Resource Allocation in SVD-assisted Broadband MIMO Systems Using Polynomial Matrix Factorization

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Abstract: Removing channel interference in broadband multiple-input multiple-output (MIMO) systems is a task which can be solved by applying a spatio-temporal vector coding (STVC) channel description and using singular value decomposition (SVD) in combination with signal pre- and post-processing. In this contribution a polynomial matrix factorization channel description in combination with a specific SVD algorithm for polynomial matrices is analyzed and compared to the commonly used STVC SVD. This comparison points out the analogies and differences of both equalization methods. Furthermore, the bit error rate (BER) performance is evaluated for two different channel types and is optimized by applying bit-allocation schemes involving a power loading strategy. Our results, obtained by computer simulation, show that polynomial matrix factorization such as polynomial matrix SVD could be an alternative signal processing approach compared to conventional SVD-based MIMO approaches in frequency-selective MIMO channels.

1 INTRODUCTION

The strategy of placing multiple antennas at the transmitter and receiver sides, well-known as multiple-input multiple-output (MIMO) system, improves the performance of wireless systems by the use of the spatial characteristics of the channel. MIMO systems have become the subject of intensive research over the past 20 years as MIMO is able to support higher data rates and shows a higher reliability than single-input single-output (SISO) systems. Singular-value decomposition (SVD) is well-established in MIMO signal processing where the whole MIMO channel is transferred into a number of weighted SISO channels. The unequal weighting of the SISO channels has led to intensive research to reduce the complexity of the required bit- and power-allocation techniques (Ahrens and Lange, 2008; Ahrens and Benavente-Peces, 2009; Kühn, 2006). The polynomial matrix singular-value decomposition (PMSVD) is a signal processing technique which decomposes the MIMO channel into a number of independent frequency-selective SISO channels so called layers (McWhirter et al., 2007). The remaining layer-specific interferences as a result of the PMSVD-based signal processing can be easily removed by further signal processing such as zero-forcing equalization as demonstrated in this work.

The novelty of our contribution is that we demonstrate the benefits of amalgamating a suitable choice of MIMO layers activation and number of bits per layer along with the appropriate allocation of the transmit power under the constraint of a given fixed data throughput. Here, bit- and power-loading in both SVD- and PMSVD-based MIMO transmission systems are elaborated. Assuming a fixed data rate, which is required in many applications (e.g., real time video applications), a two stage optimization process is proposed. Firstly, the allocation of bits to the number of SISO channels is optimized and secondly, the allocation of the available total transmit power is studied when minimizing the overall bit-error rate (BER) at a fixed data rate. Our results, obtained by computer simulation, show that PMSVD could be an alternative signal processing approach compared to conventional SVD-based MIMO approaches in frequency-selective MIMO channels.

The remaining part of this paper is structured as follows: Section 2 introduces the state of the art SVD-based MIMO system model. The polynomial matrix singular-value decomposition is analysed in section 3. In section 4 the well-know quality criteria is briefly reviewed and applied to our problem. The proposed power allocation solutions are discussed in section 5, while the associated performance results are

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presented and interpreted in section 6. Finally, section 7 provides some concluding remarks.

2 STATE OF THE ART

A frequency selective MIMO link, composed of n_{T} transmit and n_{R} receive antennas is given by

$$\mathbf{u} = \mathbf{H} \cdot \mathbf{c} + \mathbf{n} \ . \tag{1}$$

In (1), $\mathbf{c}$ is the $(N_{\mathrm{T}} \times 1)$ transmitted data signal vector containing the complex input symbols transmitted over n_{T} transmit antennas in K consecutive time slots, i. e., $N_{\mathrm{T}} = K\, n_{\mathrm{T}}$. The vector $\mathbf{u}$ describes the $(N_{\mathrm{R}} \times 1)$ received signal vector, of the length $N_{\mathrm{R}} = (K + L_{\mathrm{c}})\, n_{\mathrm{R}}$, which is extended if compared to the transmitted signal vector based on the $(L_{\mathrm{c}} + 1)$ non-zero elements of the resulting symbol rate sampled overall channel impulse response between the μth transmit and νth receive antenna. Finally, the $(N_{\mathrm{R}} \times 1)$ vector $\mathbf{n}$ in (1) describes the noise term (Ahrens and Benavente-Peces, 2009).

The $(N_{\mathrm{R}} \times N_{\mathrm{T}})$ system matrix $\mathbf{H}$ of the block-oriented system model, introduced in (1), results in

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{1\,1} & \cdots & \mathbf{H}_{1\,n_{\mathrm{T}}} \\ \vdots & \ddots & \vdots \\ \mathbf{H}_{n_{\mathrm{R}}\,1} & \cdots & \mathbf{H}_{n_{\mathrm{R}}\,n_{\mathrm{T}}} \end{bmatrix} \ , \tag{2}$$

and consists of $n_{\mathrm{R}} \cdot n_{\mathrm{T}}$ SISO channel matrices $\mathbf{H}_{\nu\mu}$ (with $\nu = 1, \ldots, n_{\mathrm{R}}$ and $\mu = 1, \ldots, n_{\mathrm{T}}$). The system description, called spatio-temporal vector coding (STVC), was introduced by RALEIGH (Raleigh and Cioffi, 1998; Raleigh and Jones, 1999). Each of these matrices $\mathbf{H}_{\nu\mu}$ with the dimension $((K + L_{\mathrm{c}}) \times K)$ describes the influence of the channel from transmit antenna μ to receive antenna ν including transmit and receive filtering. The channel convolution matrix $\mathbf{H}_{\nu\mu}$ between the μth transmit and νth receive antenna is obtained by taking the $(L_{\mathrm{c}} + 1)$ non-zero elements of resulting symbol rate sampled overall impulse response into account and results in

