

# Low-rank and Sparse Matrix Decomposition with a-priori Knowledge for Dynamic 3D MRI Reconstruction

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**Abstract:** It has been recently shown that incorporating priori knowledge significantly improves the performance of basic compressive sensing based approaches. We have managed to successfully exploit this idea for recovering a matrix as a summation of a Low-rank and a Sparse component from compressive measurements. When applied to the problem of construction of 4D Cardiac MR image sequences in real-time from highly under-sampled  $k$ -space data, our proposed method achieves superior reconstruction quality compared to the other state-of-the-art methods.

## 1 INTRODUCTION

A fundamental problem in dynamic MRI, such as real-time cardiac MRI (rtCMR), is the limitation of spatial and temporal resolution which is due to the slow data acquisition process of this modality. This problem is even more profound when dealing with 4D MR volumes. Compressive Sensing (CS) has been shown to be able to overcome these challenges and recover MRI images from much smaller  $k$ -space measurements than conventional reconstruction methods. To achieve this, earlier CS-based methods assumed that the MRI images have a sparse representation in some known transform domain (Zonoobi et al., 2014; Hu et al., 2012; Lustig et al., 2008; Zonoobi et al., 2011) and the idea was easily extended to the reconstruction of dynamic MRI images data by jointly reconstructing the entire sequence by treating it as higher dimensional data (Gamper et al., 2008; Venkatesh et al., 2010). In other works, the high spatiotemporal correlation was utilized to recover dynamic images by solving a low rank matrix completion problem in which each temporal frame is a column of the recovered matrix (Zhao et al., 2010), (Halldar and Liang, 2011).

Some studies have reported much improved results that were obtained by combining rank deficiency and transform domain sparsity. These include proposals to recover the image as a solution which is both sparse and low rank (Majumdar and Ward, 2012b), (Gao et al., 2012); and other proposals that decompose the data in two low-rank ( $\mathbf{L}$ ) and sparse ( $\mathbf{S}$ ) com-

ponents (Majumdar and Ward, 2012a), (Gao et al., 2012; Goud et al., 2010), where  $\mathbf{L}$  models the correlated information between frames and  $\mathbf{S}$  represents the rapid change of data over time.

More recently, it has been shown that incorporation of the priori knowledge into the reconstruction of sparse signals can significantly improve their performance (Zonoobi and Kassim, 2013; Vaswani and Lu, 2010a; Zonoobi and Kassim, 2014b). This idea have been used in the Modified-CS (Vaswani and Lu, 2010b) to *recursively* reconstruct a time sequence of MRI images in real-time from highly under sampled measurements by using a-priori knowledge obtained from the previous reconstructed image. The Modified-CS uses the support of the previous time instance as a partially known part of the current support and finds a signal which satisfies the observations and is sparsest outside the support of the previous time instant. The a-priori based methods, only model the signal of interest as one sparse component. However, it is observed that the  $\mathbf{L}$  and  $\mathbf{S}$  decomposition can model dynamic MRI data significantly better than a low-rank or a sparse model alone, or than a model in which both constraints are enforced simultaneously (Otazo et al., 2013; Zonoobi and Kassim, 2012; Ensafi et al., 2014). To the best of our knowledge, no previous work has been done to incorporate the priori information into image recovery while the image is modelled as a summation of a low rank and a sparse component.

In this paper we first propose a re-formulation of the  $\mathbf{L}$  and  $\mathbf{S}$  decomposition to take into account some

