Bragg Grating Solitons in Semilinear Dual-core System with Cubic-Quintic Nonlinearity

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Abstract: The existence and stability of Bragg grating solitons in a coupler, where one core is equipped with a Bragg grating (BG) and has cubic-quintic nonlinearity and the other is linear, are studied. When the group velocity term in the linear core is zero (i.e. $c = 0$), the system's linear spectrum contains two separate bandgaps. It is found that soliton solutions exist throughout both bandgaps. On the other hand, when the group velocity term in the linear core is nonzero ($c \neq 0$), the spectrum consists of three gaps: a genuine central gap and upper and lower gaps that overlap with one branch of continuous spectrum. In this case, soliton solutions exist throughout the upper and lower gaps but not in the central gap. The system supports two disjoint families of solitons (referred to as Type 1 and Type 2) that are separated by a boundary. Stability of solitons is investigated by means of systematic numerical stability analysis. It is found that Type 2 solitons are always unstable. On the other hand, there exist vast regions in the upper and lower bandgaps where stable Type 1 solitons exist.

1 INTRODUCTION

Fiber Bragg gratings (FBGs) are periodic optical structures that have been the focus of much theoretical and experimental research in the last few decades due to their applications in optical communications, filtering, sensing, signal processing, high speed switching and pulse compression in the nonlinear regime (Sankey et al., 1992; Radic et al., 1995; Loh et al., 1996; Kashyap, 1999).

One of the main characteristics of the FBGs is their strong dispersion due to the cross coupling between the forward- and backward- propagating waves (Desterke and Sipe, 1994). At sufficiently high intensities, the FBG-induced strong dispersion may be counterbalanced by the nonlinearity giving rise to Bragg grating (BG) solitons. BG solitons have been studied extensively in Kerr media, both theoretically (Christadoulides and Joseph, 1989; Aceves and Wabnitz, 1989; Desterke and Sipe, 1994; Mak et al., 2003; Neill and Atai, 2006; Neill et al., 2007) and experimentally (Eggleton et al., 1997; Taverner et al., 1998; Mok et al., 2006). The existence and stability of BG solitons have also been investigated in different nonlinear media such as quadratic (Conti et al., 1997) and cubic-quintic nonlinearities (Atai and Malomed, 2001; Atai, 2004; Dasanayaka and Atai, 2010; Dasanayaka and Atai, 2013a; Dasanayaka and Atai, 2013b) as well as dual core fibers, where the Bragg grating is written in one or both cores (Mak et al., 1998; Atai and Malomed, 2000).

In the case of a semilinear dual-core fiber, where one core has Kerr nonlinearity and is equipped with a Bragg grating and the other is linear, it has been shown that stability of BG solitons is dependent on the relative group velocity in the linear core (Atai and Malomed, 2000). In this paper, we analyze the existence and stability of BG solitons in a generalized model of semilinear dual core system, where one core has BG with cubic-quintic nonlinearity and the other core is linear.

2 THE MODEL

Starting with the model outlined in (Atai and Malomed, 2000) and following the procedures described in (Atai and Malomed, 2001), we can derive the following normalized model for two linearly coupled cores, assuming BG is present only in the nonlinear core:

$$iu_t + iu_x + \left[|v|^2 + \frac{1}{2}|u|^2\right]u -$$
$$q\left[\frac{1}{4}|u|^4 + \frac{3}{2}|u|^2|v|^2 + \frac{3}{4}|v|^4\right]u + v + \kappa\phi = 0$$

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$$
\begin{aligned}
iv_t - iv_x + \left[|u|^2 + \frac{1}{2}|v|^2\right] v - \\
q\left[\frac{1}{4}|v|^4 + \frac{3}{2}|v|^2|u|^2 + \frac{3}{4}|u|^4\right] v + u + \kappa\psi = 0 \quad (1) \\
i\phi_t + ic\phi_x + \kappa u = 0 \\
i\psi_t - ic\psi_x + \kappa v = 0
\end{aligned}
$$