$$\mathbf{H}_{\nu\mu} = \begin{bmatrix} h_0 & 0 & 0 & \cdots & 0 \\ h_1 & h_0 & 0 & \cdots & \vdots \\ h_2 & h_1 & h_0 & \cdots & 0 \\ \vdots & h_2 & h_1 & \cdots & h_0 \\ h_{L_{\mathrm{c}}} & \vdots & h_2 & \cdots & h_1 \\ 0 & h_{L_{\mathrm{c}}} & \vdots & \cdots & h_2 \\ 0 & 0 & h_{L_{\mathrm{c}}} & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & h_{L_{\mathrm{c}}} \end{bmatrix} \ . \tag{3}$$

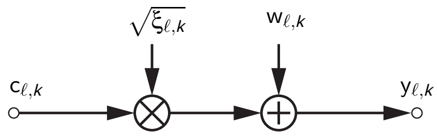

Figure 1: Resulting layer-specific SVD-based broadband MIMO system model (with $\ell = 1, 2, \ldots, L$ and $k = 1, 2, \ldots, K$).

The removal of the interferences between the different antenna's data streams, which are introduced by the non-zero off-diagonal elements of the channel matrix $\mathbf{H}$, requires appropriate signal processing strategies. Singular-value decomposition (SVD) can be considered as a promising solution for transferring the whole MIMO system into a system with non-interfering channels, so called layers.

Using SVD the system matrix $\mathbf{H}$ can be written as $\mathbf{H} = \mathbf{S} \cdot \mathbf{V} \cdot \mathbf{D}^{\mathrm{H}}$, where $\mathbf{S}$ and $\mathbf{D}^{\mathrm{H}}$ are unitary matrices and $\mathbf{V}$ is a real-valued diagonal matrix of the positive square roots of the eigenvalues of the matrix $\mathbf{H}^{\mathrm{H}}\mathbf{H}$ sorted in descending order[1]. For removing the interferences, the MIMO data vector $\mathbf{c}$ is now multiplied by the matrix $\mathbf{D}$ before transmission. In turn, the receiver multiplies the received vector $\mathbf{u}$ by the matrix $\mathbf{S}^{\mathrm{H}}$. Thereby neither the transmit power nor the noise power is enhanced given $\mathbf{S}$ and $\mathbf{D}$ are unitary. The overall transmission relationship is defined as

$$\mathbf{y} = \mathbf{S}^{\mathrm{H}} (\mathbf{H} \cdot \mathbf{D} \cdot \mathbf{c} + \mathbf{n}) = \mathbf{V} \cdot \mathbf{c} + \mathbf{w} \, . \tag{4}$$

As a consequence of the processing in (4), the channel matrix $\mathbf{H}$ is transformed into independent, non-interfering layers having unequal gains.

With the proposed system structure, the SVD-based equalization leads to different number of MIMO layers ℓ (with $\ell = 1, 2, \ldots, L$) at the time k (with $k = 1, 2, \ldots, K$). Here it is worth noting that the number of parallel transmission layers L at the timeslot k is limited by $\min(n_{\mathrm{T}}, n_{\mathrm{R}})$. The complex-value data symbol $c_{\ell,k}$ to be transmitted over the layer ℓ at the time k is now weighted by the corresponding positive real-valued singular-value $\sqrt{\xi_{\ell,k}}$ and further disturbed by the additive noise term $w_{\ell,k}$.

3 POLYNOMIAL MATRIX FACTORIZATION

In contrast to the STVC, the polynomial matrix factorization exploits a description of the channel impulse responses in the z-domain. Thus, each frequency selective channel impulse response $h_{\nu\mu}(k)$ be-

[1]The transpose and conjugate transpose (Hermitian) of $\mathbf{D}$ are denoted by $\mathbf{D}^{\mathrm{T}}$ and $\mathbf{D}^{\mathrm{H}}$, respectively.

tween the μth transmit and the νth receive antenna of a $(n_{\mathrm{R}} \times n_{\mathrm{T}})$ MIMO system is given by

$$\underline{h}_{\nu\mu}(z) = \sum_{k=0}^{L_c} h_{\nu\mu}[k]\, z^{-k} \quad , \tag{5}$$

where the underscore denotes a polynomial. Consecutively, the broadband MIMO channel is formed by grouping these impulse responses into the polynomial channel matrix and thus it can be described as multiple non-polynomial matrices $\mathbf{H}_k$ multiplied with their respective delay z^{-k} as follows

$$\begin{aligned} \underline{\mathbf{H}}(z) &= \sum_{k=0}^{L_c} \mathbf{H}_k\, z^{-k} \\ \underline{\mathbf{H}}(z) &= \begin{bmatrix} \underline{h}_{1\,1}(z) & \underline{h}_{1\,2}(z) & \cdots & \underline{h}_{1\,n_{\mathrm{T}}}(z) \\ \underline{h}_{2\,1}(z) & \underline{h}_{2\,2}(z) & \cdots & \underline{h}_{2\,n_{\mathrm{T}}}(z) \\ \vdots & \vdots & \ddots & \vdots \\ \underline{h}_{n_{\mathrm{R}}\,1}(z) & \underline{h}_{n_{\mathrm{R}}\,2}(z) & \cdots & \underline{h}_{n_{\mathrm{R}}\,n_{\mathrm{T}}}(z) \end{bmatrix} \end{aligned} \tag{6}$$

