Adaptive Segmentation by Combinatorial Optimization

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Abstract: In this paper we present an iterative segmentation. At the beginning it is using a stochastic method called Kangaroo in order to speed up the regions construction. Later the problem will be presented as non-oriented graph then reconstructed by linear software as entire number. Next, we use the combinatorial optimization to solve the system into entire number.
Finally, the impact of this solution became apparent by segmentation, in which the edges are marked with special manner; hence the results are very encouraging.

1 INTRODUCTION

The techniques of images segmentation have seen considerable development these last years, because we have passed the split-and-merge (H.Yang, 1997), to the use of watersheds, to active edges minimization, and finally to multi-agents segmentation (S.Mazouzi, 2007).

The segmentation methods by increasing regions based to measurement related to gray level, or to probability measurement, bring a good initial identification for regions of interest, but suffer from the major inconvenient of non precise localization of regions edges. The approach of segmentation by the active edges presents good results concerning the localization of regions edges of interest, providing that the initialization of these edges will not be far from the final edges. The hybrid approaches can combine information coming from several methods looking very promising.

These diversities of methods have made that we do not have a universal method of segmentation, but we have an algorithm of segmentation to be used for each application and its evaluation depends on the obtained results. This complexity is related to the principle of segmentation, because we are looking for a compromise between an over segmentation in which there are so much details and an under segmentation in which there are a lack of details (W.Eziddin, 2012).

Finally, as a conclusion of the image method segmentation, we can underline that the iterative method of segmentation lead, in general, to the best results than the non-iterative methods.

In the present work we have developed an iterative method of segmentation, which treat directly the compromise of segmentation, it is intended performance by the following strengths:

1. Self adaptive: it does adapt automatically with the sample in order to avoid huge number of iteration. This self-adaptive sentence will be constituted of two other sentences:
   (a) One first sentence in which we have used a heuristic to estimate the step used during the constitution of regions, in order to reduce the number of iteration and consequently the execution time of this sentence.
   (b) A second sentence which is of practical use, this heuristic will be applied on images of different nature, in which the gray level inside each region is almost constant, more or less constant and completely variable.
2. Optimal: the problem formulation as entire then the use of the graphs theory and the combinatorial optimization enable the fine segmentation and avoid the oversegmentation or the undersegmentation.

Its application on diversity of samples, contain at the same time regions of close levels, and regions of too far levels, giving good results. By this fact our method turns effective, and it is valid for a large variety of applications.

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2 THEORETICAL TOOLS

2.1 Definitions

The segmentation of an image A, regarding the homogenous criteria H (gray level, texture,..) , is a partition of A on homogenous regions $r_1, r_2, \ldots, r_n$.
H have an argument for one or several regions close to the initial image A to return a decision of its homogeneity, $H((r_i, r_j, \ldots) \in Z^2) \to \{true, false\}$. In our work, we introduce several parameters on the region that we would like to study, these latter will be developed in the section 2.3.

$$\bigcup_{i=1}^{n}(r_i) = A \tag{1}$$

$$\forall i \in \{1, 2, \ldots, n\} \quad r_i \quad is \quad related \tag{2}$$

$$\forall i \in \{1, 2, \ldots, n\} \quad H(r_i) \quad is \quad true \tag{3}$$

$$\forall r_i, r_j \quad voisines \quad H(r_i, r_j) \quad is \quad false. \tag{4}$$

Regarding this definition, the segmentation depends on the criteria H used. The choice of this criterion is really primordial.

2.2 Regions Training

Let A the image to make segmentation with $a(i, j)$, the gray level of pixels and C the image label (the image of labeled regions) with $c(i, j)$, the gray level of pixels, the pixels of each region have the same level noted code. To go from A to C, we proceed with double scan as follow. Each pixel $c(i, j) \in C$ is computed by the Algorithm Code.

Algorithm Code

Initialization: we initialize the $code = 1$, the gap $\varepsilon = 0$, we start with the pixel high-left of the image $c(1, 1) = code$.

