Mathematical Model of a Human Leg
The Switched Linear System Approach

Artur Babiarz, Adam Czornik, Michał Niezabitowski and Radosław Zawiski
Institute of Automatic Control, Silesian University of Technology, 16 Akademicka St., 44-100 Gliwice, Poland

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Abstract: This article presents a novel approach to modelling of the human leg with the use of linear switched systems. Second order differential equations forming a two-segments leg model moving in a vertical plane are shown. State space linear equations describing given model are derived. A linear switched system for such model is presented, where the switching function is modelled as state-dependent. Based on this approach a linear system is presented, which is composed of four subsystems between which switching occurs depending on two state variables. These variables represent angular displacements. As a consequence, a state space division is shown together with a linear system describing human leg in this setting. Finally, a set of simulations presents differences between standard linear modelling approach and a switched linear system approach.

1 INTRODUCTION

Hybrid systems have become very popular during the last decade (Sun, 2006), (Sun and Ge, 2011). There are numerous adaptations of these systems into practical solutions. They own their popularity to possibilities of applying them to both, continuous and discrete dynamical systems. Switched systems are subgroup of hybrid systems, where the switching signal may depend on the logic-based switching function. Switched systems can be categorized into two groups: autonomous and controlled (Liberzon, 2003). In addition, each of these types of systems can be classified into state-dependent and time-dependent. In this paper, we consider a switched linear systems:

$$\begin{aligned} \dot{x} &= A_{\sigma(\cdot)}x + B_{\sigma(\cdot)}u, \\ y &= C_{\sigma(\cdot)}x + D_{\sigma(\cdot)}u \end{aligned} \tag{1}$$

defined for all $t \geq 0$ where $x \in \mathcal{R}^n$ is a state, $u \in \mathcal{R}^m$ is control, $y \in \mathcal{R}^q$ is output, $\sigma(\cdot) : \mathcal{P} \to \{1,2,\ldots,N\}$ is a switching rule and A_i, B_i, C_i, D_i, $i = 1,2,\ldots,N$ are constant matrices.

The authors are not aware of any traces in the literature of modelling of human leg in the above framework. In the paper (Bai et al., 2001), authors present dynamics of a human arm, as a second order continuous object with delay. Such mathematical model is convenient from the point of view of generating a trajectory and executed computer simulation. The approach presented in (Zhao et al., 2008) is based on classical mechanics, which approximates the limb as a combination of rigid links. Using the Euler-Lagrange formalism a matrix second order differential equation can be obtained. Matrices in this equation describe moments of inertia, Coriolis forces and gravity forces. Depending on the individual case (Zhao et al., 2008) it is possible to take into account various additional forces from external factors. In the same article, the authors also take into account a ground reaction force during the movement of the foot.

Recently, researchers devote an increasing attention to mechanical systems such as exoskeletons. Their main purpose is to aid and strengthen movement attributes of a human body (Pons et al., 2007), (Kong and Tomizuka, 2009) and to support motion of people with reduced mobility (Sekine et al., 2013). The authors of (Csercsik, 2005) describe the attempt to create a control system of the lower limb exoskeleton with a PID controller. They also perform a stability analysis by means of Nyquist criterion. The paper contains a full description of the experiments for one degree of freedom only. Obviously, such approach at the design of control systems is appropriate (Zawiski and Błachuta, 2012), (Błachuta et al., 2014), but it does not fully reflect the dynamics of the movement having seven degrees of freedom, and in a further perspective is a significant limitation. Another group of articles tackles the research focusing on impedance control (Burdet et al., 2006), (Chang et al., 2013). In

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the paper (Chang et al., 2013), authors present an application of the theoretical results of impedance control for trajectory generation and control of robot motion imitating the movement of a human arm with two degrees of freedom.
By scrutinizing the literature describing a human leg the problem of geometric description of the human limbs should be distinguished, along with the problem of stability during movement, the optimal control problem and the control problem with limitations on the value of the driving torque as a control signal (Ueyama and Miyashita, 2014). The information from (Burdet et al., 2006), (Chen, 2011), (Lee et al., 2010), (Neumann et al., 2013) became the inspiration to develop the idea of application of hybrid systems.