where $\underline{\mathbf{H}}(z) \in \mathbb{C}^{n_{\mathrm{R}} \times n_{\mathrm{T}}}$. Using this polynomial description in the z-domain a MIMO system is described in analogy to (1) by

$$\underline{\mathbf{u}}(z) = \underline{\mathbf{H}}(z)\, \underline{\mathbf{c}}(z) + \underline{\mathbf{n}}(z) \quad , \tag{7}$$

where $\underline{\mathbf{c}}(z)$ is the $(n_{\mathrm{T}} \times 1)$ transmit signal vector, $\underline{\mathbf{u}}(z)$ is the $(n_{\mathrm{R}} \times 1)$ receive signal vector and $\underline{\mathbf{n}}(z)$ describes the additive white Gaussian noise (AWGN) component in polynomial notation.

The polynomial channel matrix $\underline{\mathbf{H}}(z)$ can be orthogonalized by calculating the polynomial matrix singular value decomposition (PMSVD) with the help of the second-order sequential best rotation (SBR2) algorithm as presented in (McWhirter et al., 2007; Foster et al., 2010). The decomposition of the polynomial channel matrix results in $\underline{\mathbf{H}}(z) = \underline{\mathbf{S}}(z)\, \underline{\mathbf{V}}(z)\, \underline{\widetilde{\mathbf{D}}}(z)$, where $\widetilde{(\cdot)}$ denotes the para-conjugate operator. The matrices $\underline{\mathbf{S}}(z) \in \mathbb{C}^{n_{\mathrm{R}} \times n_{\mathrm{R}}}$ and $\underline{\widetilde{\mathbf{D}}}(z) \in \mathbb{C}^{n_{\mathrm{T}} \times n_{\mathrm{T}}}$ are paraunitary matrices and $\underline{\mathbf{V}}(z) \in \mathbb{C}^{n_{\mathrm{R}} \times n_{\mathrm{T}}}$ is assumed to be a diagonal matrix, because the off-diagonal elements are negligibly small when the SBR2 algorithm is set up accordingly. The diagonal matrix has the following form $(n_{\mathrm{T}} = n_{\mathrm{R}})$

$$\underline{\mathbf{V}}(z) = \begin{bmatrix} \underline{v}_1(z) & 0 & \cdots & 0 \\ 0 & \underline{v}_2(z) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \underline{v}_L(z) \end{bmatrix} \quad , \tag{8}$$

where the diagonal polynomial elements are described by $\underline{v}_\ell(z) = \sum_{k=0}^{L_{\mathrm{v}}} v_{\ell,k}\, z^{-k}$. In contrast to the singular values $\sqrt{\xi_{\ell,k}}$ using SVD, the polynomial coefficients of $\underline{v}_\ell(z)$ are complex. In analogy to the SVD model, the maximal number of activated layers L using PMSVD is $\min\{n_{\mathrm{R}}, n_{\mathrm{T}}\}$. For removing the interference signal pre-processing at the transmitter and post-processing at the receiver is applied in analogy to the classical SVD. Consequently, the transmit data vector $\underline{\mathbf{c}}(z)$ is multiplied by $\underline{\mathbf{D}}(z)$ so that

$$\begin{aligned} \underline{\mathbf{u}}(z) &= \underline{\mathbf{H}}(z)\, \underline{\mathbf{D}}(z)\, \underline{\mathbf{c}}(z) + \underline{\mathbf{n}}(z) \\ \underline{\mathbf{u}}(z) &= \underline{\mathbf{S}}(z)\, \underline{\mathbf{V}}(z)\, \underline{\widetilde{\mathbf{D}}}(z)\, \underline{\mathbf{D}}(z)\, \underline{\mathbf{c}}(z) + \underline{\mathbf{n}}(z) \quad , \end{aligned} \tag{9}$$

with $\underline{\widetilde{\mathbf{D}}}(z)\, \underline{\mathbf{D}}(z) = \mathbf{I}$ and $\mathbf{I}$ describing the identity matrix. The receive vector $\underline{\mathbf{u}}(z)$ is multiplied by $\underline{\widetilde{\mathbf{S}}}(z)$ resulting in

$$\begin{aligned} \underline{\mathbf{y}}(z) &= \underline{\widetilde{\mathbf{S}}}(z)\, \underline{\mathbf{u}}(z) = \underline{\widetilde{\mathbf{S}}}(z) \left(\underline{\mathbf{S}}(z)\, \underline{\mathbf{V}}(z)\, \underline{\mathbf{c}}(z) + \underline{\mathbf{n}}(z) \right) \\ &= \underline{\widetilde{\mathbf{S}}}(z)\, \underline{\mathbf{S}}(z)\, \underline{\mathbf{V}}(z)\, \underline{\mathbf{c}}(z) + \underline{\widetilde{\mathbf{S}}}(z)\, \underline{\mathbf{n}}(z) \quad . \end{aligned} \tag{10}$$

where $\underline{\widetilde{\mathbf{S}}}(z)\, \underline{\mathbf{S}}(z)$ simplifies to the identity matrix $\mathbf{I}$. Therefore, the orthogonalized system is given by

$$\underline{\mathbf{y}}(z) = \underline{\mathbf{V}}(z)\, \underline{\mathbf{c}}(z) + \underline{\mathbf{w}}(z) \quad . \tag{11}$$

