Conics Detection Method based on Pascal's Theorem

Musfequs Salehin, Lihong Zheng and Junbin Gao
School of Computing & Mathematics, Charles Sturt University, Wagga Wagga, Australia

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Abstract: This paper presents a novel conics detection method that can be applied for real images. The existing methods usually detect either circular or elliptical, or parabolic shape at one operation. Most of them need the information about center, radius, major axis, minor axis, vertex, and more. In our proposed method, the tangents on curve segments, conic parts, and conics are constructed using Pascal's theorem. The conic parts can be used to detect different types of conic sections from an image. The performance of the proposed method has been tested on the sample images selected from Caltech-256 database and various types of conic sections can be identified from the real images compared to other method.

1 INTRODUCTION

Conics are the curves formed by the intersection of a plane with a right circular cone. They include circles, ellipses, parabolas, and hyperbolas. Among conics, the most common geometric shapes are circles, ellipses, and parabolas to present various conic shaped objects for detection in computer vision. Many traditional methods can detect only circles or ellipses, or parabolas individually. Though such methods are robust to noise and show good performance, it is hard to find an existing method that can detect all types of conic shaped objects in a single operation. Again, conics detection from an image is still a very challenging problem due to various reasons. Not every edges of an image are smooth and clear. Some edges might be blurry and disconnected due to reflections or varied illumination. In addition to this, object contours may be distracted in the presence of clatters, occlusion, and perspective errors. Consequently, in most of these cases, the conic shape of a desired object is distorted, disconnected, or even overlapped by other objects.

In this paper, we aim at detecting conic shaped edges in an image based on Pascal's theorem. Firstly, we apply Triangular Area Ratio (TAR) (Alajlan et al. 2007) to break each edge contour into two or more segments based on sharp turns or inflection points. Secondly, we calculate two tangents at two ending points of a curve segment using Pascal's theorem (Coxeter & Greitzer 1967). From the two tangents, conic part is constructed and fitted with edge segments using least squares fitting method. Lastly, the validated edge segments are grouped together to match the conic shaped object in an image. We apply Pascal's theorem to detect conics in the image. Our contribution is that the proposed method does not need to calculate the center, major axis, or minor axis to detect conic shaped object. Simply, it identifies conic shaped object by verifying the best matching between conic shaped object and grouped edge segments.

The organization of this paper is as follows. The background of conics detection is presented in Section 2. The proposed approach is discussed in Section 3; the visual results are displayed in Section 4. Lastly, we conclude the paper in Section 5.

2 BACKGROUND

Conics detection from an image is an important and recurring research problem. Various methods for the detection or fitting of different types of conics have been developed in the last few decades. These algorithms can be categorized into three main approaches. They are (1) Hough Transform (HT) based approaches, (2) optimization based approaches and (3) edge following based approaches.

In general, Hough Transform based method is a standard technique for detecting any parameterized curve. One of the key advantages of HT based

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method is that it can detect fragmented ellipses. The reason of this is that, it does not require the edge pixels to be connected consecutively on the ellipse. This method maps two dimensional image space into the higher dimensional parameter space. For example, an ellipse can be defined by five parameters, such as its center (c_x, c_y), the major axis a, the minor axis b, and the orientation θ. Therefore, $O(N^5)$ space is required for an ellipse to accumulate the parameter space, where N is the size of each dimension of the parameter space. As a result, this is a combinatory complexity. Moreover, in the basic HT based method, the accumulator's bin sizes are determined by "windowing and sampling the parameter space in a heuristic way" (Xu et al. 1990). However, a large window size is necessary to detect curves from the image with different sizes and a high dimensional parameter space to increase accuracy. Again, these obviously lead to large storage and more processing time. Additionally, in case of poorly defined accumulator, (Xu et al. 1990) identified four difficulties may occur, such as (a) problems in finding an optimal local maxima, (b) low precision, (c) large storage, and (d) low speed. For example, coarse quantization may have poor influence on accurate ellipse detection while fine quantization may lead to missing the true ellipses. Besides this, the accuracy of HT decreases at the increased number of ellipses in an image (McLaughlin 2000).