1.1 Background and Significance

The applications of hybrid systems are based on the results in (Pavlovic et al., 2001), (Lee et al., 2010), (Neumann et al., 2013). According to the information in (Lee et al., 2010) and (Neumann et al., 2013), shape of upper and lower limbs is changing strongly during the execution of any movement. In addition, the construction of a single muscle and the occurrence of muscle synergism is precisely described. As a result, we can assume that the matrix of inertia and the distance from the center of gravity of each link, are changing. However, changes of these parameters depend on the configuration of the leg.

Furthermore, research results published in (Babiarz et al., 2013) justify the application of hybrid systems for modeling objects with complex biomechanical structure. On the other hand, results presented in the work by (Burdet et al., 2006) and (Chen, 2011) indicate that the human arm is unstable in the considered range of motion. In addition, the state space of the object can be naturally divided and one can get a family of subsystems. Those subsystems form a basis for the modeling of the human leg by means of the switched systems. In this setting a switching function depends on the state vector. Consequently, we conclude that utilising of switching between the subsystems designated by given states of the limb's motion is a novel approach modeling and analysis of dynamic properties of human limbs.

The structure of this paper is as follows. We begin by presenting mathematical model of vertical human two-link leg. Next section describes proposed switched linear systems. In this section, we show the partition of the state space and switching functions. Section 3 presents a comparison of human leg linear and switched model. Moreover, the obtained results are shown for two simulation experiments. Finally, we conclude our proposition of mathematical model.

2 A HUMAN LEG MODEL

2.1 A State-space Model

In general terms the motion of a rigid body can be described by second-order differential nonlinear equation resulting from Euler-Lagrange formalism

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = u, \qquad (2)$$

where: $M(q)$ is an inertia matrix, $C(q,\dot{q})$ is a Coriolis and centrifugal forces matrix, $G(q)$ is a gravity forces vector, u are forces and moments acting on the system, q is an angular displacement.

The human leg model is presented in Figure 1. The equation describing dynamics of a two-link leg

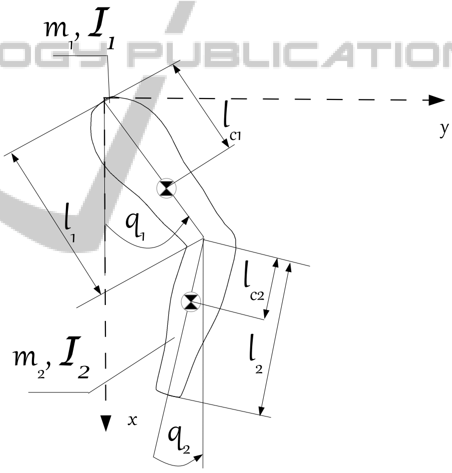

Figure 1: The model of two–link human leg.

in nonlinear state equation form (2) is

$$M(q) = \begin{bmatrix} c_1 & c_2\cos(q_1-q_2) \\ c_2\cos(q_1-q_2) & c_3 \end{bmatrix},$$

$$C(q,\dot{q}) = \begin{bmatrix} 0 & c_2\sin(q_1-q_2)\dot{q}_2 \\ -c_2\sin(q_1-q_2)\dot{q}_1 & 0 \end{bmatrix},$$

$$G(q) = \begin{bmatrix} -\sin q_1 \\ -c_5\sin q_2 \end{bmatrix},$$

$$c_1 = m_1 l_{c1}^2 + m_2 l_1^2 + I_1, \quad c_2 = m_2 l_1 l_{c2}, \quad c_3 = m_2 l_{c2}^2 + I_2,$$
$$c_4 = (m_1 l_{c1} + m_2 l_1)g, \quad c_5 = m_2 l_{c2} g.$$

where m is a mass, l is a link's length, l_c is a distance from the joint to the center of mass, I is a moment of inertia, g is a gravity acceleration.

Physical parameters of a modelled leg are presented in Table 1. Namely, the dynamics of model in

Table 1: The parameters of human leg.