Hereinafter, the resulting system is described by multiple parallel SISO channels, so called layers. The layer based discrete-time description is expressed as

$$y_\ell(k) = v_\ell(k) * c_\ell(k) + w_\ell(k) \quad , \tag{12}$$

where $*$ denotes discrete convolution such that $v_{\ell,k} * c_{\ell,k} = \sum_{\kappa=0}^{L_{\mathrm{v}}} v_{\ell,\kappa} \cdot c_{\ell,k-\kappa}$. Herein the parameter $L_{\mathrm{v}} + 1$ describes the number of non-zero coefficients of the layer-specific impulse response. The layer-specific model is depicted in Fig. 2. Here in each layer the input symbols $c_\ell(k)$ are influenced by a finite impulse response filter $v_\ell(k) = (v_{\ell,0}, v_{\ell,1}, \ldots, v_{\ell,L_{\mathrm{v}}})$ and hence inter symbol interference (ISI) occurs on each layer. In order to remove the ISI a corresponding T-spaced equalizer $f_\ell(k)$ is applied to the received signal $y_\ell(k)$ on each layer so that $z_\ell(k) = y_\ell(k) * f_\ell(k)$ as depicted in Fig. 3. The equalizer is designed as an FIR filter with coefficients as described in (Bingham, 2000) or (Tse and Viswanath, 2005) and therefore comes as close as possible to the following condition

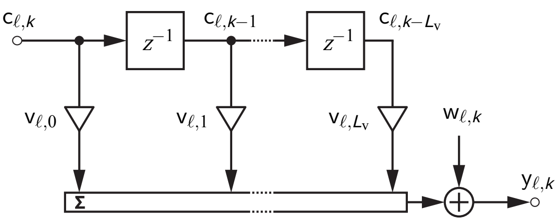

Figure 2: Resulting layer-specific PMSVD-based broadband MIMO system model (with $\ell = 1, 2, \ldots, L$ and $k = 1, 2, \ldots, K$) assuming $L_{\mathrm{v}} + 1$ non-zero coefficients of the layer-specific impulse response.

$$v_\ell(k) * f_\ell(k) = i_\ell(k) \quad , \tag{13}$$

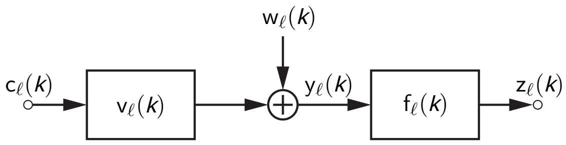

Figure 3: Layer-specific PMSVD-based transmission model applying a T-spaced equalizer with the coefficients $f_\ell(k)$ specifically designed for each layer.

with $i_\ell(k) = (0,\ldots,0,1,0,\ldots,0)$, where the position of the 1 in $i_\ell(k)$ is a degree of freedom in the equalizer design process. Accordingly, the equalized receive signal results in

$$z_\ell(k) = c_\ell(k) + w_\ell(k) * f_\ell(k) \quad . \tag{14}$$

The corresponding layer-specific ISI free system model is shown in Fig. 4, where the transmitted symbols are received unchanged and the noise $w_\ell(k)$ is weighted by the equalizer coefficients $f_\ell(k)$. The PMSVD-based broadband MIMO system model with layer-specific T-spaced equalization is henceforth referred to as T-PMSVD system model (Sandmann et al., 2014).

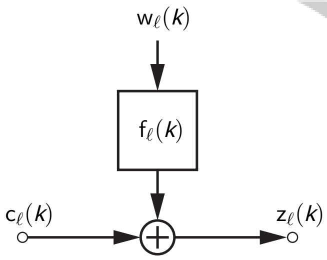

Figure 4: ISI free layer-specific T-PMSVD-based broadband MIMO system model.

4 TRANSMISSION QUALITY CRITERION

In general the quality criterion for transmission systems can be expressed with using the signal to noise ratio (SNR) at the detector input as follows

$$\rho = \frac{(\text{half vertical eye opening})^2}{\text{noise power}} = \frac{(U_\mathrm{A})^2}{P_\mathrm{R}} \quad , \tag{15}$$

where U_A and P_R correspond to one quadrature component. Considering a layer based MIMO system with a given SNR $\rho^{(\ell,k)}$ for each layer ℓ and time k and a M-ary quadrature amplitude modulation (QAM) the bit error rate (BER) probability is given by (Proakis, 2000)

$$P_\mathrm{BER}^{(\ell,k)} = \frac{2}{\log_2 M_\ell}\left(1 - \frac{1}{\sqrt{M_\ell}}\right)\mathrm{erfc}\left(\sqrt{\frac{\rho^{(\ell,k)}}{2}}\right) \tag{16}$$

This BER is averaged at each time slot over all activated layers taking different constellation sizes at each layer into account and results in

$$P_\mathrm{BER}^{(k)} = \frac{1}{\sum_{\ell=1}^{L}\log_2 M_\ell}\sum_{\ell=1}^{L}\log_2(M_\ell)\,P_\mathrm{BER}^{(\ell,k)} \quad . \tag{17}$$

In order to obtain the average BER of one data block consisting of K transmitted symbols the time slot dependent BER has to be averaged as follows

$$P_\mathrm{BER} = \mathrm{E}\left\{P_\mathrm{BER}^{(k)}\right\} \qquad \forall k \quad , \tag{18}$$

where $\mathrm{E}\{\cdot\}$ denotes the expectation functional. Finally, when considering time-variant channel conditions, rather than an AWGN channel, the BER can be derived by considering the different transmission block SNRs.

