region-based
	F-histogram	core-pair-based	region-based
specific	angle histogram	pixel-pair-based	region-based
	force histogram	point-pair-based (φ-histogram)	region-based
	Allen histograms	core-pair-based (F-histogram)	region-based
	R-histogram	pixel-pair-based	boundary-based
	R*-histogram	pixel-pair-based	region-based
	spread histogram	pixel-pair-based	region-based
	visual area histogram	pixel-pair-based	boundary-based
	radial line model	core-based	region-based
	ratio histogram	core-pair-based (F-function-based)	region-based

Moreover, a relative position descriptor may be defined using a *region-based* approach (all the pixels, or all the points of the objects are considered) or a *boundary-based* approach (only the boundary pixels, or the boundary points, are considered).

2.2 Properties

Table 2 summarizes the properties and characteristics of the relative position descriptors considered in this paper. More about the properties and characteristics of each descriptor can be found in Section 3. The information necessary to fully understand these table and section is presented below.

An object as defined in Section 2.1 is a 2D crisp object. From now on, unless otherwise specified, objects will be assumed to be raster objects. All relative position descriptors can handle (2D crisp raster) objects without having to vectorize them. Likewise, we say that a descriptor can handle (2D crisp) vector objects if there is no need to rasterize them. We say that a descriptor can handle (2D) fuzzy (raster) objects if there is no need for a general and computationally expensive approach like the double sum or simple sum scheme (Dubois and Jaulent, 1987) (Krishnapuram et al., 1993).

The symbol N refers to the number of pixels in the image (case of raster objects), and n is the number of directions θ considered (case of histograms defined on the set $(-\pi,\pi]$ of directions in the plane). When dealing with raster objects, n is $O(\sqrt{N})$ at worst and O(1) at best. Practically, there does not seem to be any interest in considering more than a few hundred directions, whatever N.

A relative position descriptor is usually designed so that specific spatial relationship information can be extracted. The target relationships can be directional, topological, or distance relationships. Note that topological relationships include set relationships. For example, the condition $A\cap B\neq\emptyset$ defines the set (and hence topological) relationship *intersects*, while $A\cap B\neq\emptyset$ and $int(A)\cap int(B)=\emptyset$ define the topological (but non-set) relationship *touches*. Surroundedness is treated independently. The fact that the target relationships are, say, topological, does not mean of course that information about every possible topological relationship can be extracted, and there is no implication about the quality (e.g., completeness, meaningfulness) of the extracted information. Moreover, a descriptor may allow some information about non-target relationships to be extracted.

Table 2: Properties. An empty cell means that, to our knowledge, the property has not been investigated, and that, as far as we can tell, there is no straightforward evidence towards the property.

DESCRIPTORS	**handling of objects**				**target relationships**					**semantic inverse retrieval**	**sim(ilarities) and AFF(inities)**		
	2D crisp raster	vector (2D crisp)	fuzzy (2D raster)	3D (crisp raster)	directional	topological	set	distance	surround		invariance under	solution to the direct problem wrt	solution to the inverse problem wrt
angle histogram	$\mathcal{O}(\text{N} \log \text{N})$		✔	✔	✔					✔	**sim**	**sim**	**sim**
force histogram	$\mathcal{O}(\text{N} \log \text{N})$	✔	✔	✔	✔					✔	**sim**	**AFF**	**sim**
Allen histograms	$\mathcal{O}(n\,\text{N}^2)$	✔		✔	✔	✔	✔			✔	**sim**	**AFF**	**sim**
R-histogram	$\mathcal{O}(\text{N}^2)$		✔	✔	✔		✔	✔		✔	**sim**	**sim**	**sim**
R*-histogram	$\mathcal{O}(n\,\text{N} \log \text{N})$		✔	✔	✔		✔	✔		✔	**sim**	**sim**	**sim**
spread histogram	$\mathcal{O}(\text{N}^2)$						✔		✔		**sim**	**sim**	
visual area histogram	$\mathcal{O}(\text{N}^2)$			✔	✔					✔	**sim**	**sim**	**sim**
radial line model	$\mathcal{O}(\text{N})$	✔	✔	✔	✔		✔			✔			
ratio histogram	$\mathcal{O}(\text{N})$	✔	✔	✔						✔	**AFF**	**AFF**	