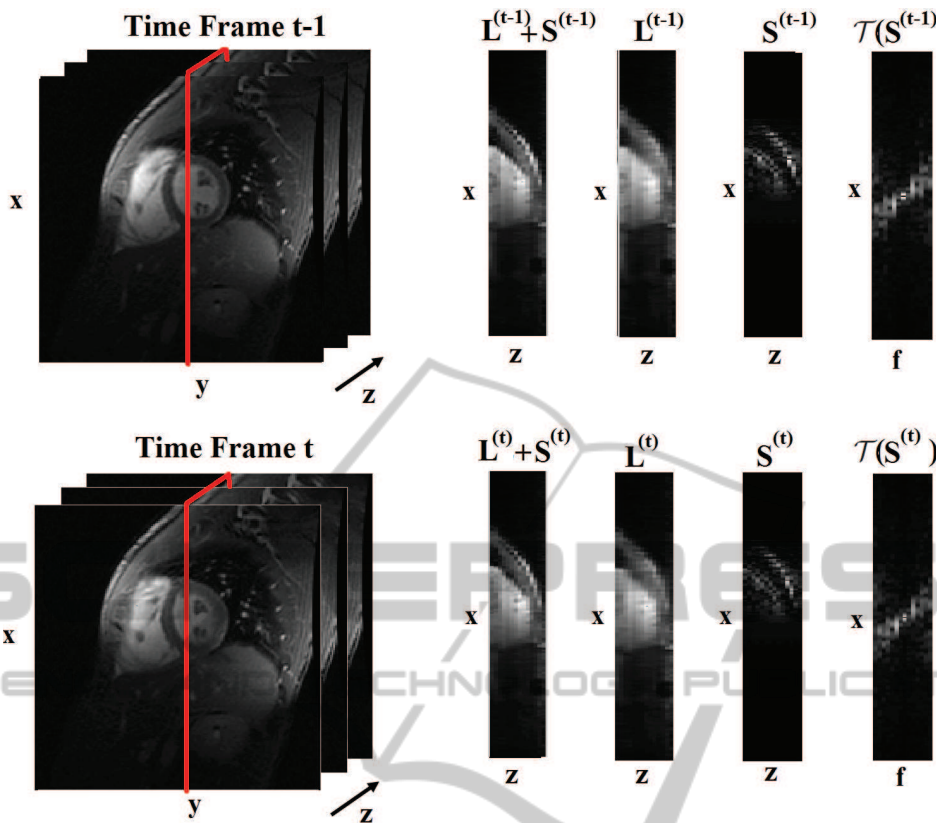


Figure 1: Illustration of L+S decomposition of fully-sampled 3D cardiac cine at time  $t$  and  $t + 1$ .

priori knowledge and then use a soft-thresholding based algorithm to efficiently solve it. The algorithm is then employed to reconstruct a time sequence of 3D cardiac MRI volumes from highly undersampled measurements.

The rest of this paper is organized as follows: this section ends with a description of the notations used. Section 2 presents the problem of reconstruction of 3D dynamic MRI volumes and the current state-of-the-art CS-based approaches that address this problem. In section 3, we provide details of our proposed algorithm which we call *Priori L+S*. Finally, we present and analyze our experimental results in section 4 before providing the concluding remarks in section 5.

**Notations:** Throughout the paper, matrices are denoted by boldface letters (e.g.  $\mathbf{X}, \mathbf{S}$ ) while scalars are shown by small regular letters (e.g.  $n, m, k, r$ ) and linear maps and operators are denoted by bold calligraphic uppercase letters ( $\mathcal{T}, \mathcal{A}, \Sigma$ ) and  $\mathcal{A}^{-1}$  denotes the adjoint of the operator. Superscript ( $t$ ) added to a matrix refers to that of time  $t$ . For a matrix, the notation  $\mathbf{M}|_S$  forms a sub-matrix that contains elements with indices in  $S$ .

## 2 PROBLEM FORMULATION

The low rank and sparse matrix decomposition (L+S) is particularly suitable to the problem of dynamic imaging, where the low rank component models the temporally correlated background and the sparse component represents the dynamic information that lies on top of the background (Gao et al., 2012; Lingala et al., 2011). To apply the low-rank and sparse matrix decomposition to 3D dynamic MRI, let's assume that the 3D volume of interest is of size  $[n_x \times n_y \times n_z]$ , ( $n_x, n_y > n_z$ ) which is changing with time. At each time instance  $t$  the 3D volume is converted to a matrix  $\mathbf{X}^{(t)} \in \mathcal{R}^{(n_x n_y) \times n_z}$ , where each column is consist of a frame. This matrix could be then decomposed into a low rank matrix  $\mathbf{L}^{(t)}$  and a matrix  $\mathbf{S}^{(t)}$ , which we assume to have a sparse representation in some known basis  $\mathcal{T}$  (such as Wavelets (Kassim et al., 2008)), as  $\mathbf{X}^{(t)} = \mathbf{L}^{(t)} + \mathbf{S}^{(t)}$ .