	$m\,[kg]$	$l\,[m]$	$l_c\,[m]$	$I\,[kgm^2]$
Link 1	5.7	0.32	0.14	0.061
Link 2	2.65	0.40	0.17	0.038

terms of the state vector $\left[q^T, \dot{q}^T\right]^T$ can be expressed as (Babiarz et al., 2013), (Babiarz et al., 2014b)

$$\frac{d}{dt}\begin{bmatrix} q \\ \dot{q} \end{bmatrix} = \begin{bmatrix} \dot{q} \\ M(q)^{-1}[u - C(q,\dot{q})\dot{q} - G(q)] \end{bmatrix}. \quad (3)$$

Now, a new set of variables can be assigned to each of the state variables. In accordance with (3), the new set of state variables and their equivalences can be expressed as

$$\begin{aligned} x_1 = q_1, x_2 = q_2, \\ x_3 = \dot{x}_1 = \dot{q}_1, x_4 = \dot{x}_2 = \dot{q}_2. \end{aligned} \quad (4)$$

We can write the general state and output equations in the following way

$$\dot{x} = Ax + Bu, \quad (5)$$

$$y = Cx + Du, \quad (6)$$

where

$$\dot{x} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix}, x = \begin{bmatrix} q_1 \\ q_2 \\ \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}, \quad y = \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix}, u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}. \quad (7)$$

2.2 An Approach Switched Linear System

According to section 1 and subsection 1.1, we can design switched linear system based on (3) and (7). Analysing the activity of the human leg, we can assume that the switching times depend on state vector and the switched linear system is controlled (Liberzon, 2003). According to these assumptions the mathematical model can be described by equations (Babiarz et al., 2014a)

$$\dot{x}(t) = A_{\sigma(x)}x(t) + B_{\sigma(x)}u(t), \quad (8)$$

$$y(t) = C_{\sigma(x)}x(t) + D_{\sigma(x)}u(t). \quad (9)$$

We consider the switched linear systems with state-dependent switching. For the switched system described above we propose a state-dependent switching function

$$\dot{x} = \begin{cases} A_1x + B_1u & \text{if} \quad x_1 = 0, x_2 \geq 0 \\ A_2x + B_2u & \text{if} \quad x_1 > 0, x_2 > 0 \\ A_3x + B_3u & \text{if} \quad x_1 < 0 \quad or \quad x_1 > 0, x_2 = 0 \\ A_4x + B_4u & \text{if} \quad x_1 < 0, x_2 > 0 \end{cases} \quad (10)$$

$$y = \begin{cases} C_1x + D_1u & \text{if} \quad x_1 = 0, x_2 \geq 0 \\ C_2x + D_2u & \text{if} \quad x_1 > 0, x_2 > 0 \\ C_3x + D_3u & \text{if} \quad x_1 < 0 \quad or \quad x_1 > 0, x_2 = 0 \\ C_4x + D_4u & \text{if} \quad x_1 < 0, x_2 > 0 \end{cases} \quad (11)$$

Under the above assumption about the division of state space, the model's description can be made using four linear subsystems forming a linear switched system (equations (10) and (11)). Switching between any dynamics depends only on angular displacement because only this parameter influences leg's shape and configuration during motion. The direct consequence is that for each switching the angular velocity may be arbitrary.

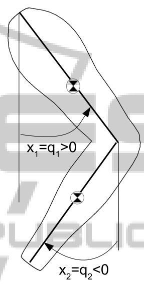

Figure 2: The first case.

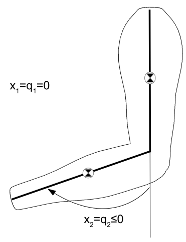

Figure 3: The second case.

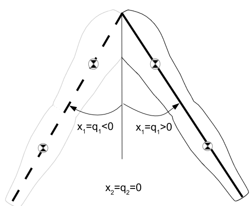

Figure 4: The third case.

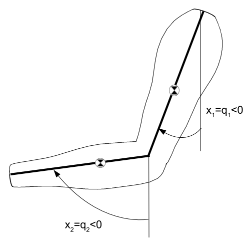

Figure 5: The fourth case.

3 SIMULATION STUDY

This paragraph presents human leg model's simulation conditions for linear as well as for switched system. All simulations were performed with the use of Matlab Simulink package. A block diagram is given in Figure 6.

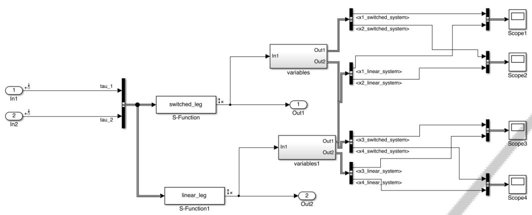

Figure 6: The Simulink model.