For QAM modulated signals the average transmit power per layer can be expressed as

$$P_{\mathrm{s},\ell} = \frac{2}{3}\,U_{\mathrm{s},\ell}^2\,(M_\ell - 1) \quad , \tag{19}$$

assuming that all M symbols are equally distributed. Intuitively the total available transmit power P_s is equally split between the L activated layers and hence the layer-specific transmit power is given by: $P_{\mathrm{s},\ell} = P_\mathrm{s}/L$. This guarantees that the condition

$$P_\mathrm{s} = \sum_{\ell=1}^{L} P_{\mathrm{s},\ell} \tag{20}$$

is complied. With rearranging (19) the half-level transmit amplitude for each layer results in

$$U_{\mathrm{s},\ell} = \sqrt{\frac{3\,P_\mathrm{s}}{2\,L\,(M_\ell - 1)}} \quad . \tag{21}$$

Considering the SVD layer model the noise power is unchanged at the receiver. However, the half vertical eye opening U_A at each time slot k and layer ℓ is influenced by the singular values so that $U_\mathrm{A}^{(\ell,k)} = \sqrt{\xi_{\ell,k}}\,U_{\mathrm{s},\ell}$. Using the T-PMSVD model the equalizer fully removes the ISI and thus for each layer the half vertical eye opening $U_{\mathrm{A},\ell}$ of the receive signal equals the half-level amplitude of the transmitted symbol $U_{\mathrm{s},\ell}$. The drawback of the T-PMSVD is that the noise and hence the noise power is weighted differently on each layer by the equalizer coefficients expressed by the factor θ_ℓ so that the noise power on each layer results in

$$P_{\mathrm{R},\ell} = \theta_\ell\,P_\mathrm{R} \quad , \quad \text{where } \theta_\ell = \sum_{\forall k}|f_{\ell,k}|^2 \quad . \tag{22}$$

Taking the influence of the singular values $\sqrt{\xi_{\ell,k}}$ at each time slot k in the SVD based layer model into account and considering the weighing factor of the noise power θ_ℓ induced by the T-spaced equalizer coefficients in the PMSVD based layer model the corresponding SNR values become

$$\rho_{\mathrm{SVD}}^{(\ell,k)} = \frac{\xi_{\ell,k}\, U_{\mathrm{s},\ell}^2}{P_{\mathrm{R}}} = \frac{3\,\xi_{\ell,k}}{L\,(M_\ell - 1)}\,\frac{E_{\mathrm{s}}}{N_0} \qquad (23)$$

and

$$\rho_{\mathrm{T-PMSVD}}^{(\ell)} = \frac{U_{\mathrm{s},\ell}^2}{\theta_\ell\, P_{\mathrm{R}}} = \frac{3}{\theta_\ell\, L\,(M_\ell - 1)}\,\frac{E_{\mathrm{s}}}{N_0} \quad , \qquad (24)$$

where E_{s} is the signal energy of the transmit signal.

5 POWER ALLOCATION

The overall bit error rate of a decomposed MIMO system is largely determined by the layer with the highest BER. In order to balance the bit error rates on all layers the mean of choice is to equalize the SNR values $\rho^{(\ell,k)}$ over all layers. This is clearly not the optimal solution for minimizing the overall BER but it is is easy to implement and not far away from the optimum as shown in (Ahrens and Benavente-Peces, 2009; Ahrens and Lange, 2008).

Therefore, the half-level transmit amplitude $U_{s,\ell}$ is adjusted on each layer by multiplying it with $\sqrt{p_{\ell,k}}$ so as to apply the power allocation (PA) scheme. Consequently the half vertical eye opening of the received symbols for the SVD-based model becomes

$$U_{\mathrm{A,PA}}^{(\ell,k)} = \sqrt{p_{\ell,k}}\,\sqrt{\xi_{\ell,k}}\, U_{s,\ell} \quad , \qquad (25)$$

whereas in the T-PMSVD model the factor $\sqrt{\xi_{\ell,k}}$ is dropped due to the ZF-equaliser. With this adjustment the SNR values result in

$$\rho_{\mathrm{PA}}^{(\ell,k)} = p_{\ell,k}\, \rho^{(\ell,k)} \quad . \qquad (26)$$

The respective system models for T-PMSVD and SVD equalization including PA are depicted in Fig. 5 and 6.

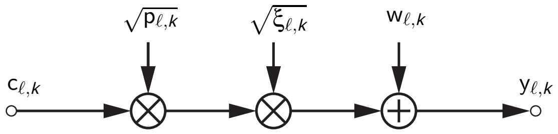

Figure 5: Resulting layer-specific SVD-based model with power allocation by adjusting the half-level amplitude of the transmit symbols $c_{\ell,k}$ with the square root of the PA factor $p_{\ell,k}$.