Optimization based approaches include Genetic Algorithms (GA), Least Squares Fitting (LSF) method, or Robust Regression (RR) based approach for ellipse detection. GA based methods represent each potential solution as a chromosome and the population of chromosomes are generated iteratively. This process terminates when some predefined conditions are satisfied. However, finding the best possible evaluation criteria and thresholds for fitness function are often very hard (Qiao & Ong 2007). Both LSF and RR methods extract ellipse by optimizing an objective function to fit edge pixels to a standard ellipse. However, in presence of outliers, these methods may produce false or missing detection (Qiao & Ong 2007).

Edge following based approaches extract some arc fragments and group them together based on geometric properties of ellipse. Recently, a method for ellipse detection based on edge following is proposed by (Chia & Rahardja 2011). In their method, line segments are formed from the edge map and elliptical-arcs are constructed from these line segments. These arcs are then grouped to form ellipses and false ellipses are neglected by the developed feedback loop method. This method performs very well both in synthetic as well as in real-world images. However, it is computationally very expensive (Wong & Lin 2012).

Another edge following method for ellipse detection is proposed by (Prasad et al. 2012). This method uses edge curvature and convexity instead of continuity as a constraint for the ellipse detection. Hough Transform is then applied to assign a relationship score to the edge contours for grouping. Three robust non-heuristic saliency criteria are used for generating the good elliptic hypotheses. This method does not require any threshold or control parameter. Although this method achieves a high accuracy, it is also very time consuming in the presence of outliers and it often provides false or missing detection (Akinlar & Topal 2013).

Akinlar and Topal proposed a real-time and parameter free method (Akinlar & Topal 2013) to detect both circles and near-circular ellipses. This algorithm first extracts edge segments from a given image by implementing Edge Drawing Parameter Free (EDPF) edge detector. The detected edge segments are converted into line segments. Circular arcs are then detected from these line segments based on line direction and angle between two lines. Candidate circles and near-circular ellipses are detected based on the constraint of center distance and radius difference with the root mean square error. Finally, the candidates are validated using Helmholtz principle (Akinlar & Topal 2013). However, its success depends on the accurate edges detected by EDPF and the presence of noise in the image (Akinlar & Topal 2013).

In this paper, we propose a novel method based on the edge following method. We extract edge or part of edge that can be a part of a conic section. To construct the conic part by Pascal's theorem, two tangents, and a point on the curve are required. We construct tangent using the theorem proposed by Pascal (Coxeter & Greitzer 1967). Instead of applying LSF method or HT-based method for detecting circle or ellipse or parabola, we apply Pascal's theorem for detecting different types of conic sections. Method based on LSF is designed for detecting only one specific type of conic section at a time. For example, LSF method proposed for detecting for circle cannot detect parabola and vice-versa. In contrast, our method based on Pascal's theorem can detect any sorts of conics. Again, for grouping edge fragments to construct conics, we apply conic part construction method by Pascal theorem.

Thus, the contributions of the proposed method are listed below:

1. A novel technique for detecting conic part using Pascal's theorem
2. A novel method for constructing tangent on a curve using the theorem proposed by Pascal
3. A novel approach for grouping edge fragments.

3 THE PROPOSED METHOD

The proposed method follows three main steps to detect conic shaped objects in an image. The flow chart of the proposed method is presented in Figure 1. The first step is to extract smooth conic edge curves from the edge map obtained by Canny edge detector; the second step is to generate the conic hypothesis; and the last step is to detect conics from the conic hypothesis. Each step of the proposed method is discussed in details in the following subsections.

3.1 Extraction of the Conic Edge Segment

Firstly, an edge map is generated from a given image using Canny edge detection method. Then, connected edge contours and line segments are obtained from the edge map using Kovesi's codes (Kovesi 2000).