3.1 A Linear Model

For the purpose of obtaining a linear model equation 2 was linearized about equilibrium point $(x^0;u^0) = ([0\,rad, 0\,rad, 0\,\frac{rad}{s}, 0\,\frac{rad}{s}];[0\,Nm, 0\,Nm])$. Parameters taken for linearization are given in Table 1. The moment of inertia was calculated for truncated cone approximating the shape of a human leg.

$$A_L = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -61.44 & 21.14 & 0 & 0 \\ 77.27 & -65.14 & 0 & 0 \end{bmatrix}, \quad (12)$$

$$B_L = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 3.806 & -4.8 \\ -4.8 & 14.75 \end{bmatrix}, \quad (13)$$

$$C_L = \begin{bmatrix} -61.44 & 21.14 & 0 & 0 \\ 77.27 & -65.14 & 0 & 0 \end{bmatrix}, \quad (14)$$

$$D_L = \begin{bmatrix} 3.806 & -4.8 \\ -4.8 & 14.75 \end{bmatrix}. \quad (15)$$

3.2 A Switched Linear System

For a linear switched system equation (2) was linearized about arbitrary selected working points which are contained within particular regions Ω_i, $i = 1,2,3,4$ of a state space. Parameters used for calculation of matrices A_i, B_i, C_i, D_i, $i = 1,2,3,4$ in state equation (8) and output equation (9) are gathered in Table 2. Values of parameter l_c and moments of inertia for each Ω_i region are in the same table. The moment of inertia was calculated also for a truncated cone, the radii of which were changing depending on the values of state vector elements x_1 and x_2.

Table 2: The parameters of switched model.

		Link 1	Link 2
$m\,[kg]$		5.7	2.65
$l\,[m]$		0.32	0.40
The case I	$l_c\,[m]$	0.14	0.17
The case II	$l_c\,[m]$	0.14	0.17
The case III	$l_c\,[m]$	0.14	0.17
The case IV	$l_c\,[m]$	0.14	0.17
The case I	$I\,[kgm^2]$	0.061	0.038
The case II	$I\,[kgm^2]$	0.061	0.038
The case III	$I\,[kgm^2]$	0.061	0.038
The case IV	$I\,[kgm^2]$	0.061	0.038

A First Example

3.2.1 Case I (Figure 2)

Matrices A_1, B_1, C_1, D_1 calculated for $(x^0;u^0) = ([0.349\,rad, -0.349\,rad, 0\,\frac{rad}{s}, 0\,\frac{rad}{s}];[0\,Nm, 0\,Nm])$ working point belonging to Ω_1 region are presented below:

$$A_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -31 & -1.8 & 0 & 0 \\ 14.52 & -19.3 & 0 & 0 \end{bmatrix}, \quad (16)$$

$$B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 3.05 & -3.02 \\ -3.02 & 13.9 \end{bmatrix}, \quad (17)$$

$$C_1 = \begin{bmatrix} -31 & -1.8 & 0 & 0 \\ 14.52 & -19.3 & 0 & 0 \end{bmatrix}, \quad (18)$$

$$D_1 = \begin{bmatrix} 3.05 & -3.02 \\ -3.02 & 13.9 \end{bmatrix}. \quad (19)$$

3.2.2 Case II (Figure 3)

From the Ω_2 region a working point $(x^0;u^0) = ([0\,rad, -0.523\,rad, 0\,\frac{rad}{s}, 0\,\frac{rad}{s}];[0\,Nm, 0\,Nm])$ was selected, for which matrices A_2, B_2, C_2, D_2 were obtained:

$$A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 43 & 4.65 & 0 & 0 \\ 44.1 & -35.5 & 0 & 0 \end{bmatrix}, \quad (20)$$

$$B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 3.04 & -3.4 \\ -3.4 & 14.7 \end{bmatrix}, \quad (21)$$

$$C_2 = \begin{bmatrix} 43 & 4.65 & 0 & 0 \\ 44.1 & -35.5 & 0 & 0 \end{bmatrix}, \quad (22)$$