where u and v are the forward- and backward-propagating waves in the nonlinear core and $\phi(x,t)$ and $\psi(x,t)$ are their counterparts in the linear core, respectively. $q > 0$ is a parameter that controls the strength of the quintic nonlinearity, and κ is the coefficient of linear coupling between the cores. It should be noted that cubic-qunitic nonlinearity has been measured in chalcogenide glass (Boudebs et al., 2003) and some transparent organic materials (Lawrence et al., 1994; Zhan et al., 2002). Using the values of nonlinear coefficients from these references and assuming a typical value of $\Delta n = 5 \times 10^{-4}$, q is found to be in the range $0.05 < q < 0.6$. Therefore, in the following analysis, without loss of generality, we are going to assume that q varies in the range $0 \leq q \leq 1$. c represents the relative group velocity in the linear core (group velocity in the nonlinear core has been set to 1). In the nonlinear core, the normalized coefficient of the self-phase and cross-phase modulation parts in front of the cubic terms assumed to be the usual 1 : 2 for Kerr nonlinearity (Agrawal, 1995) and 1: 6: 3 for the quintic nonlinearity as derived in (Maimistov et al., 1999).

To analyze the system, it is necessary to first determine the system's linear spectrum within which the solitons may exist. Substituting $(u,v,\phi,\psi) \sim \exp(ikx - i\omega t)$ into the system of Eqs. (1) and linearizing, the following dispersion relation can be derived (Atai and Malomed, 2000):

$$
\begin{aligned}
\omega^4 - \left[1 + 2\kappa^2 + (1+c^2)k^2\right]\omega^2 + \kappa^4 + \\
(c^2 - 2c\kappa^2)k^2 + c^2k^4 = 0
\end{aligned} \quad (2)
$$

For $c = 0$, the spectrum contains two disjoint bandgaps; one in the upper half and the other in the lower half of the spectrum and are respectively given by

$$
\begin{cases}
-\frac{1}{2} + \sqrt{\frac{1}{4} + \kappa^2} \leq \omega \leq \frac{1}{2} + \sqrt{\frac{1}{4} + \kappa^2} & \omega > 0 \\
-\frac{1}{2} - \sqrt{\frac{1}{4} + \kappa^2} \leq \omega \leq \frac{1}{2} - \sqrt{\frac{1}{4} + \kappa^2} & \omega < 0
\end{cases} \quad (3)
$$

Note that, the width of the gaps remain constant irrespective of the values of the coupling coefficient κ. The upper and lower gaps merge into a single gap when $\kappa \to 0$. When $c \neq 0$, the shapes of the branches of the dispersion diagram change, and as a result a central gap (which is a genuine gap) is formed. In this case, the lower and upper gaps overlap with one branch of continuous spectrum and therefore they are not genuine bandgaps. Examples of typical dispersion diagrams are shown in Figure 1 for different values of c and κ .

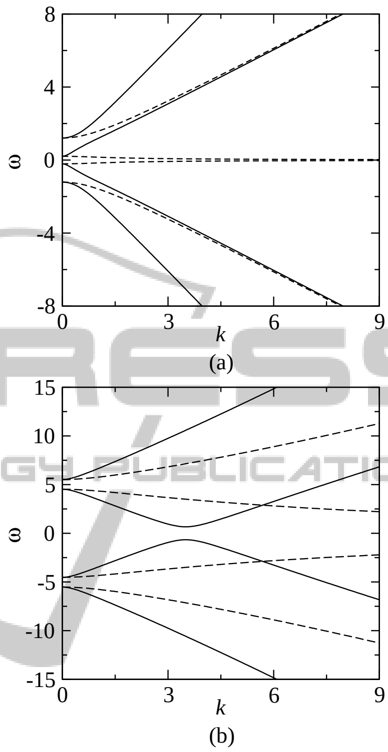

Figure 1: Examples of the dispersion diagrams for (a) $\kappa = 0.5$ and (b) $\kappa = 5.0$. Solid and dashed curves correspond to $c = 2.0$ and$c = 0$, respectively.

3 STABILITY ANALYSIS

Exact analytical solutions for Eqs. (1) can only be found for $c = 0$. In the general case $c \neq 0$, no analytical solution is available and the soliton solutions must be determined numerically. We sought for stationary solutions of Eqs. (1) as $\{u(x,t), v(x,t), \phi(x,t), \psi(x,t)\} = \{U(x), V(x), \Phi(x), \Psi(x)\}\exp(-i\omega t)$. Substitution of these expressions into Eqs. (1) results in a set of ordinary differential equations that can be solved using the relaxation algorithm. A key finding is that, similar to the model of single core Bragg grating with cubic-quintic nonlinearity (Atai and Malomed, 2001), the model admits two disjoint families of BG solitons that are separated by a border, at which solitons do not exist. The soliton families differ in their shape, phase structure and parities of the real and imaginary