Figure 1 shows a cross section of the low rank and sparse components of cardiac data sets for two adjacent time instances ( $t$  and  $t + 1$ ). It can be seen that  $\mathbf{L}^{(t)}$  represents the background component and  $\mathbf{S}^{(t)}$  corresponds to the changes from a frame to another, e.g., organ motions or contrast -enhancement,

etc (Majumdar and Ward, 2012b; Zonoobi and Kasim, 2014a; Feng et al., 2012).

With this the problem can be posed as follows: let  $\mathcal{A}$  be the acquisition/sampling operator that performs a frame-by-frame k-space under-sampling of the  $t^{\text{th}}$  volume ( $\mathcal{A} : \mathcal{R}^{(n_x n_y) \times n_z} \rightarrow \mathcal{R}^{m \times n_z}$ , where  $m \ll n_x n_y$ ). Using this operator, the under-sampled acquisition of  $\mathbf{X}^{(t)}$  can be expressed as:

$$\mathbf{d}^{(t)} = \mathcal{A}(\mathbf{X}^{(t)}) + \boldsymbol{\eta}$$

where  $\mathbf{Y}^{(t)}$  is the observation matrix of size  $m \times n_z$ , and is assumed to be incoherent with respect to the sparsity basis. Also  $\boldsymbol{\eta}$  is the measurement noise with finite energy (i.e.  $\|\boldsymbol{\eta}\|_2 \leq \epsilon_1$ ), which can be modelled as a complex Gaussian noise. The problem, at each time instance  $t$ , is then to recover the original  $\mathbf{X}^{(t)}$ , from the corresponding compressive samples  $\mathbf{Y}^{(t)}$  assuming that the signal of interest is can be decomposed into low rank and sparse components. The problem of recovering each volume from the compressive measurements can be then formulated as:

$$(\mathbf{L}^{(t)}, \mathbf{S}^{(t)}) = \arg \min \{ \|\Sigma(\mathbf{L}^{(t)})\|_0 + \|\mathcal{T}(\mathbf{S}^{(t)})\|_0 \} \quad (1)$$

subject to  $\|\mathbf{Y}^{(t)} - \mathcal{A}(\mathbf{L}^{(t)} + \mathbf{S}^{(t)})\|_2 \leq \epsilon_1$ ,

where  $\mathcal{T}$  is a sparsifying transform for  $\mathbf{S}$ , and  $\Sigma$  is an operator that maps any matrix to the vector of its singular values (i.e.  $\|\Sigma(\mathbf{L}^{(t)})\|_0 = \text{rank}(\mathbf{L}^{(t)})$ ).

Solving the above minimization problem is known to be computationally unwieldy in view of its combinatorial nature. As a consequence, we are compelled to resort to an alternative convex approximation as follows:

$$(\mathbf{L}^{(t)}, \mathbf{S}^{(t)}) = \arg \min \{ \|\Sigma(\mathbf{L}^{(t)})\|_1 + \|\mathcal{T}(\mathbf{S}^{(t)})\|_1 \} \quad (2)$$

subject to  $\|\mathbf{Y}^{(t)} - \mathcal{A}(\mathbf{L}^{(t)} + \mathbf{S}^{(t)})\|_{\ell_2} \leq \epsilon_1$ ,

where  $\|\Sigma(\mathbf{L}^{(t)})\|_1$  is the nuclear norm of  $\mathbf{L}^{(t)}$ . This convex problem can be solved efficiently using an iterative algorithm, thereafter referred to as L+S method (Otazo et al., 2013), which is closely related to (Beck and Teboulle, 2009) and (Cai et al., 2010) for sparse ( $\mathcal{T}(\mathbf{S})$ ) and low-rank matrix recovery ( $\mathbf{L}$ ), respectively. The L+S method starts from a signal proxy and then at each iteration proceeds through three steps to update its estimates of the low rank matrix and the sparse component using a soft-thresholding operator. This operator is defined as:

$$\mathbb{S}\{x, \lambda\} = \frac{x}{|x|} \max(|x| - \lambda, 0)$$

in which  $x$  could be a complex number and the threshold  $\lambda$  is real valued. This is extended to matrices by applying it to each element of that matrix.