$$D_2 = \begin{bmatrix} 3.04 & -3.4 \\ -3.4 & 14.7 \end{bmatrix}. \quad (23)$$

3.2.3 Case III (Figure 4)

Working point $(x^0;u^0) = ([-0.523\,rad, 0\,rad, 0\,\frac{rad}{s}, 0\,\frac{rad}{s}];[0\,Nm, 0\,Nm])$ belonging to Ω_3 region was used for linearization resulting with matrices A_3, B_3, C_3, D_3:

$$A_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -32.06 & 2.54 & 0 & 0 \\ 20.11 & -26,8 & 0 & 0 \end{bmatrix}, \quad (24)$$

$$B_3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 3.33 & -3.8 \\ -3.8 & 15.5 \end{bmatrix}, \quad (25)$$

$$C_3 = \begin{bmatrix} -32.06 & 2.54 & 0 & 0 \\ 20.11 & -26,8 & 0 & 0 \end{bmatrix}, \quad (26)$$

$$D_3 = \begin{bmatrix} 3.33 & -3.8 \\ -3.8 & 15.5 \end{bmatrix}. \quad (27)$$

3.2.4 Case IV (Figure 5)

Matrices A_4, B_4, C_4, D_4 present in the state and output equations of subsystem four of switched system are obtained by linearization about $(x^0;u^0) = ([-0.523\,rad, -0.523\,rad, 0\,\frac{rad}{s}, 0\,\frac{rad}{s}];[0\,Nm, 0\,Nm])$ working point:

$$A_4 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -45 & 12.3 & 0 & 0 \\ 58.2 & -50.1 & 0 & 0 \end{bmatrix}, \quad (28)$$

$$B_4 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 3.02 & -3.9 \\ -3.9 & 16 \end{bmatrix}, \quad (29)$$

$$C_4 = \begin{bmatrix} -45 & 12.3 & 0 & 0 \\ 58.2 & -50.1 & 0 & 0 \end{bmatrix}, \quad (30)$$

$$D_4 = \begin{bmatrix} 3.02 & -3.9 \\ -3.9 & 16 \end{bmatrix}. \quad (31)$$

3.2.5 The Results

Figures 7-10 present time history of four elements of state vector. Each figure shows comparison between consecutive elements of state vector of a simple linear system and a switched system. Initial condition used for every simulation was $x(t=0) = [0.349\,rad, -0.349\,rad, 0\,\frac{rad}{s}, 0\,\frac{rad}{s}]$ and simulation time was equal to 20 seconds.

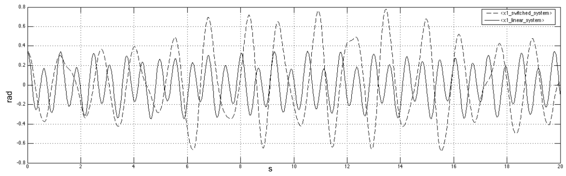

Figure 7: Result of the first example - x_1 signal.

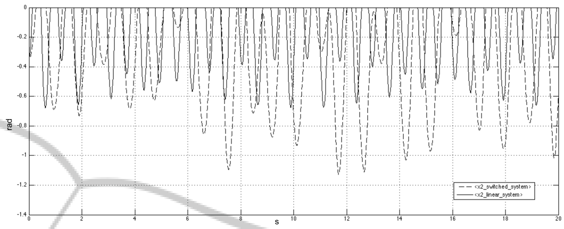

Figure 8: Result of the first example - x_2 signal.

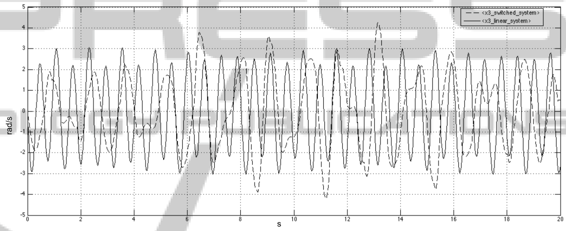

Figure 9: Result of the first example - x_3 signal.

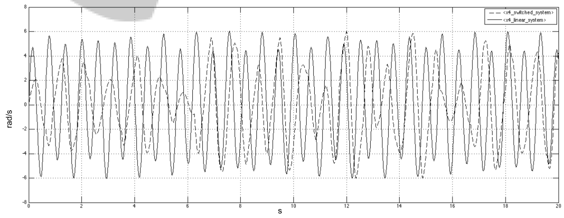

Figure 10: Result of the first example - x_4 signal.