Affine or similarity invariant descriptors play an important role in image processing and computer vision. Examples of similarity invariant colour, texture, and shape descriptors abound in the literature. Consider a geometric transformation *t*. We say that the relative position descriptor H is *invariant* under *t* if for any objects *A* and *B* we have $H^{t(A)t(B)} = H^{AB}$. If H is not invariant under *t*, there may be a *normalization* procedure $H^{AB} \mapsto \overline{H^{AB}}$ such that $\overline{H^{t(A)t(B)}} = \overline{H^{AB}} = H^{A'B'}$, where the objects A' and B' can be derived from *A* and *B* using some invertible transformation. The normalized descriptor is then invariant under *t*. Note that all the descriptors considered in this paper are invariant under translations.

It may be possible to find $H^{t(A)t(B)}$ knowing *t* and H^{AB}, without having to rely on any computational optimization technique. We then say that the descriptor offers a solution to the *direct problem* with respect to *t*, or that the behaviour of the descriptor under *t* is known. It may also be possible to find *t* (up to a translation) knowing H^{AB} and $H^{t(A)t(B)}$. We then say that the descriptor offers a solution to the *inverse problem* with respect to *t*. Finally, it may be possible to find H^{BA} knowing H^{AB}. We then say that the descriptor allows *semantic inverse retrieval*.

3 DESCRIPTORS

Here, we briefly present the descriptors mentioned in Section 2. We also comment on some of their properties (which are listed in Table 2).

3.1 Angle Histogram

Consider two raster objects *A* and *B* (Fig. 1*a*). For any pixels p of *A* and q of *B*, with p≠q, let ∠(p,q) be the angle between the x-axis and the directed line that passes through the center of q and then of p. This angle belongs to $(-\pi,\pi]$. Partition $(-\pi,\pi]$ into *n* intervals Θ_1, Θ_2, etc. (the *direction bins*). The histogram value $H^{AB}(i)$ is the number of pixel pairs $(p,q) \in A \times B$ such that $\angle(p,q) \in \Theta_i$. See (Krishnapuram et al., 1993) (Miyajima and Ralescu, 1994).

Note

The histogram of angles may be the first true relative position descriptor. The original algorithm runs in $O(N^2)$ time. To shorten processing times, it is of course possible to downsize the image before computing the histogram. A similar approach is to partition each object into rectangular blocks of pixels and to consider that the center of any pixel in a given block is the center of the block. This is the principle of a variant of the histogram of angles called the *quadtree histogram* (Wang, 2013) (Zhang et al., 2014). Also note that the histogram of angles can be computed in $O(N \log N)$ time using the same $O(N \log N)$ approach as for force histograms (Matsakis, Wendling and Ni, 2010). Nonetheless, processing times are usually much longer than for other relative position descriptors.

Directional relationship information can be extracted using, e.g., the aggregation (Krishnapuram et al., 1993) or the compatibility method (Miyajima and Ralescu, 1994).

At first glance, the behaviour of the histogram under similarity transformations seems easy to determine and similarity invariance seems easy to obtain. This, however, may not be the case. One issue is how to choose the number of bins, *n*. For example, if the bins are too narrow then the histogram of angles inherits the anisotropy of the grid of pixels. Let *rot* be a π/4-angle rotation. Assume $0 \in \Theta_i$ and $\pi/4 \in \Theta_j$. We should have $H^{rot(A)rot(B)}(j) \approx H^{AB}(i)$. Instead, we get $H^{rot(A)rot(B)}(j) \approx H^{AB}(i)/\sqrt{2}$.