Hereinafter, the strategy for choosing the PA factors $p_{\ell,k}$ is elucidated. As mentioned above, the aim

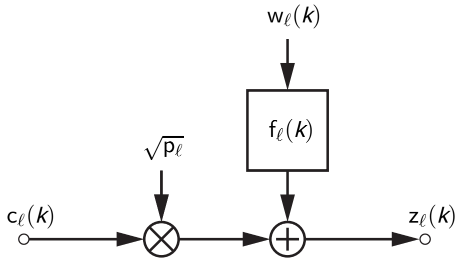

Figure 6: Resulting layer-specific T-PMSVD-based model with power allocation by adjusting the half-level amplitude of the transmit symbols $c_{\ell,k}$ with the square root of the PA factor $p_{\ell,k}$.

is to achieve equal SNRs over all activated layers ℓ at the time k and hence

$$\rho_{\mathrm{PA}}^{(\ell,k)} = \text{constant} \quad \forall \ell \qquad (27)$$

has to be fulfilled for all activated MIMO layers. Additionally, the overall transmit power after PA needs to be the same as without PA and thus the second condition

$$P_{\mathrm{s}} = \sum_{\ell=1}^{L} p_{\ell,k}\, P_{\mathrm{s},\ell} = \frac{P_{\mathrm{s}}}{L} \sum_{\ell=1}^{L} p_{\ell,k} \qquad \forall k \quad , \qquad (28)$$

has to be guaranteed. By combining these two requirements the PA factor $p_{\ell,k}$ for SVD and T-PMSVD based systems can be calculated as follows (Ahrens and Lange, 2008; Ahrens and Benavente-Peces, 2009)

$$p_{\ell,k}^{(\mathrm{SVD})} = \frac{(M_\ell - 1)}{\xi_{\ell,k}}\, \frac{L}{\sum_{\lambda=1}^{L} \frac{(M_\lambda - 1)}{\xi_{\lambda,k}}} \qquad (29)$$

and

$$p_{\ell}^{(\mathrm{T-PMSVD})} = \theta_\ell\,(M_\ell - 1)\, \frac{L}{\sum_{\lambda=1}^{L} \theta_\lambda\,(M_\lambda - 1)} \quad . \qquad (30)$$

6 RESULTS

Hereinafter, the BER quality is studied by using fixed transmission modes with a spectral efficiency of 8 bit/s/Hz. The analyzed QAM constellations, equivalent to how many bits are allocated to each layer, are shown in Tab. 1.

In order to analyse the T-PMSVD, a two-path time-invariant (2×2) MIMO system is investigated. The polynomial channel matrix is chosen as

$$\underline{\mathbf{H}}(z) = \mathbf{H}_0 + \mathbf{H}_1\, z^{-1} \qquad (31)$$

with

$$\mathbf{H}_0 = \frac{4}{5} \begin{bmatrix} 1 & 0.6 \\ 0.5 & 0.8 \end{bmatrix} \quad \text{and} \quad H_1 = \frac{\mathbf{H}_0}{2} \quad . \qquad (32)$$

Table 1: Transmission modes.

throughput	layer 1	layer 2	layer 3	layer 4
8 bit/s/Hz	256	0	0	0
8 bit/s/Hz	64	4	0	0
8 bit/s/Hz	16	16	0	0
8 bit/s/Hz	16	4	4	0
8 bit/s/Hz	4	4	4	4

The factor $4/5$ is chosen so to guarantee that the channel is not amplifying in any power allocation situation. In addition, the number of equalizer coefficients within the T-PMSVD model is chosen to be 10 and thus the factors by which the noise power is weighted on each layer come out as $\theta_1 = 0.9768$ and $\theta_2 = 17.7729$. Therefore, assuming equal QAM constellations on all layers the modified noise power affects the SNR of the second layer approximately 18 times more than the SNR of the first one. The calculated BER results as a function of the signal energy to noise power spectral density E_s/N_0 for both equalization types are depicted separately in Fig. 7 and 8.

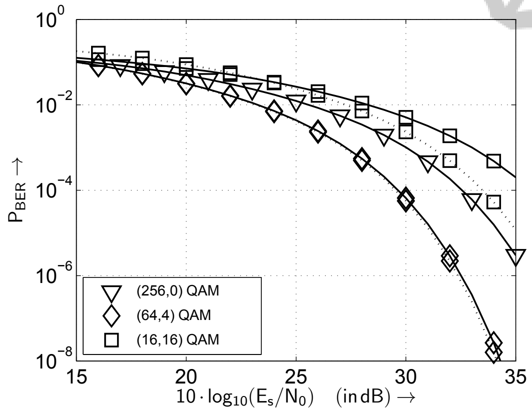

Figure 7: BER with PA (dotted line) and without PA (solid line) applying SVD-based equalization when transmitting over the time-invariant (2×2) MIMO channel given by (31) and (32) with 8 bit/s/Hz using the transmission modes introduced in Table 1.

Here the (64,4) QAM transmission mode shows the best results. Furthermore, comparing the SVD and T-PMSVD results indicate that the quality ranking of the transmission modes is similar for both equalization types. Also, the benefits of using the equal SNR power allocation method are visible. A direct comparison between the SVD and T-PMSVD results is depicted in Fig. 9 and shows that the T-PMSVD quality outperforms the SVD results.