3.1.1 Sharp Turn and Zero Curvature Detection

Using two consecutive line segments with points (x_{pre}, y_{pre}), (x_{curr}, y_{curr}) and (x_{next}, y_{next}) in counter clockwise direction, a triangle is formed. The angle between two consecutive line segments is obtained by the following equation

$$Angle(curr) = Cos^{-1}\left(\frac{b^2+c^2-a^2}{2bc}\right), \qquad (1)$$

where a, b, c are triangle sides between (x_{pre}, y_{pre}) & (x_{curr}, y_{curr}), (x_{curr}, y_{curr}) & (x_{next}, y_{next}) and (x_{next}, y_{next}) & (x_{pre}, y_{pre}) respectively. If the *Angle(Curr)* is less than 100° or greater than 175°, line segments are divided into two parts at (x_{curr}, y_{curr}) point. The reason is that angle less than 100° suppose to be a corner point and angle greater than 175° is almost similar to a straight line (see Figure 2). Again, if the value of a or b exceeds 100 pixels, the line segment is divided at (x_{curr}, y_{curr}).

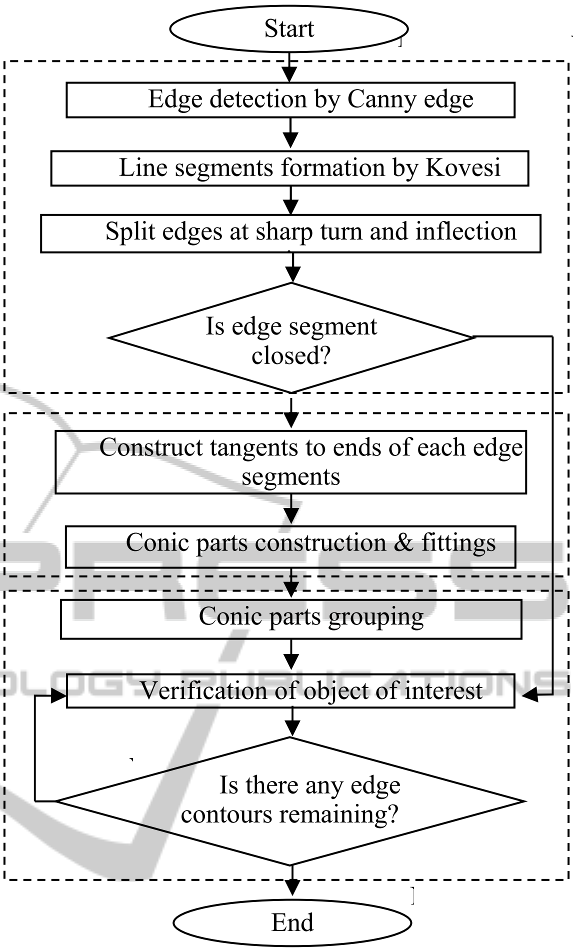
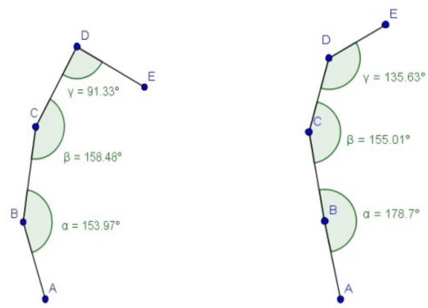


Figure 1: The flow chart of the proposed method.



(a) (b)

Figure 2: The sharp turn and zero curvature elimination. (a) sharp turn point is *D*, (b) zero curvature point is *B*.

3.1.2 Inflection Point Detection

If (x_{curr}, y_{curr}) is not considered as a sharp turn point, the TAR is calculated by the following equation

$$TAR(curr) = \frac{1}{2} Det \begin{bmatrix} x_{pre} & y_{pre} & 1 \\ x_{curr} & y_{curr} & 1 \\ x_{next} & y_{next} & 1 \end{bmatrix}, \tag{2}$$

where, *Det* presents the matrix determinant. The negative, positive, and zero values of *TAR(curr)* represent concave, convex and straight-line respectively (Alajlan et al. 2007). Here, we split edge segment at a point containing zero *TAR(curr)* value. As a result, the extracted line-segments only contain negative (n) or positive (p) values. Let, $\{T_1, T_2, T_3, \ldots T_M\}$ be the sequence of line segments where M represents the total number of positive and negative values within an edge segment. T_i $\{i$=1, 2, 3, … $M\}$ contains either n or p values. To identify the inflection points, the following criteria are applied.