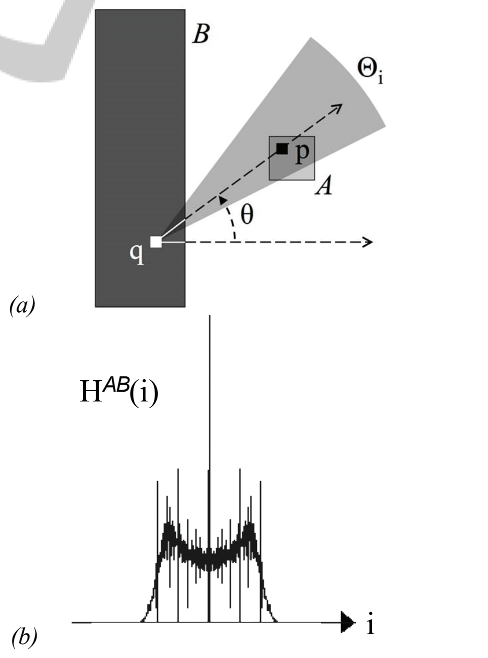

Figure 1: Histogram of angles. *(a)* $H^{AB}(i)$ is the number of pixel pairs (p,q) such that θ falls into Θ_i. *(b)* Example.

3.2 Force Histogram

The *histogram of forces* (Matsakis and Wendling, 1999) is a φ-histogram. Consider two objects *A* and *B* and two points $p \in A$ and $q \in B$, as in Fig. 2*a*. Let qp be the vector from q to p and let |qp| be its length. A function φ, denoted by φ_r, is attached to the real number *r*. It maps (θ,p,q) to $1/|qp|^r$ if θ is the direction of qp and to 0 otherwise. p and q are seen as particles and the vector $qp/|qp|^{r+1}$ as an elementary force exerted by p on q. The histogram value $\varphi_r^{AB}(\theta)$ is then the scalar resultant of all the elementary forces in direction θ.

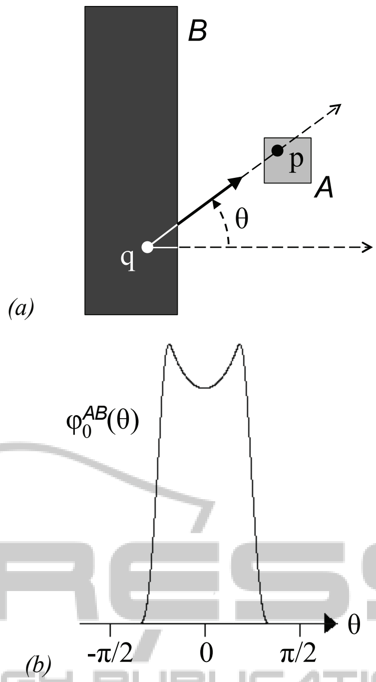

Figure 2: Histogram of forces. *(a)* $\varphi_r^{AB}(\theta)$ is the sum (integral) of all the elementary forces in direction θ. *(b)* Example.

Note

When *r*=2, the forces are gravitational-like. When *r*=0, the forces are distance-independent, and the histogram of forces coincides with (but does not have the weaknesses of) the histogram of angles.

The original algorithm runs in $O(nN\sqrt{N})$ time, where *n* is the number of directions θ considered. The best case performance (convex objects) is $O(nN)$. A more recent algorithm (Matsakis, Wendling and Ni, 2010) runs in $O(N \log N)$, but the processing times are usually much longer, unless *n* is very large or the objects are fractal-like.

Directional relationship information can be extracted using the same methods as for the histogram of angles, or using a method based on force categorization (Matsakis, Wendling and Ni, 2010).

Fuzzy objects and 3D objects are best handled by the $O(N \log N)$ algorithm. Vector objects can be handled as well (Recoskie et al., 2012). However, the time complexity of the algorithm has been severely underestimated and is $O(n\eta^3)$ instead of $O(n\eta \log \eta)$, where η is the total number of object vertices. The best case performance (convex objects) is $O(n\eta^2)$.