The previous channel is now extended to a two-path time-invariant (4×4) MIMO system with the

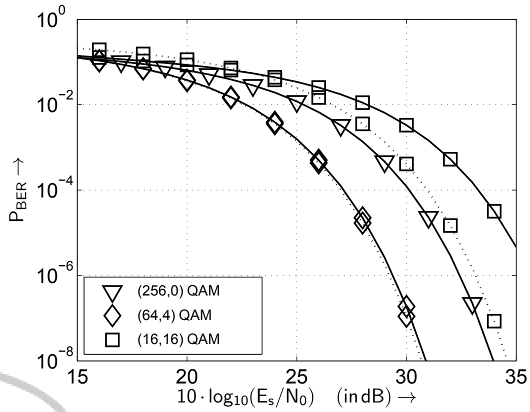

Figure 8: BER with PA (dotted line) and without PA (solid line) applying T-PMSVD equalization when transmitting over the time-invariant (2×2) MIMO channel given by (31) and (32) with 8 bit/s/Hz using the transmission modes introduced in Table 1.

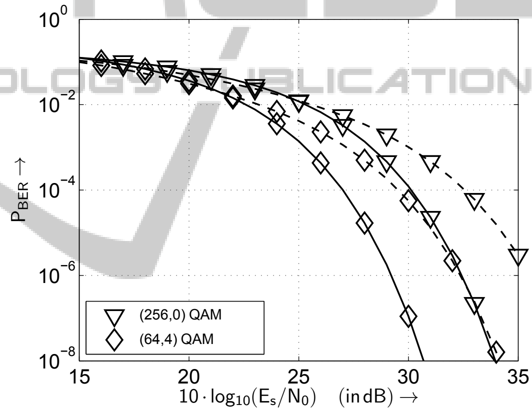

Figure 9: BER comparison between the SVD-based (dashed line) and T-PMSVD-based equalization results (solid line) when transmitting over the time-invariant (2×2) MIMO channel given by (31) with 8 bit/s/Hz using the transmission modes introduced in Table 1 and applying equal SNR PA.

polynomial channel matrix

$$\underline{\mathbf{H}}(z) = \mathbf{H}_0 + \mathbf{H}_1 z^{-1} \tag{33}$$

with

$$\mathbf{H}_0 = \sqrt{\frac{8}{15}} \begin{bmatrix} 1 & 0.6 & 0.5 & 0.3 \\ 0.5 & 0.8 & 0.6 & 0.4 \\ 0.4 & 0.5 & 0.7 & 0.5 \\ 0.3 & 0.4 & 0.5 & 0.6 \end{bmatrix} \tag{34}$$

and

$$\mathbf{H}_1 = \frac{\mathbf{H}_0}{2} \ . \tag{35}$$

The corresponding BER results are shown in Fig. 10 and 11 for SVD as well as T-PMSVD processing. The results show, that the (64,4,0,0) configura-

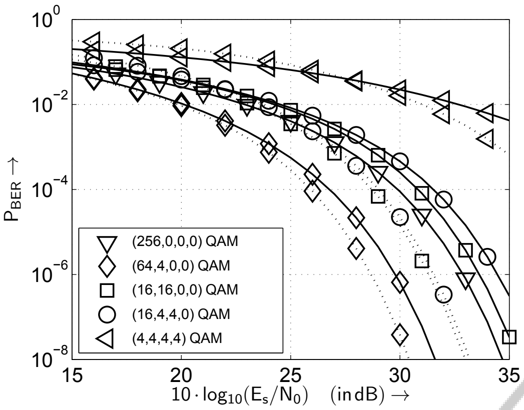

Figure 10: BER with PA (dotted line) and without PA (solid line) applying SVD-based equalization when transmitting over the time-invariant (4×4) MIMO channel given by (33) and (35) with 8 bit/s/Hz using the transmission modes introduced in Table 1.

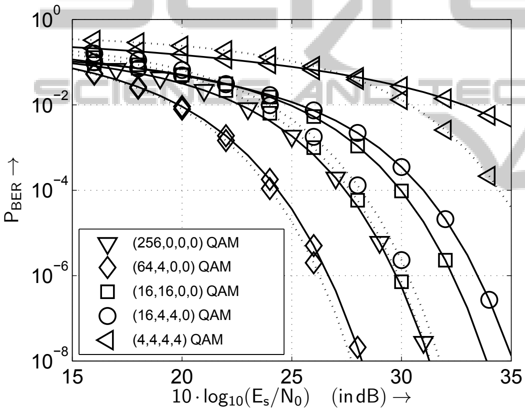

Figure 11: BER with PA (dotted line) and without PA (solid line) applying T-PMSVD equalization when transmitting over the time-invariant (4×4) MIMO channel given by (33) and (35) with 8 bit/s/Hz using the transmission modes introduced in Table 1.