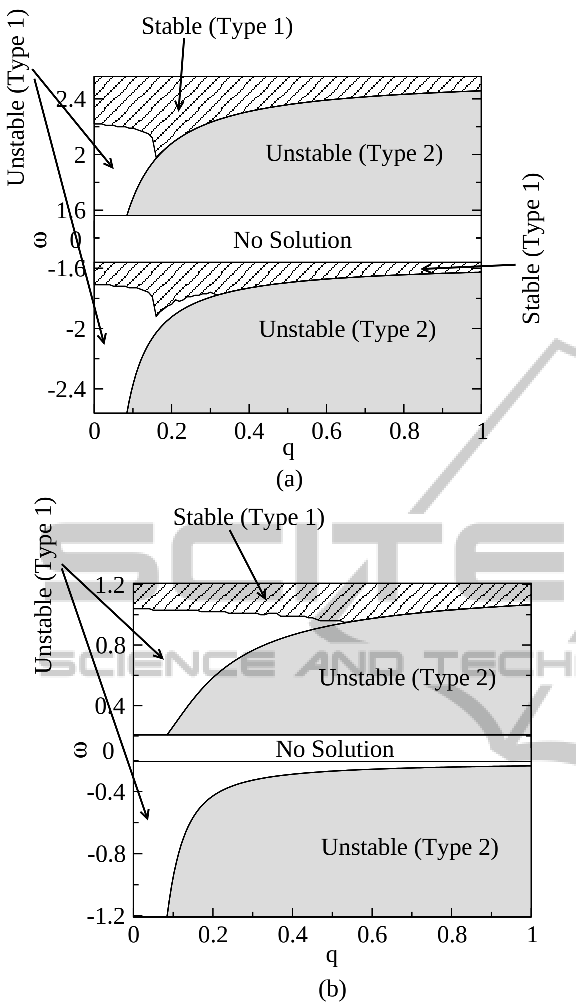

Figure 2: Stability diagrams for (a) $\kappa = 2.0$, $c = 0.0$ and (b) $\kappa = 0.5$, $c = 0.2$.

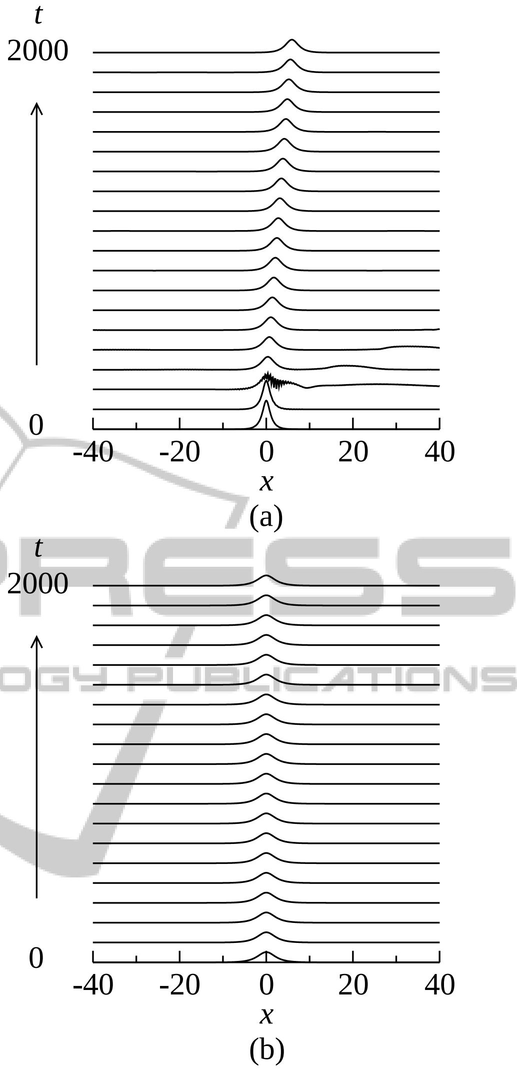

Figure 3: Examples of Type 1 soliton evolution. (a) Evolution of an unstable soliton corresponding to $c = 0.0$, $\kappa = 5.0$, $q = 0.10$, $\omega = 4.90$ resulting in the formation of a moving Type 1 soliton; (b) stable soliton with $c = 0.2$, $\kappa = 0.5$, $q = 0.20$, $\omega = 1.05$.

parts. One family (Type 1) can be regarded as the generalization of the BG solitons in the semilinear model with cubic nonlinearity (i.e. the model of (Atai and Malomed, 2000)) and in the other family (Type 2), the quintic nonlinearity is dominant. In the case of $c = 0$, soliton solutions exist throughout the upper and lower gaps. On the other hand, when $c \neq 0$, no solutions exist in the genuine central gap. However, Type 1 and Type 2 solitons are found within the upper and lower gaps (as was mentioned above, these are not genuine gaps because they overlap with one branch of the continuous spectrum). To investigate the stability of the BG solitons, we have performed a systematic stability analysis by numerically solving Eqs. (1). Through the stability analysis we have been able to identify nontrivial stability borders in the (q, ω) plane.