The histogram of forces has found many applications, including human-robot interaction (Skubic et al., 2004), geospatial information retrieval and indexing (Shyu et al., 2007), and map-to-image conflation (Buck et al., 2013).

3.3 Allen Histograms

The *Allen histograms* are a tuple of 13 F-histograms (Malki et al., 2002) (Matsakis and Nikitenko, 2005). Allen's logic considers 13 jointly exhaustive and pairwise disjoint relations for convex temporal intervals (Allen, 1983). Each relation *r* corresponds to a topological relationship between two segments on a directed line. A function F, denoted by F_r, is attached to *r*. It extends *r* from pairs of segments to pairs of cores, while the Allen histogram F_r^{AB} extends *r* from pairs of segments to the pair *(A,B)* of objects. $F_r^{AB}(\theta)$ measures the extent to which *r* holds, in direction θ, between *A* and *B*. See Fig. 3.

First, *r* is fuzzified. *r*(I,J), where I and J are two segments on a directed line, denotes a real number between 0 (it is totally false that *r* holds between I and J) and 1 (it is totally true). For example, if two segments are disjoint but very close to each other, then we want to be able to say that they almost touch. Next, the cores of the objects are fuzzified as well. The idea is to consider that if two segments in a core are very close, then they should be seen, to a certain extent, as a single segment. Now, consider a line *L* in direction θ. Any α-cut of the fuzzy core $A \cap L$ is the union of pairwise disjoint segments I_i^α. Likewise, any α-cut of $B \cap L$ is the union of segments J_j^α. The function F_r maps (θ, $A \cap L$, $B \cap L$) to a weighted average of all the $r(I_i^\alpha, J_j^\alpha)$.

Note

The idea of using 13 histograms, 1 per Allen relation, to describe the relative position of objects was first proposed by Malki et al. (2002). Only convex objects can be handled, and there is no consistency between the histograms—which are f-histograms. The idea was revisited by Matsakis and Nikitenko (2005) to address these flaws. The f-histograms are replaced with F-histograms. The worst case performance is $O(nN^2)$. The best case performance (convex objects) is $O(nN)$.

The definition of the Allen F-histograms was simplified and adapted to the handling of vector objects by Salamat and Zahzah (2012*a*). However, the time complexity of the algorithm was severely underestimated. It is $O(n\eta^3)$ instead of $O(n\eta\log\eta)$, where η is the total number of object vertices. The best case performance (convex objects) is $O(n\eta^2)$.

There is a straightforward extension to 3D objects, but processing times are prohibitive.

Meaningful directional and topological relationship information can be extracted when the objects are convex (Salamat and Zahzah, 2012*b*), but is much harder to extract when the objects are concave (Matsakis, Wawrzyniak and Ni, 2010) because of the inadequacy of describing 2D spatial relationships in terms of Allen's 1D temporal relationships (Cohn et al., 1997).

The Allen histograms have been used for linguistic scene description (Matsakis, Wawrzyniak and Ni, 2010), spatio-temporal reasoning (Salamat and Zahzah, 2012*c*) and the modeling of motion classes (Salamat and Zahzah, 2012*d*).

3.4 R-Histogram

Consider two raster objects *A* and *B*. For any pixels p of *A* and q of *B*, let $\angle(p,q)$ be the angle between the x-axis and the directed line that passes through the center of q and then of p, let $d(p,q)$ be the distance from p to q, and let $l(p,q)$ be the integer as defined in Table 3. See Fig. 4. The angle $\angle(p,q)$ is an element of $(-\pi,\pi]$ while $d(p,q)$ belongs to some interval $[0, d_{max}]$. Partition $(-\pi,\pi]$ into *n* intervals Θ_1, Θ_2, etc. (the direction bins), and partition $[0, d_{max}]$ into *m* intervals D_1, D_2, etc. (the distance bins). The histogram value $R^{AB}(i,j,k)$ is the number of pixel pairs (p,q) such that:

p is a boundary pixel of *A*,
q is a boundary pixel of *B*,
$\angle(p,q)\in\Theta_i$ and $d(p,q)\in D_j$ and $l(p,q)=k$.