1. if $T_i = n$ and $T_{i-1} = T_{i+1} = p$, the splitting point is T_i
2. if $T_i = T_{i-1} = n$ and $T_{i-2} = T_{i+1} =$ p, the splitting point is T_i and T_{i-1}
3. if $T_i =$ n and $T_{i-3} = T_{i-2} = T_{i-1} =$ p, the splitting point is T_i

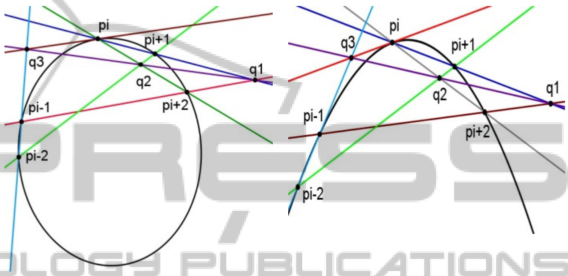
Table 1: Representation of the inflection points.

T_{i-2}	T_{i-1}	T_i	T_{i+1}	T_{i+2}
p	p	n	p	p
n	n	p	n	n
p	n	n	p	p
n	p	p	n	n
p	p	p	n	n
n	n	n	p	p

Table 1 summaries the possible cases to reflect the selective criteria that find out the inflection points. After splitting edge segments at the inflection points, edge segments containing at least three n or p values are selected. Later, the closed edge contours and the selected edge segments are considered for the next step to generate conic hypothesis.

3.2 Generation of the Conic Hypothesis

To validate an edge segment as a conic part, tangents are drawn at each endpoint of an edge segment using Pascal's theorem (Coxeter & Greitzer 1967). This is one of the main contributions of the paper. Consider p_{i-2}, p_{i-1}, p_i, p_{i+1}, and p_{i+2} are five points of an ellipse (see Figure 3(a)) and a parabola (see Figure 3(b)). To avoid exceptional case of this theorem, these five points are selected in such a way that no three collinear and no parallel lines can be formed using these points. Point q_1 is the intersection of line p_ip_{i+1} and line $p_{i-1}p_{i+2}$, and q_2 is the intersection of p_ip_{i+2} and $p_{i-2}p_{i+1}$ respectively. Again, $p_{i-1}p_{i-2}$ and q_1q_2 must meet at point q_3. The expected tangent line of the conic at point p_i is obtained by connecting p_i and q_3. Similarly, we can get tangent line at point p_{i+1} or other points. The pseudo-code for tangent detection is described in Figure 4.



(a) (b)

Figure 3: The construction of tangent on a point of the conic sections using Pascal's theorem. Tangents (p_iq_3) are drawn on the point p_i of (a) an ellipse and (b) a parabola.