It is known from the literature that recovery of sparse vectors and low-rank matrices can be accomplished when the measurement operator  $\mathcal{A}$  satisfies the appropriate RIP or RRIP conditions (Candes et al., 2009). The above formulation, however, does not take into account any priori information that may be available about the low rank/sparse components.

### 3 LOW-RANK AND SPARSE MATRIX DECOMPOSITION WITH a-priori INFORMATION

To reconstruct images from even fewer number of samples than L+S method (Otazo et al., 2013), we aim to use the  $L$  and  $S$  components of the previously reconstructed volume to guide the reconstruction of the current time volume. The idea is based on the observation that the  $\mathbf{L}$  and  $\mathbf{S}$  components of each MRI volume is very closely related to those of the adjacent time instances. This is not surprising as it is known that dynamic images are highly redundant in space and time (Jung et al., 2009). To illustrate this, figure 1 shows a cross-section of the low rank and sparse components of a fully-sampled cardiac data set for two adjacent time instances. From the figure it can be seen that  $\mathbf{L}^{(t-1)}$  and  $\mathbf{L}^{(t)}$  are quite similar, in fact  $\|\Sigma(\mathbf{L}^{(t)}) - \Sigma(\mathbf{L}^{(t-1)})\|_2 = 0.04$ . This means that vector of singular values of  $\mathbf{L}^{(t-1)}$  and  $\mathbf{L}^{(t)}$  are very close in Euclidean space. Similarly, support of  $\mathcal{T}(\mathbf{S}^{(t)})$  is much the same as the one of  $\mathcal{T}(\mathbf{S}^{(t-1)})$ . In this case, for instance, the support change turns out to be less than 5% of the support size. Therefore, support of  $\mathcal{T}(\mathbf{S}^{(t-1)})$  can be viewed as an a-priori knowledge of the partial support of  $\mathcal{T}(\mathbf{S}^{(t)})$ . Based on the above observations, to recover  $\mathbf{X}^{(t)}$ , we modify the formulation of the problem to incorporate the information of  $\mathcal{T}(\mathbf{S}^{(t-1)})$  and  $\mathbf{L}^{(t-1)}$  as follows:

$$(\mathbf{L}^{(t)}, \mathbf{S}^{(t)}) = \arg \min \{ \|\Sigma(\mathbf{L}^{(t)})\|_1 + \|\mathcal{T}(\mathbf{S}^k)|_{\bar{T}^{(t-1)}}\|_1 \}$$

subject to  $\|\mathbf{Y}^{(t)} - \mathcal{A}(\mathbf{L}^k + \mathbf{S}^k)\|_2 \leq \epsilon_1$ ,

$$\|\Sigma(\mathbf{L}^k) - \Sigma(\mathbf{L}^{k-1})\|_2 \leq \epsilon_2 \quad (3)$$

where  $T^{(t-1)}$  denotes the support of  $\mathcal{T}(\mathbf{S}^{(t-1)})$  and  $\bar{T}^{(t-1)}$  is the complement of  $T^{(t-1)}$ . Basically we are searching for an image which satisfies the observations, its  $\mathbf{S}$  component is sparsest outside  $T^{(t-1)}$  and at the same time it has  $\Sigma(\mathbf{L}^{(t)})$  closest to  $\Sigma(\mathbf{L}^{(t-1)})$ .

The above formulation is convex and therefore it has a unique solution, however using convex-based optimization methods may not be practical for large-scale problems due to their considerable computational complexity and memory requirements (Needell