See (Wang and Makedon, 2003).

Note

While the worst case performance is $O(N^2)$, the best case performance (convex objects) is $O(N)$.

The R-histogram obviously supports extraction of some directional, set and distance relationship information, but extraction methods and models of spatial relationships based on the R-histogram have not been investigated.

The R-histogram can only handle objects that are homeomorphic to a 2-ball. There are straightforward extensions to fuzzy objects and 3D objects, but processing times may be prohibitive.

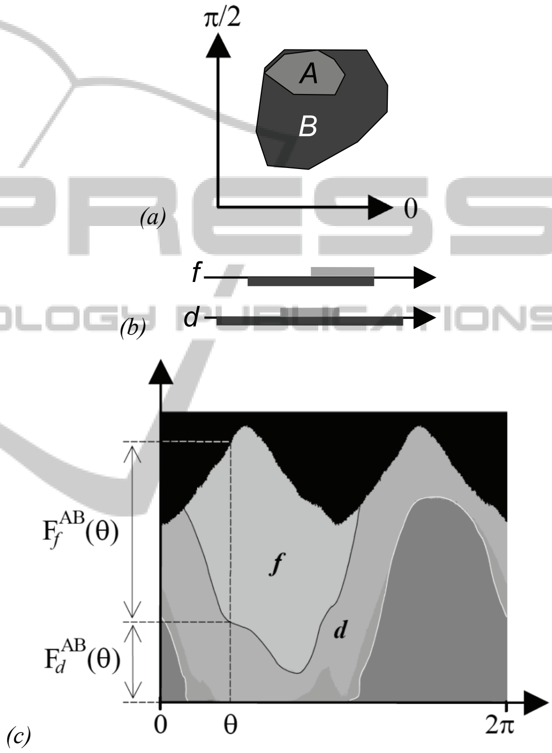

Figure 3: Allen histograms. *(a)* A pair of objects. *(b)* The Allen relations *f* (finishes) and *d* (during). *(c)* The 13 Allen histograms associated with *(A,B)* and stacked on top of each other.

The behaviour of the R-histogram under similarity transformations seems easy to determine and similarity invariance seems easy to obtain. However, see last paragraph of Section 3.1.

Table 3: The label $l(p,q)$.

p is a pixel of *B*	q is a pixel of *A*	$l(p,q)$
false	false	0
true	false	1
false	true	2
true	true	3

The R-Histogram has been applied to similarity-based image retrieval.

3.5 R*-Histograms

Consider two raster objects *A* and *B* and an element θ of the interval $(-\pi,\pi]$. The image is partitioned into raster lines running in direction θ, as shown in Fig. 5. For any pixels p of *A* and q of *B*, let *d*(p,q) be the distance from p to q, and let *l*(p,q) be the integer as in Table 3. The distance *d*(p,q) belongs to some interval $[0,d_{\max}]$. Partition $[0,d_{\max}]$ into *m* intervals D_1, D_2, etc. (the distance bins). The histogram value $R^{*AB}(\theta,j,k)$ is the number of pixel pairs $(p,q)\in A\times B$ such that:

> q is before p on the same raster line,
> $d(p,q)\in D_j$ and $l(p,q)=k$.

See (Wang et al., 2004).

Note

A first algorithm runs in $O(n N\sqrt{N})$ time, where *n* is the number of directions θ considered. A second algorithm runs in $O(n N \log N)$. No comparative study of the two algorithms is available.

Extension to vector objects may be possible. There are straightforward extensions to fuzzy objects and 3D objects, but processing times may be prohibitive.