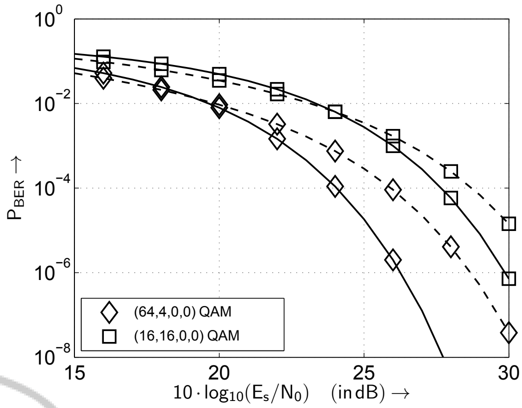

Figure 12: BER comparison between the SVD-based (dashed line) and T-PMSVD-based equalization results (solid line) when transmitting over the time-invariant (4×4) MIMO channel given by (33) with 8 bit/s/Hz using the transmission modes introduced in Table 1 and applying equal SNR PA.

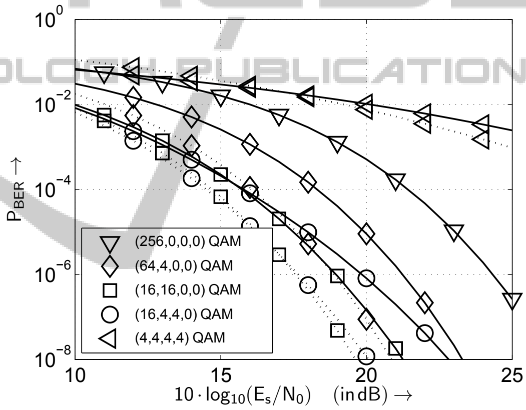

Figure 13: BER with PA (dotted line) and without PA (solid line) applying SVD-based equalization when transmitting over a Rayleigh distributed (4×4) MIMO two path channel with 8 bit/s/Hz using the transmission modes introduced in Table 1.

tion shows the best performance. Comparing these results with the (4,4,4,4) transmission mode, it turns out that activating all MIMO layers results in a high BER, based on activating layers with low quality. Directly comparing both equalization types as shown in Fig. 12 highlights the superior BER performance of the PMSVD-based equalization.

So far only time-invariant channels were studied. These investigations are now extended to time-variant wireless channels. Here, a two path (4×4) MIMO channel without a line-of-sight component is analyzed ($L_c = 1$), where the amplitudes are modeled as Rayleigh distributed. The BER results are shown in Fig. 13 and 14. Here the (16,16,0,0) QAM transmission mode performs best for SVD as well as for T-PMSVD equalization. Thus, not all layers have to be activated for achieving the best BER performance results. The transmission mode performance for both equalization types is also similar. Applying the easy to implement equal SNR PA results in a significant improvement of the BER. The direct BER performance comparison depicted in Fig. 15 shows that the T-PMSVD BER quality is superior to the SVD BER quality.

7 CONCLUSION

In this paper broadband MIMO systems have been described using polynomial matrix factorization. In order to remove the MIMO channel interference

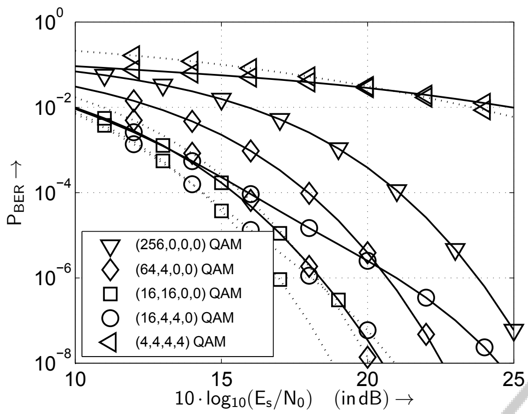

Figure 14: BER with PA (dotted line) and without PA (solid line) applying T-PMSVD equalization when transmitting over a Rayleigh distributed (4×4) MIMO two path channel with 8 bit/s/Hz using the transmission modes introduced in Table 1.

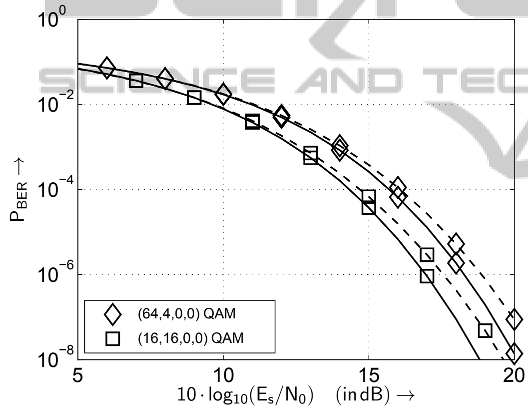

Figure 15: BER comparison between the SVD-based (dashed line) and T-PMSVD-based equalization results (solid line) when transmitting over a Rayleigh distributed (4×4) MIMO two path channel with 8 bit/s/Hz using the transmission modes introduced in Table 1 and applying equal SNR PA.

a particular singular value decomposition algorithm for polynomial matrices (PMSVD) including layer-specific T-spaced equalization for eliminating the remaining inter symbol interference has been studied. This T-PMSVD technique has been compared in terms of bit error rate performance with the well-known spatio-temporal vector coding description applying SVD equalization. The simulation results demonstrate that using T-PMSVD equalization the BER performance is superior compared with applying SVD. For both equalization types bit loading schemes have been combined with equal SNR power allocation so as to optimize the BER performance. The equal SNR criteria for power allocation seems to be a good sub-optimum solution to improve the channel performance. Furthermore, the bit and power loading analogies between both equalization types have been shown. The two different analyzed channels clarify that the optimal QAM transmission mode largely depends upon the used channel type and that the activation of all transmission layers doesn't always lead to the best performance.

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