Horizontal: for two horizontal pixels close:

If $|a(i, j) - a(i, j+1)| \leq \varepsilon$
  then $c(i, j+1) = c(i, j)$
  else $code = code + 1$, $c(i, j+1) = code$;
EndIf
If (end of line)
  then $code = code + 1$
EndIf

Vertical: for two vertical pixels close:

If $(|a(i, j) - a(i+1, j)| \leq \varepsilon)$ then
  $c(i, j) = min(c(i, j), c(i+1, j))$
  $c(i+1, j) = min(c(i, j), c(i+1, j))$;
EndIf

Equality: Iterate the vertical scan untill not equality of image C.

Let B the vector of codes used in the Algorithm Code so $lentgh(B) = n$.

A propagation with equality levels $\varepsilon = 0$ give regions only for perfect images, but in the general case the levels of the same region are variables, they vary from one sample to the other. So we have to make compromise between one ε weak giving over-segmentation and another big giving under-segmentation. To this stage two approaches are possible:

1. The first approach consists of starting from the equality $\varepsilon = 0$, then increment it gradually, till the regions training, but this procedure is boring in time, because the propagation process is repeated each iteration. This drawback is appearing usually for far regions levels.
2. A second approach consists of starting from the equality $\varepsilon = 0$, then find an estimation function which adapt the increment step according to the sample. This self adaption increase the quickly the step ε, decrease the number of iteration and speed up the convergence process to regions training. A detailed method of speed up will be presented in section 3.1 .

Finally the two approaches require a stop condition; this latter is fixed according to the classification rate of pixels in regions.

2.3 Connection Graph

Having A, C and B, we proceed to the regions extraction one by one. For each region, we determine the following parameters (A.Herbulot, 2007); for example for one region r_i coded by $B(i)$,

- s_{r_i} the surface of r_i ,
- μ_{r_i} the average level,
- h_{r_i} the histogram,
- $\rho_{r_i} = \{x : h_{r_i}(x) = max(h_{r_i})\}$ the mode,
- $I_{r_i} = [a, b]$ such as $\Sigma_{x=a}^{b} h_{r_i}(x) > 0.8 * \Sigma_{x=0}^{255} h_{r_i}(x)$ the confidence interval .

The coded image C leads to non oriented graph $G = (V, E)$ (E.Fleury, 2009; F.Khadar, 2009).

- The set of nodes V present the regions r_i with $i \in V$ (1) in which each node is characterized by its parameters $(s_{r_i}, \mu_{r_i}, h_{r_i}, \rho_{r_i}, I_{r_i})$ and $\|V\| = n$.
- A link between i and j, $e = (i, j) \in E$ such as $i, j \in V$ show the regions r_i and r_j are close. This link does not exist unless the parameters of regions are distinct (3) and (4).

3 PROBLEM FORMULATION

3.1 The Kangaroo Method to Speed up the Convergence

The stochastic method of Kangaroo (G.Fleury, 1993) , is metaheuristic used in the NP-difficult problems, that we have adopted to our problem to accelerate the process of regions constitutions. As indicated by its name, it has a variable step, thus we have experimentally used an empiric formula to update the step of each iteration (5, 6). Following the step of propagation, the trained regions of some pixels form an over-segmentation and their pixels are not classified.

Let ψ the pixel number non classified and $S = size$, the ratio of non classification τ is expressed as:

$$\tau = \frac{\psi}{S} \tag{5}$$

Since this ratio is expressing the number of non-classified pixels, so it does show good sign of the increment step of Kangaroo and consequently the estimation function ε. Because a high τ means that the majority of pixels are not classified, which means that we are far from the convergence, so we have to increment the step ε and inversely for low τ means that the process of propagation is close to the convergence, consequently we have to reduce the step ε. In practice several estimation functions of this step of increment ε have been tested we have kept the one given by (6).

$$\varepsilon = \varepsilon + 10.\tau \tag{6}$$

The regions construction is estimated achieved when $\tau < 10\%$, the non-classified left pixels are assigned to the close region.