The R*-histogram obviously supports extraction of some directional, set and distance relationship information, but extraction methods and models of spatial relationships based on the R*-histogram have not been investigated.

The R*-histogram probably offers a solution to the direct and inverse problems with respect to affine transformations (as long as the distance $d_{\max}$ is determined independently for each direction θ and recorded). A normalization procedure should then allow affinity invariance. These issues deserve further investigation.

3.6 Spread Histogram

Consider two raster objects *A* and *B*, as in Fig. 6. For any pixel p of *A*, the half-lines originating from the centre of p and passing through the centres of the pixels of *B* divide the plane into sectors. Let $\angle(p,B)$ be the central angle of the largest sector. This angle belongs to $[0,2\pi]$. Now, partition $[0,2\pi]$ into *n* intervals Θ_1, Θ_2, etc. The value $H^{AB}(i)$ is the number of pixels $p\in A$ such that $\angle(p,B)\in\Theta_i$. See (Kwasnicka and Paradowski, 2005).

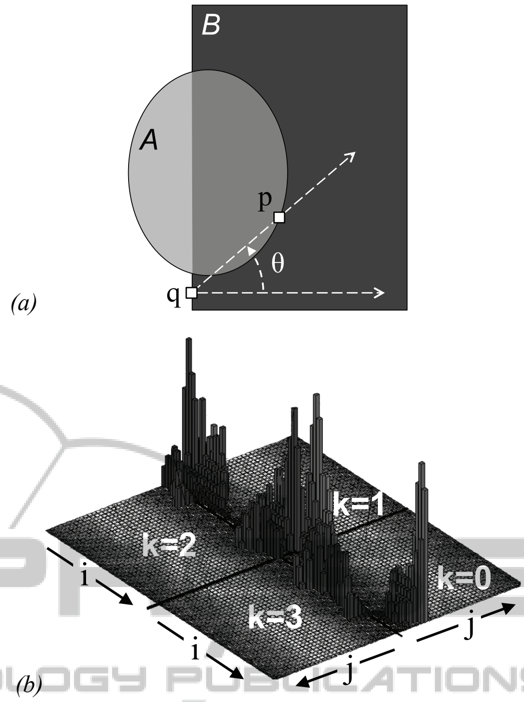

Figure 4: R-histogram. *(a)* $R^{AB}(i,j,k)$ is the number of boundary pixel pairs (p,q) such that the angle θ falls into the bin Θ_i, the distance *d*(p,q) falls into the bin D_j and *l*(p,q)=k. *(b)* Representation.

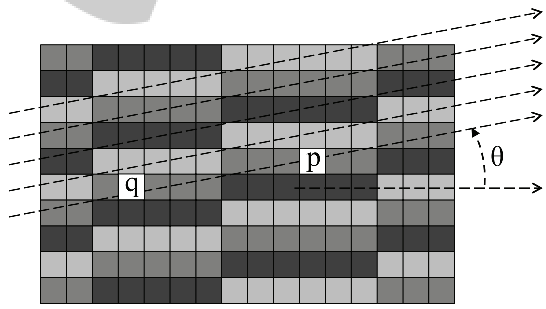

Figure 5: R*-histogram. The image is partitioned into raster lines running in direction θ. Here, the pixels p and q are on the same raster line, and q is before p.

Note

The spread histogram is meant to be used along with the histogram of angles. The two can be computed simultaneously in $O(N^2)$ time and allow extraction of directional relationship information as well as information on the relationships *inside* ($A\subseteq B$), *outside* ($A\cap B=\emptyset$) and *surround*. Note that, here, surroundedness should be understood as visual surroundedness (Rosenfeld and Klette, 1984), not as topological surroundedness. See Fig. 7.

The behaviour of the spread histogram under similarity transformations seems easy to determine and similarity invariance seems easy to obtain. However, see last paragraph of Section 3.1.