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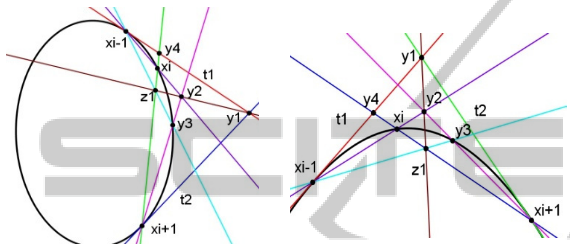
Tangent (p_{i-2}, p_{i-1}, p_i, p_{i+1}, p_{i+2})
// p_{i-2}, p_{i-1}, p_i, p_{i+1}, p_{i+2} are five
//points. Tangent will be constructed
//on point p_i
   Begin
     1. Find the intersecting point (q_1)
        between p_i p_{i+1} and p_{i-1} p_{i+2}
     2. Find the intersecting point (q_2)
        between p_i p_{i+2} and p_{i-2} p_{i+1}
     3. Find the intersecting point (q_3)
        between p_{i-1} p_{i-2} and q_1 q_2
     4. Draw the tangent on p_i by
        connecting p_i and q_3
   End
```

Figure 4: Pseudocode of the proposed tangent detection algorithm.

These tangents are then used to construct conic part using the Pascal's Theorem. In our method, these five points are obtained from an edge segment so that these points can divide it into four equal parts. We follow this approach as it represents the conic more accurately than the random sampling.

Now, another main contribution of this work to construct conic parts is discussed here. To construct a conic part, two tangent lines (t_1 and t_2 on x_{i-1} and x_{i+1} respectively) and a point (x_i) on the conic part are required (see Figure 5). First, t_1 and t_2 intersect at point y_1. The Point z_1 is taken from line $x_i\,x_{i+1}$. The intersection of y_1z_1 and $x_{i-1}x_i$ is y_2. Connecting $x_{i-1}z_1$ and $x_{i+1}y_2$, y_3 is crossing the conic at point y_3. If z_1 moves from x_{i+1} to y_4 (intersection of t_1 and x_ix_{i+1}), the conic part $x_{i-1}x_ix_{i+1}$ will be constructed. Here, we obtained points, x_{i-1}, x_i *and* x_{i+1} from the two ends and the midpoint of the edge segment. In this way, we get rid of the influence of traditional construction of tangent line on the unsmoothed digital curves. The edge segments fitted with the corresponding conic parts using LSF method with residual two pixels are kept for further processing. Otherwise, the edge segments are neglected.

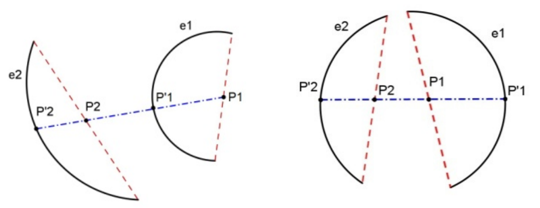


(a) (b)

Figure 5: Conic part construction using two tangents and a point on the conic (a) ellipse part construction and (b) parabola part construction.

3.3 Detection of the Conics

Following the previous step, the obtained edge segments are sorted according to their length in a descending order. After selecting an edge segment from the list, other edge neighboring fragments satisfying the convexity constraints are grouped with edge segment to form a conic. Here, we follow the method applied by (Prasad et al. 2012). Consider e_1 and e_2 are two edge segments and P_1 and P_2 are the midpoints obtained from two ends of e_1 and e_2 respectively (Figure 6). Suppose line P_1P_2 intersects



(a) (b)

Figure 6: The condition of associated convexity (a) *e1* and *e2* cannot be grouped together, (b) *e1* and *e2* can be grouped together.

e_1 and e_2 at P'_1 and P'_2 respectively. The edge segments e_1 and e_2 can be grouped if they satisfy the following condition