3.2 Individual Regions Processing

Now as the regions are marked, each region is extracted separately, and their parameters μ_{r_i}, ρ_{r_i} and I_{r_i} they are calculated. Then the two next logical variables are deduced:

$$y_i = \begin{cases} 1 & if \quad |\mu_{r_i} - \rho_{r_i}| < s1 \\ 0 & otherwise \end{cases} \tag{7}$$

$$z_i = \begin{cases} 1 & if \quad I_{r_i} < s2 \\ 0 & otherwise \end{cases} \tag{8}$$

Practically we have fixed the parameters, $s1 = 9$, in order that the statistical average and the region mode are close, so significant and $s2 = 30$, so that the pixels of the same region will have a uniform visual aspect, because beyond the human eye can make differences.

The edge $e = (i, j) \in E$ will be presented by a logical variable $x_{ij} = x_{ji}$ this latter at the time of segmentation optimization.

$$x_{ij} = \begin{cases} 1 & if \quad r_i \neq r_j \\ 0 & otherwise \end{cases} \tag{9}$$

3.3 Formulation of the Problem into Entire Number

Following the regions construction, according to (G.L.Nemhauser and L.A.Wolsey, 1988; M.Baiou and F.Barahona, 2011), we have the following constraint :

Region: each region i give two logical variables, y_i and z_i such as:

$$y_i + z_i \geq 0 \tag{10}$$

The region i is not effective unless $y_i + z_i = 2$.

Edge: $e = (i, j) \in E$ between the regions i and j presented by the variable $x_{ij} = x_{ji}$ is not effective unless the regions i and j are also so. Therefore it should verify the following constraints:

$$\begin{cases} x_{ij} \geq 0 & \text{(false)} \\ x_{ij} \leq 1 & \text{(true)} \\ 2x_{ij} \leq y_i + y_j & \text{(modes)} \\ 2x_{ij} \leq z_i + z_j & \text{(distributions)} \end{cases}$$

Estimation: to have effective differences,

$$Maximize \qquad \Sigma_{i=1}^{\|V\|}(y_i + z_i) \tag{11}$$

Relaxation: each region i having $y_i + z_i < 2$, it is decomposed into two regions u and v such as $y_u + z_u \geq 1$ and $y_v + z_v \geq 1$, then we refresh the parameters of the graph $G = (V, E)$ and finally we rewrite the corresponding constraints to the recently created or modified regions.

4 SEGMENTATION ALGORITHM

Our algorithm of segmentation uses the principle of top-down iterative cutting. Having the image to be segmented A, we proceed as follow:

i- Regions construction:
Let A the image to be segmented and C the image of the coded regions. We denote $C = SPPpropag(A, \varepsilon)$ the routine enabling this code with an error gap ε within the gray levels of the same regions, we have used the Kangaroo method as follow:

1 - Initialization $\varepsilon = 1$
2 - $C = SPPpropag(A, \varepsilon)$
3 - Calculation of τ then $\varepsilon = \varepsilon + Round(10.\tau)$
4 - **If** $\tau > 0.1$
then back to the step 2
Endif

The use of this stochastic algorithm, has allowed us to reduce with considerable manner the number of iteration of the propagation regardless the image sample.

ii - The graph calculation :
Starting from the image of the regions code C, the one of the departure A, a graph $G = (V, E)$ is being calculated. The logical variables y_i, z_i of each region are calculated as well as the variables edges x_{ij} with a number of regions $\|V\|$.

iii - Estimation :
calculation of the combinatorial optimization function $H = \Sigma_{i=1}^{\|V\|}(y_i + z_i)$.

If H is maximum
then end of process
else following the step of relaxation
Endif

iv - Relaxation :
for each region i having $y_i + z_i < 2$, an optimum cutting (K.Chehdi, 1991) is applied to decompose this region into two sub-regions $r_i = r_u + r_v$ then back to the step $ii-$.

The synoptic of figure 1 summarize the steps of our algorithm of segmentation.

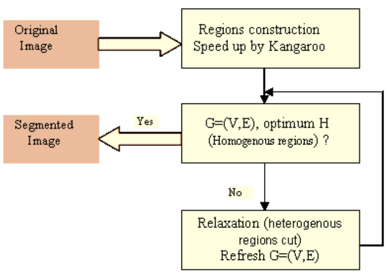

Figure 1: the principle of the algorithm of segmentation.