3.7 Visual Area Histogram

Consider two disjoint raster objects *A* and *B*, as in Fig. 8. For any pixels p of *A* and q of *B*, if a raster segment whose endpoints are p and q does not contain any other pixel of *A* or *B* then p is a boundary pixel of *A* visible from *B* and q is a boundary pixel of *B* visible from *A*. Let $\angle(p,q)$ be the angle between the x-axis and the directed line that passes through the center of q and then of p. This angle belongs to $(-\pi,\pi]$. Partition $(-\pi,\pi]$ into *n* intervals Θ_1, Θ_2, etc. The histogram value $H^{AB}(i)$ is the number of pixel pairs (p,q) such that

> p is a boundary pixel of *A* visible from *B*,
> q is a boundary pixel of *B* visible from *A*,
> and $\angle(p,q) \in \Theta_i$.

There is, however, a more general definition. Instead of contributing 1 unit to $H^{AB}(i)$, a pair (p,q) as above may contribute $[d_{min} / d(p,q)]^r$, where *r* is a real number, $d(p,q)$ is the distance from p to q, and d_{min} is the minimum distance over all pairs. See (Zhang et al., 2010).

Note

The visual area histogram can only handle disjoint objects. The algorithm runs in $O(N^2)$ time. In most cases, however, computing a visual area histogram is expected to be much faster than computing a histogram of angles, since many fewer pixel pairs are considered.

There is a straightforward extension to 3D objects, but processing times may be prohibitive.

The behaviour of the visual area histogram under

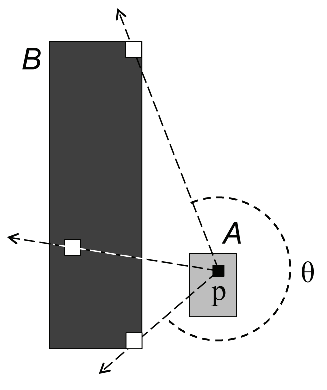

Figure 6: Spread histogram. The half-lines originating from the centre of p and passing through the centres of the pixels of *B* divide the plane into sectors. Here, the central angle $\angle(p,B)$ of the largest sector is θ.

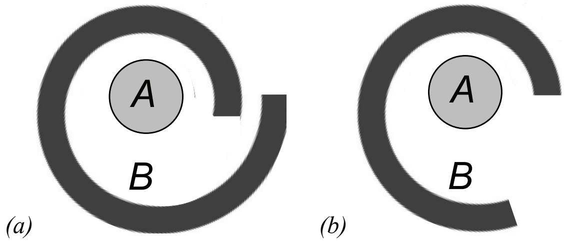

Figure 7: Visual surroundedness. *(a) A* is completely surrounded by *B*. *(b) A* is partially surrounded by *B*.

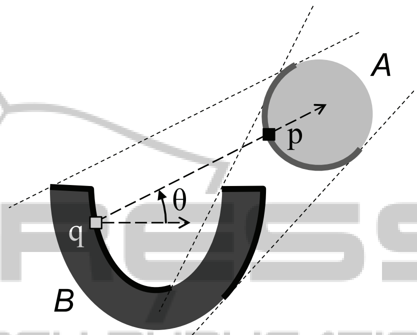

Figure 8: Visual area histogram. p is a boundary pixel of *A* visible from *B* and q is a boundary pixel of *B* visible from *A*. Here, $\angle(p,q)=\theta$.

similarity transformations seems easy to determine and similarity invariance seems easy to obtain. However, see last paragraph of Section 3.1.

3.8 Radial Line Model

Consider two objects *A* and *B*, and a reference point p determined by the minimum bounding rectangles of the objects, as suggested in Fig. 9. Partition $(-\pi,\pi]$ into *n* intervals Θ_1, Θ_2, etc. (the *direction bins*). The unbounded sector extending from p and defined by Θ_i intersects *A* (resp. *B*) in some region A_i (resp. B_i). The histogram value $H^A(i)$ (resp. $H^B(i)$) is the area of A_i (resp. B_i) over the area of *A* (resp. *B*). The position of *A* relative to *B* is represented by the pair (H^A, H^B). See (Santosh et al., 2010).