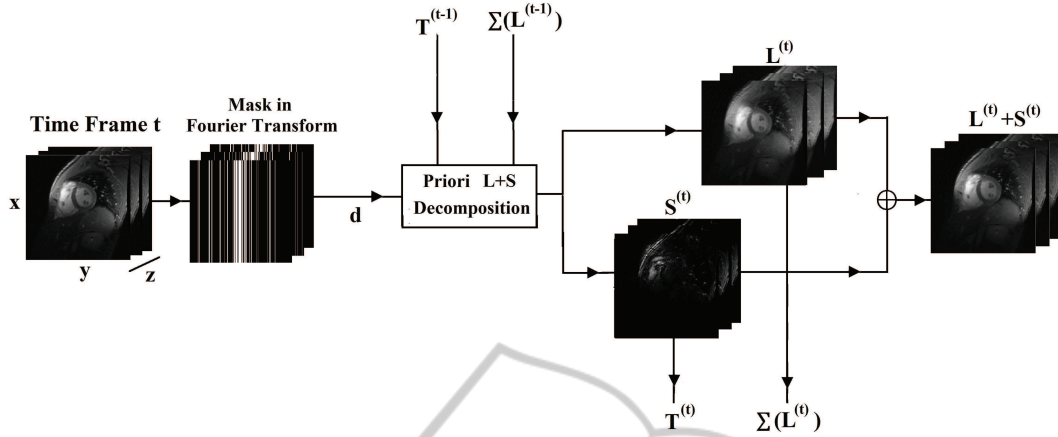


Figure 2: Overview of the priori L+S scheme.

and Tropp, 2009). Therefore we solve (1) using an iterative algorithm inline with L+S method (Otazo et al., 2013).

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**Algorithm 1:** PrioriL+S decomposition.
 

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**Input:**  $\mathbf{Y}^{(t)}, \mathcal{A}, \mathbf{S}^{(t-1)}, \mathbf{L}^{(t-1)}, \lambda_T, \lambda_S$   
 (0) Initialization:  $\mathbf{X}_0 = \mathcal{A}^{-1}(\mathbf{Y}^{(t)}), \mathbf{S}_0 = 0$ ;  
**while** not converged **do**

(1) Singular-value soft-thresholding of  $\mathbf{L}$ :

$$\mathbf{L}_{it-1} = \mathbf{X}_{it-1} - \mathbf{S}_{it-1};$$

$$\mathbf{L}_{it} = \Sigma^{-1}(\mathcal{S}\{\Sigma(\mathbf{L}_{it-1}), \lambda_L\});$$

(2) Imposing the priori knowledge on  $\mathbf{L}$ :

$$\mathbf{D}_{it} = (\Sigma(\mathbf{L}_{it}) - \Sigma(\mathbf{L}^{(t-1)}));$$

$$\mathbf{L}_{it} = \Sigma^{-1}(\Sigma(\mathbf{L}_{it}) - \lambda_p \mathbf{D}_{it})$$

(3) Imposing Sparsity and the priori knowledge on  $\mathbf{S}$ :

$$\mathcal{T}^{(t-1)} = \text{supp}(\mathcal{T}(\mathbf{S}^{(t-1)}));$$

$$\mathbf{S}_{it} = \mathcal{T}^{-1}(\mathcal{S}\{\mathcal{T}(\mathbf{S}_{it-1})|_{\mathcal{T}^{(t-1)}}, \lambda_S\});$$

(4) Update estimation to minimize error:

$$\mathbf{E}_{it} = \mathbf{Y}^{(t)} - \mathcal{A}(\mathbf{L}_{it} + \mathbf{S}_{it});$$

$$\mathbf{X}_{it} = \mathbf{L}_{it} + \mathbf{S}_{it} - \mathcal{A}^{-1}(\mathbf{E}_{it});$$

**end**

**Output:**  $\mathbf{L}^{(t)} \leftarrow \mathbf{L}_{it}, \mathbf{S}^{(t)} \leftarrow \mathbf{S}_{it}$

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Our proposed algorithm, which is summarized below, mainly differs with (Otazo et al., 2013) in two steps where we impose the available priori-knowledge into estimation of  $\mathbf{S}$  and  $\mathbf{L}$  components. **Initialization:** similar to the original L+S algorithm, we start with an initial estimation of  $\mathbf{X}^{(t)}$  and we set the sparse component to be all zeros.

**Singular-value Soft-thresholding:** to impose the low-rank property on  $\mathbf{L}^{(t)}$ , in this step at the  $i$ -th iteration the vector of singular values of  $(\mathbf{X}_{it-1} - \mathbf{S}_{it-1})$

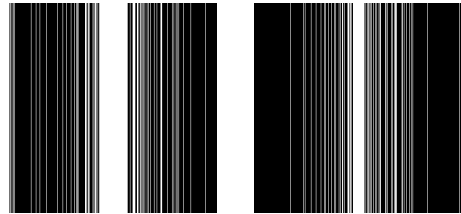
is soft thresholded.