5 EXPERIMENTAL RESULTS

5.1 Speed up the Regions Construction

By applying our method upon several images samples, we recorded the following results:
We proposed in this paper, the acceleration results of regions which are constituted by three images in which, the gray levels by region are homogenous in the first ones $(Gap \leq 4)$, more or less variables $(6 \leq Gap \leq 9)$ in the second and completely heterogeneous $(25 \leq Gap \leq 40)$ in the third one.
The algorithm starts the first iteration with a gap $\varepsilon = 1$ and τ is evaluated at the end of iteration according to (5) , if the stop condition 3.1 is not reached then ε is calculated according to (6) to start the following iteration.

Table 1: Result of speed up convergence.

images	Gap in the region	Iteration of $SPPpropag$	ε	τ in %
circles	≤ 4	1st iteration	1	6.6
alumgrns	6, 9	1st iteration	1	12.9
		2nd iteartion	2	8.6
rice	25, 40	1st iteration	1	98
		2nd iteartion	11	7.5

The results gathered in (Table 1) confirm the following:

- The first is indeed concerning the Kangoroo method, because the regions constitution is always achieved in about two or three iteration, which reduce and minimize greatly the time of this stage.
- The second shows that our empiric formula of Kangaroo step of evaluation ε is valid for large range of images samples.
- The column of step ε shows that this latter varies with respect to gray level variation inside the regions, and it adapts automatically for each situation.

The gaps in gray level inside the same image, they are variable from one region to the other. Sometimes the regions construction which the gap is high, they will be followed with fusion of close regions. This phenomenon is totally normal because it is the Kangaroo principle but it will be solved in the relaxation stage which will be discussed in section 5.2 in page 5.

5.2 Relaxation and Correction of Segmentation

In the image of figure 2 in the left, the steps $i-$ and $ii-$ of the algorithm has given the images of figure 2 in the right, presented by the graph of figure 4, high left. This latter contain 8 regions and 11 edges, so we have a graph $G = (E,V)$ with $\|V\| = 8$ and $\|E\| = 11$, having the data as follow:

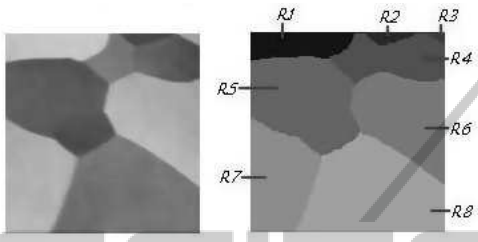

Figure 2: original (left) regions (right).

The step $iii-$ of the Algorithm has given:

Table 2: Result of the 1st iteration.

V	1	2	3	4	5	6	7	8
y_i	1	1	1	0	1	1	1	1
z_i	1	1	1	0	1	1	1	1

Thus the Table 2 illustrates the results of the first iteration and as we can notice, the region r_4 does not satisfy the H criteria.

a- An estimation function $H = 14$, whereas it's maximum have to be 16, therefore it is not optimum.

b- Edges vector E, such as:

Table 3: Link vector E after the 1st estimation.

$(i,j) \in E$	$(1,4)$	$(2,4)$	$(3,4)$	$(1,5)$
x_{ij}	0	0	0	1
$(i,j) \in E$	$(4,5)$	$(4,6)$	$(5,6)$	$(5,7)$
x_{ij}	0	0	1	1
$(i,j) \in E$	$(5,8)$	$(6,8)$	$(7,8)$	
x_{ij}	1	1	1	

The compounds to 0 in E correspond to missing edges in the graph and this means that the separation between regions in these positions is not effective. Whereas the compounds to 1 correspond to available edges in the graph and they are presented in the image by white line separating the regions in these positions as shown by figure 3 in left.

According to (9), the Table 3 shows that all the binary variables x_{ij} connected to r_4 are 0 ($\forall(i = 4$ or $j = 4) \Rightarrow x_{ij} = 0$).

c- The results of this first iteration of estimation are presented by the graph of figure 4 high right.