$$P_1'P_2' \approx P_1P_1' + P_1P_2 + P_2P_2'. \qquad (3)$$

Here, the procedure to group segments to form a conic section is presented. This is one of the key contributions of this paper. Suppose, an edge segment $A_1A_2A_3$ has two other edge segments ($B_1B_2B_3$ and $C_1C_2C_3$) that satisfy the convexity constraints (see Figure 7). These edge contours are sorted based on the distance from the ends of the edge segments $A_1A_2A_3$. After that, tangents are drawn on the midpoints of each segment as discussed in subsection 3.2 (see Figure 7(a1) & 7(b1)). It is seen that $C_1C_2C_3$ is closer to $A_1A_2A_3$ than $B_1B_2B_3$. As a result, a conic part is constructed according to subsection 3.2 shown in Figure 7(a2) & 7(b2). If the constructed conic part fits with the $A_1A_2A_3$ and $C_1C_2C_3$ by LSF method with residual two pixels, they are merged together. Next, another segments $B_1B_2B_3$ is selected and the same procedure is followed to connect two edge segments (see Figure 7(a3) & 7(b3)).

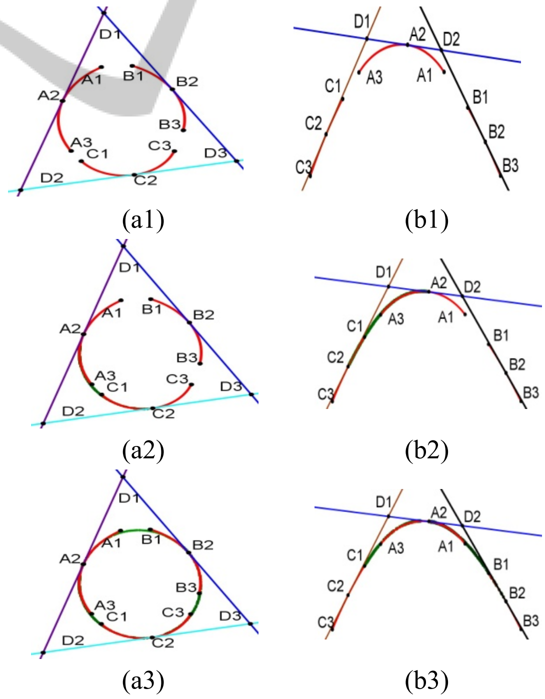


Figure 7: The procedure to merge the conic edge segments as a part of an ellipse (a1-a3) and a parabola (b1-b3).

After merging the groups of edge segments in to a single curve C_{ue}, now, we need to verify whether this curve is a conic matching the interested object.

Hence, on this curve, five equally spaced points are picked up in such a way that no three collinear and no parallel lines can be formed using these points. The points are selected from the curve at equally distance as this approach presents the curve more accurately. Based on Pascal Theorem, a conic C_{on} can be constructed crossing these five points. The difference D between the grouped curve C_{ue} and the conic C_{on} is calculated by least square distance. If D is less than a predefined threshold, this curve C_{ur} is identified as a valid conic shaped boundary of the interested object.

Next, another edge segment is selected from the list sorted in a descending order and the same process is followed. The process mentioned in this section will be iterated until there is no edge segment to process.

4 RESULTS & DISCUSSION

We selected some images containing round shaped objects from Caltech-256 database (Griffin et al. 2007) to evaluate the performance of the proposed method and compared with the recently proposed method for detecting the circle and ellipse known as EDCircles method (Akinlar & Topal 2013). The EDCircles uses two different LSF based methods for detecting both circle and ellipse. In this method, the least square fitting algorithm is applied for detecting circle and direct least square fitting of ellipse is implemented for ellipse detection from an image.

Four sample images and their comparison results are shown in Figure 8. The first column (Figure 8(a1-a4)) shows four sample images. The results of canny edge detector and the proposed method are shown in the second and third column respectively. The edge detection result obtained by EDPF is