**Imposing the Priori Knowledge on  $\mathbf{L}$ :** this step is designed to impose the available priori knowledge of the low rank component  $\mathbf{L}^{(t)}$  which is extracted from  $\mathbf{L}^{(t-1)}$ . To this end at each iteration  $it$ , it minimizes the Euclidean distance between the singular values of  $\mathbf{L}_{(it)}$  and the previously reconstructed component,  $\mathbf{L}^{(t-1)}$ , by moving into its gradient decent direction.

**Imposing Sparsity and the Priori Knowledge on  $\mathbf{S}$ :** In this step the goal is only force  $\mathcal{T}(\mathbf{S}_{it})$  to be sparse in locations not belonging to the spikes of the previous time instance ( $\mathcal{T}^{(t-1)}$ ). To this end, the algorithm only shrinks those elements not belonging to  $\mathcal{T}^{(t-1)}$ .

**Update Estimation to Minimize Error:** the new  $\mathbf{X}$  is finally obtained by enforcing measurement consistency, where the aliasing artifacts corresponding to the residual in k-space are subtracted from  $\mathbf{L}_{it} + \mathbf{S}_{it}$ . The algorithm iterates until the relative change in the solution is less than  $10^{-3}$ .

Figure 2 shows the priori L + S scheme for reconstructing the entire time sequence. At each time  $t$ , Algorithm 1 is used to recover  $\mathbf{X}^{(t)}$  except for  $t = 1$ , where the simple L + S algorithm is used as no priori knowledge is available for the reconstruction of the first volume.


 Figure 3: Cartesian sampling mask for (left)  $t=1$  and (right) subsequent frames.

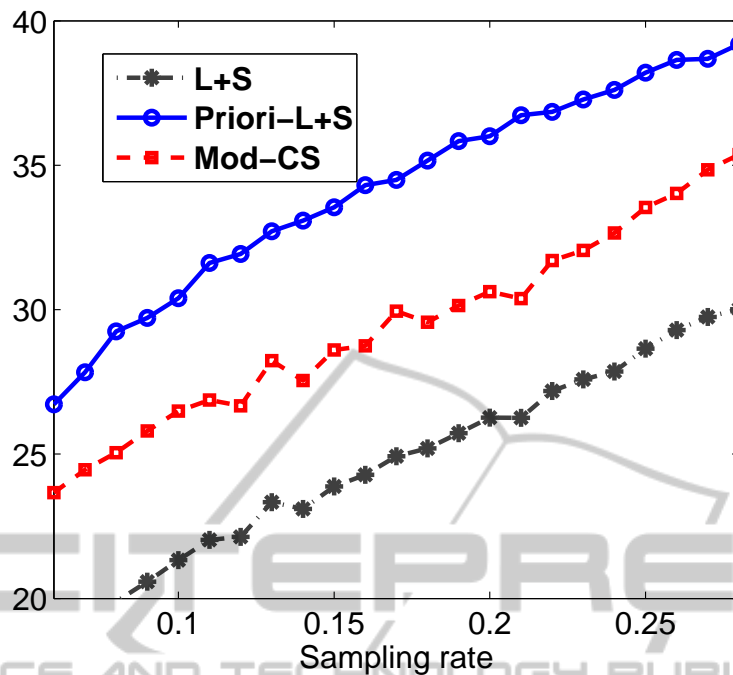


Figure 4: PSNR of the reconstructed images vs. sampling rate.

#### 4 EXPERIMENTAL RESULTS

To evaluate the performance of the proposed Priori  $L+S$  method, we applied it to the reconstruction of dynamic 3D Cardiac volumes of size  $256 \times 256 \times 14 \times 20$ . The results are then compared with that of  $L+S$  method (Otazo et al., 2013), and also with Modified-CS method (Vaswani and Lu, 2010b) (Mod-CS in figures 3 & 4). In all the experiments, we used a variable density Cartesian sampling mask which in practice is less time consuming than random sampling. However, to take the energy distribution of MR images in  $k$ -space into account, we used a variable-density sampling with denser sampling near the center. Figure 3 shows the sampling masks used in these experiments with two different. It should be noted that for the very first time-frame, since no priori information is available %50 of the  $k$ -space samples are taken and the sampling rate reported in figure 3 is for the successive frames. Moreover, the sparse domain is assumed to be the Wavelet domain and the reconstruction quality is measured using the Peak signal-to-noise ratio (PSNR).