The algorithm then follows the step $iv-$. Regarding the data, the relaxation acts on the region r_4, because it is the less homogenous.

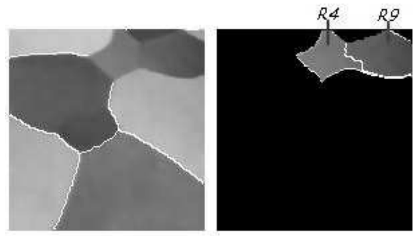

Figure 3: Before the relaxation (left) after the relaxation (right).

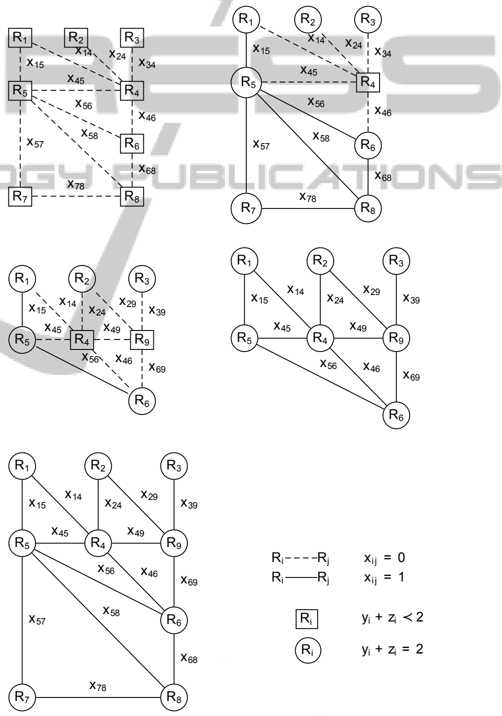

Figure 4: Algorithm in graphs.

As we can notice, the region r_4 is treated separately; the result is illustrated by figure 3 left. This step of relaxation is illustrated by the graph of figure 4 down left. It has lead to two new regions rated 4 and 9, having the parameters $y_4 = z_4 = 1$, and $y_9 = z_9 = 1$.

Following this stage, the second iteration of estimation gives new graph $G = (E,V)$, presented by figure 4 down right, such as $\|V\| = 9$ is a set of edges $\|E\| = 14$, therefore an estimation function $H = H_{Max} = 18$, then all the variables x_{ij} are set to 1.

The immediate consequence, it is an optimum estimation, therefore the algorithm is coming to the end,

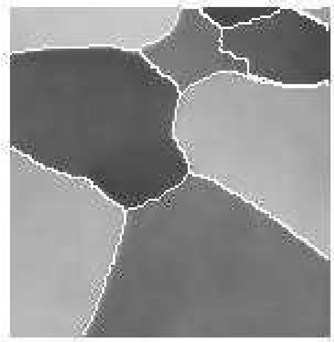

Figure 5: Final segmentation.

and the final segmentation is given by the 5.

Hence the merged regions by mistake in the Kangaroo stage are separated again in this stage of relaxation.

6 CONCLUSIONS

We have expanded in this paper, an iterative segmentation method based upon the process of segmentation by a set of regions significantly different, then to realize an iterative adjustment to converge to the exact regions.

The use of stochastic method of Kangaroo, allowed us not only to speed up the initialization phase of the process but also self-adaptation of our method with the sample image to be segmented.

Next, the representation of the problem as none oriented graph $G=(E,V)$ then its formulation by a entire linear program (PLE) which have make easy the problem study under several constraints. Further the theory of the combinatorial optimization was a considerable support in branching of segmentation.

The performance evaluation of our method was applied on a diverse set of images (small to large variations gray levels within the same regions). In these different situations,

- The initialization stage, by its self adaptation, is accelerated and it converges to the maximum after tree 3 iterations,

- The stage of the optimization refines the processing by giving precise edges and regions more homogeneous.

By this encouraging results, our segmentation method can be a new way to use other segmentation approaches involving highest semantic level of knowledge.

Now, this work is extended into two axes:

- The first one consists to refine and confirm the estimator (6) by experimentation upon a set of varied of images of great size.
- The second consists of increasing the problem complexity by inserting the color parameter among the decision parameters

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