Note

The Radial Line Model (RLM) targets directional and set relationships. However, extraction methods and models of such relationships based on the RLM have not been investigated. Besides, the RLM does not always allow us to determine whether two objects overlap, or whether one includes the other.

The behaviour of the RLM under similarity transformations is unknown, and similarity invariance cannot be obtained. These two properties would hold, however, if the model was defined as follows: choose p as the centroid of $A \cup B$; the half-line that extends from p in direction θ intersects A (resp. B) in a union (possibly empty) of pairwise disjoint segments (Fig. 9); set the histogram value $H^A(\theta)$ (resp. $H^B(\theta)$) to the total length of these segments, and represent the position of A relative to B by the pair (H^A, H^B).

The RLM has been used for graphical symbol retrieval.

3.9 Ratio Histogram

Consider two objects A and B with distinct centroids a and b. Any real number x can be mapped to a line $L(x)$ as shown in Fig. 10. This line $L(x)$ is parallel to the line that passes though a and b, it does not intersect both objects if x is less than 0 or greater than 1, and it does intersect both objects if x is 0 or 1. The core $A \cap L(x)$ is the union of a finite number of pairwise disjoint segments. Let $|A \cap L(x)|$ be the total length of these segments. The ratio histogram H^{AB} is the function $x \longmapsto |A \cap L(x)| \,/\, |B \cap L(x)|$. See (Wang et al., 2012).

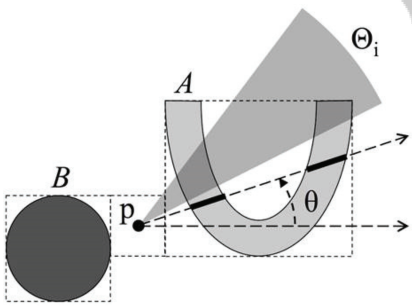

Figure 9: Radial Line Model. $H^A(i)$ is the total area of the two darker regions in A, divided by the area of A. Another option is to define $H^A(\theta)$ as the total length of the two black segments.

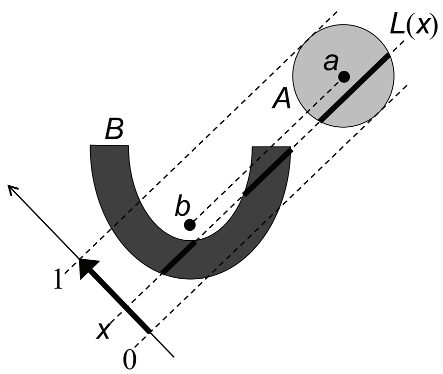

Figure 10: Ratio histogram. The line $L(x)$.

Note

The centroids of the two objects must be distinct.

The ratio histogram is designed to be invariant to affine transformations. As a result, it offers a trivial solution to the direct problem with respect to affinities, and there is no solution to the inverse problem with respect to similarities.

The ratio histogram has been used for shape matching and object recognition.

4 CONCLUSIONS

Various relative position descriptors have been considered in this review. They illustrate various approaches to relative position description, and are of interest for various reasons. For example, the Allen histograms are the only ones that really target topological relationships; the spread histogram is the only one that targets *surrounds*; the ratio histogram is the only one that is affine invariant. Every descriptor has its strengths, and its limitations: meaningful topological relationship information cannot be easily extracted from Allen histograms when the objects are concave; the spread histogram is computationally expensive; the discriminating power of the ratio histogram is low. There is a need for a more versatile descriptor, that targets all types of spatial relationships. Moreover, there is no descriptor that offers a solution to the inverse problem with respect to affine transformations, and there is no descriptor that offers a solution to the *recovery problem* (i.e., given a relative position descriptor, find all the pairs of objects that receive the same description). All these are potential areas for future work.

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