Figure 4 compares the average PSNR of the reconstructed volumes vs. percentage of the samples taken in the  $k$ -space. It can be seen that our method consistently out-performs the others in terms of the improved PSNR. To compare the visual quality of the reconstructed images, figure 5 shows a slice of the re-

constructed volume using different methods together with the difference images (reconstruction error) amplified by a factor of 4. It is evident that the reconstructed image using the Priori- $L+S$  method is perceptually better with less loss of details and significantly reduced reconstruction error.

#### 5 CONCLUSIONS

In this paper, we presented a method which utilizes a-priori knowledge for high resolution and fast reconstruction of dynamic 3D MRI image sequences from undersampled  $k$ -space data. First, the problem of recovering a MRI images as a sum of low-rank and sparse components ( $L+S$ ) has been reformulated, to incorporate the priori knowledge extracted from previous reconstructions. Then we proposed an iterative soft thresholding-based algorithm to efficiently solve this minimization problem. To evaluate its performance, we used it to reconstruct a time sequence of 3D cardiac MRI volumes from highly undersampled  $k$ -space data. Our experiments show that our proposed method is superior to the other state-of-the-art CS-based methods, in terms of both visual quality and improved PSNR. Further investigation is still needed to study the effect of the sparsifying transforms and sampling patterns on the performance of the proposed Priori- $L+S$ .

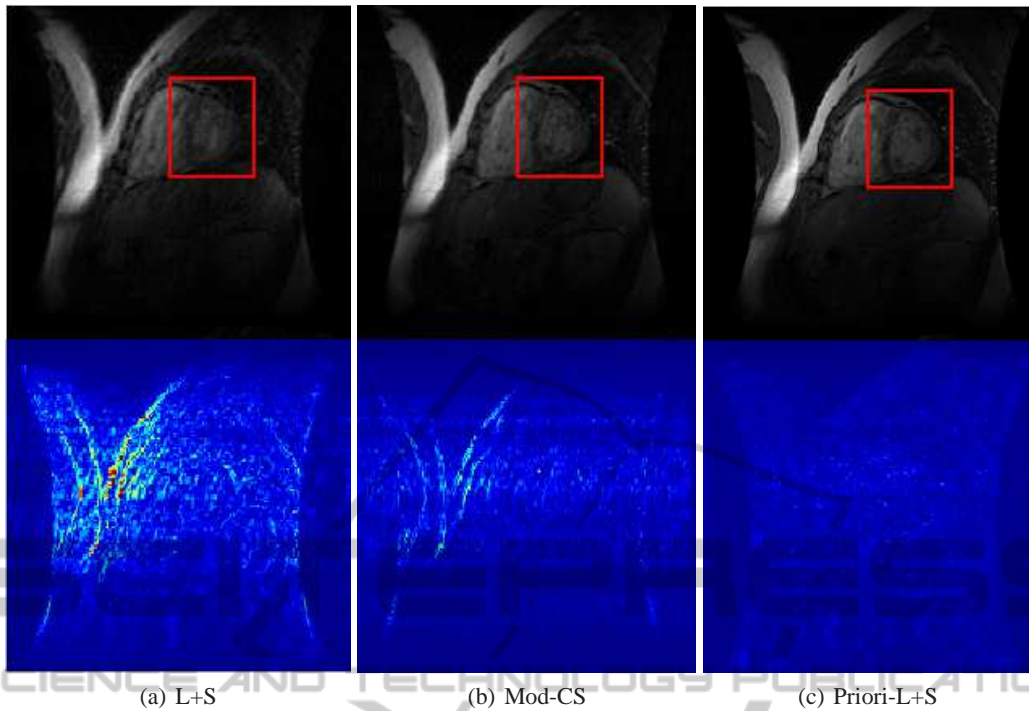


Figure 5: Comparison of the reconstructed images (1/7 of samples taken), together with the difference images that are amplified by a factor of 4.

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