3.3 Second Example

In the second simulation example the same simulation time of 20 seconds is set but initial conditions are changed. The nonlinear systems described by (2) was in this case linearize about arbitrary selected working point $(x^0, u^0) = ([0.349\,rad, -0.349\,rad, 0\,\frac{rad}{s}, 0\,\frac{rad}{s}];[0\,Nm, 0\,Nm])$. The resulting matrices A_L, B_L, C_L i D_L are of the form

$$A_L = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -31 & -1.8 & 0 & 0 \\ 14.5 & -19.3 & 0 & 0 \end{bmatrix}, \quad (32)$$

$$B_L = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 3.05 & -3.021 \\ -3.021 & 13.9 \end{bmatrix}, \qquad (33)$$

$$C_L = \begin{bmatrix} -31 & -1.8 & 0 & 0 \\ 14.5 & -19.3 & 0 & 0 \end{bmatrix}, \qquad (34)$$

$$D_L = \begin{bmatrix} 3.05 & -3.021 \\ -3.021 & 13.9 \end{bmatrix}. \qquad (35)$$

Matrices in the state and output equation of the switched system remained unchanged.

Another simulation was run, this time with the initial condition $x(t = 0) = [0\,rad, -0.349\,rad, 0\,\frac{rad}{s}, 0\,\frac{rad}{s}]$ for both, linear "standard" and switched system. Figures 11-14 depict, as previously, comparison of every member of the state vector. Based on presented simulation results for both examples we state that the modelling with the use of switched systems gives better outcome than standard approach with linear systems. As a result the modelled object exhibits smaller oscillations,

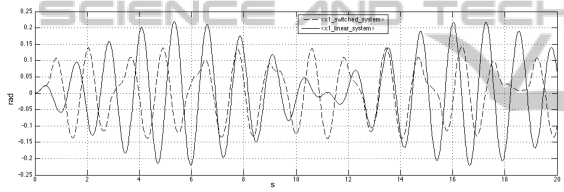

Figure 11: Result of the second example - x_1 signal.

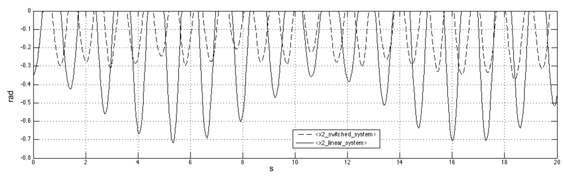

Figure 12: Result of the second example - x_2 signal.

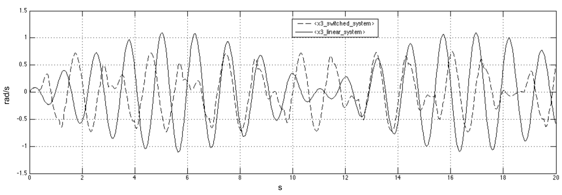

Figure 13: Result of the second example - x_3 signal.

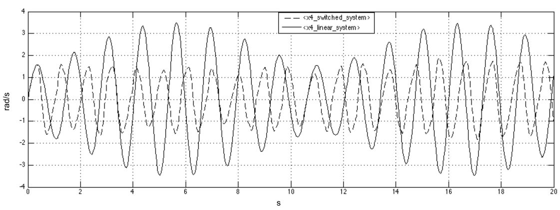

Figure 14: Result of the second example - x_4 signal.

what translates directly into the synthesis of control algorithms.

Although the presented model of a human leg is largely simplified, on the current state of work it is sufficient for the analysis of dynamical properties which will be of interest to us in further research (Czornik and Świerniak, 2004), (Czornik and Świerniak, 2005).

4 CONCLUSIONS

In future research authors aim at using linear switched system to modelling of the human leg with seven degrees of freedom. Moreover, as shown in (Babiarz et al., 2013) the subsystems of given object may be unstable or on the stability boundary and may be unobservable. For this reason we plan conduct the analysis of properties such as stability (Shorten, 2007) or controllability and observability (Czornik and Niezabitowski, 2013), (Klamka and Niezabitowski, 2013), (Klamka et al., 2013). In the next step we will incorporate systems with fractional order, as in (HosseinNia et al., 2013), (Kaczorek, 2013), (Tejado et al., 2013b), (Klamka et al., 2014) into this framework.

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