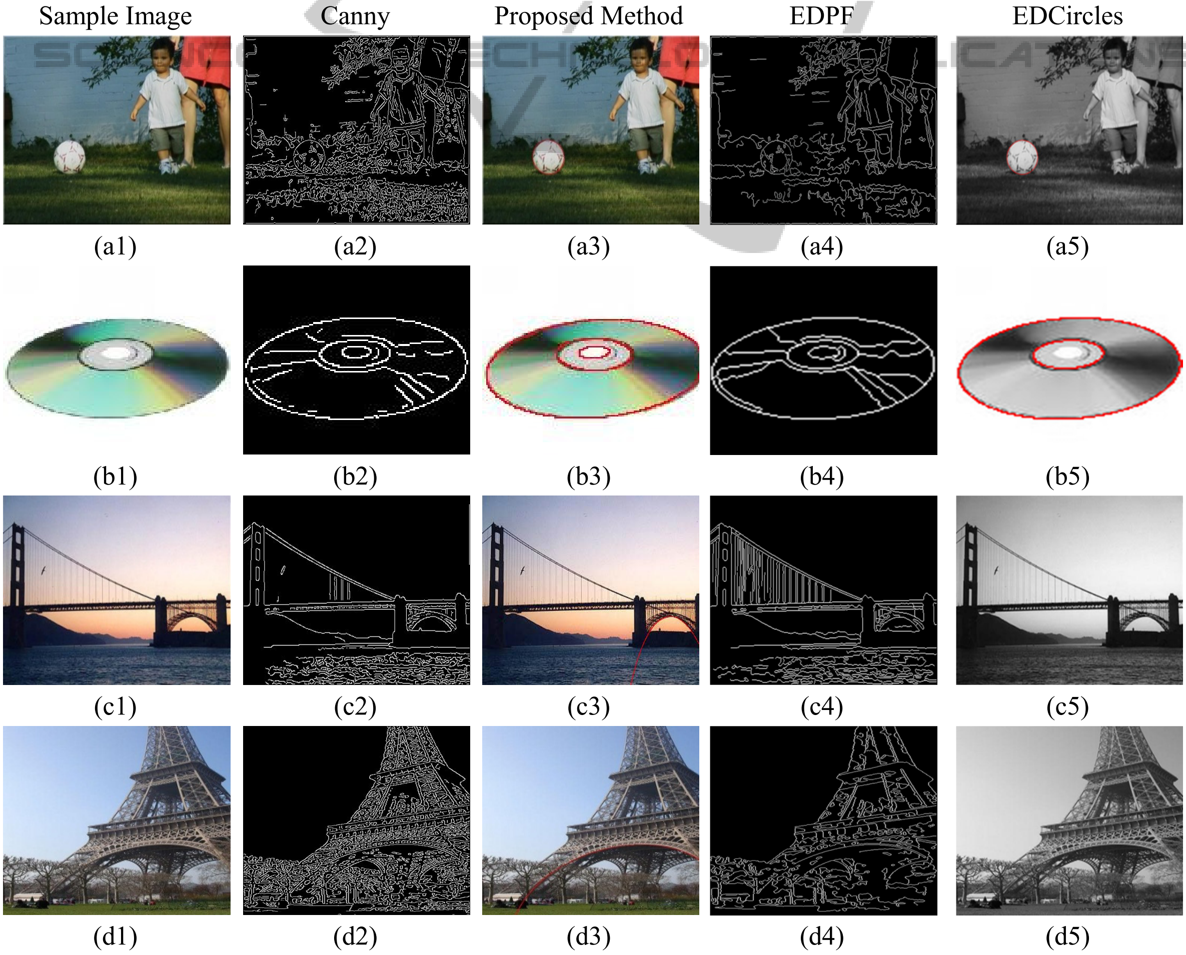


Figure 8: Comparison results by the proposed method and EDCicles. First column (a1-d1) the sample image, second column (a2-d2) Canny edge detection, third column (a3-d3) the results obtained by the proposed method, forth column (a4-d4) edge detection by EDPF and last column (a5-d5) the results obtained by EDCircles.

presented in the forth column. The last column represents the outputs of EDCircles method.
It can be seen from Figure 8(a3-d3) that our method can detect both elliptical and parabolic shape. Conversely, EDCircles detects only circular shape and misses parabolic shape although their EDPF method (Akinlar & Topal 2013) can detect the edge representing the parabolic shape (see Figure 8(c4) and 8(d4)). The reason is that circle and ellipse detection methods are applied in EDCircles. As this method does not apply any parabola detection method, it cannot detect any parabolic shaped object shown in Figure 8(c5) and 8(d5). Instead of using several different methods for detecting different conic shape, we use only a single procedure based on Pascal's theorem to detect any conic shaped object (see Figure 8(a3-d3)).

It has been noted that the result of the proposed method depends on the Canny edge detection method. If edges of a conic shaped (such as, circle, ellipse or parabola) object are not detected by Canny edge detector, this method will fail to detect that conic shaped object. Another drawback is that our method completely depends on the conic edge segments detected by subsection 3.1. If two different conic segments merged such a way that there is no sharp turn or inflection point exist, this method will consider it as a non-conic edge segment. This problem is also unsolved by other methods such as (Prasad et al. 2012).

5 CONCLUSIONS

The proposed method can detect any type of conic shaped object from the real image in a single operation by constructing tangent, conic parts and conic using Pascal's theorem. The existing method such as EDCircles (Akinlar & Topal 2013) uses separate methods (least squares circle fit and direct least square fitting of ellipses) for detecting different type of conic sections. Another traditional method for detecting conic sections is based on Hough Transform. This method detects parameter for detecting conic shape. For example, Hough Transform constructs three-dimensional, five-dimensional, four-dimensional parameter space for detecting circle, ellipse and parabola respectively. As a result, a single Hough Transform method cannot be used to detect any type of conics. In future work, we will compare other geometric shape based methods and Pascal's theorem based method to detect the conic shaped objects.

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