The outcomes of stability analysis for two sets of parameters are summarized in Figure 2. An important finding is that, unlike the case of a single core Bragg grating with cubic-quintic nonlinearity, the Type 2 solitons are always unstable. On the other hand, vast regions of Type 1 stable solitons have been found for different values of κ and c in both upper and lower gaps. An interesting feature shown in Figure 2 is that the stable regions in both the upper and lower gaps enlarge as the strength of quintic nonlinearity increases. Also, the stable region in the upper gap is generally larger than that in the lower gap.

Examples of the propagation of Type 1 and Type 2 solitons are displayed in Figures 3 and 4, respectively, for different values of κ, c, q and ω. Figure 3(a) shows that Type 1 solitons belonging to the the

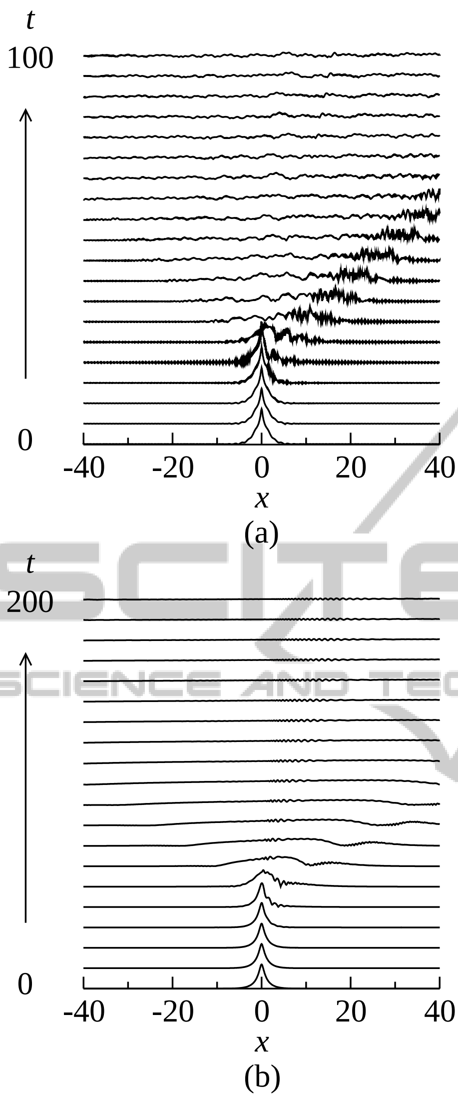

Figure 4: Examples of Type 2 unstable soliton evolution. (a) $c = 0.2$, $\kappa = 0.5$, $q = 0.50$, $\omega = 0.80$, (b) $c = 0.0$, $\kappa = 2.0$, $q = 0.85$, $\omega = -2.35$.

unstable region initially shed some energy and consequently evolve into moving solitons belonging to the same family. Solitons in the stable region are highly robust as shown in Figure 3(b). However, the Type 2 solitons are highly unstable and upon propagation they radiate significant amount of energy and subsequently are destroyed (see Figure 4).

4 CONCLUSIONS

In this paper, we have put forward a model of semilinear dual core system, where one core has a cubic-quintic nonlinearity and is equipped with a Bragg grating and the other is linear. We have investigated the existence and stability of quiescent Bragg grating solitons in this model. We have derived exact analytical soliton solutions for the limiting case of $c = 0$. In the case of $c \neq 0$, soliton solutions have been determined numerically. Similar to the case of a single core Bragg grating in cubic-quintic nonlinearity, the model supports two disjoint families of BG solitons. We have conducted a systematic numerical stability analysis for various values of c and κ and identified nontrival stability borders in the (q, ω) plane. The analysis reveals that, for the both $c = 0$ and $c \neq 0$, there exist vast regions within the (q, ω) plane where Type 1 solitons are stable. On the other hand, Type 2 solitons are always unstable.

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