Computing with Perceptions for the Linguistic Description of Complex Phenomena through the Analysis of Time Series Data

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1 RESEARCH PROBLEM

Nowadays, new technologies allow acquiring and archiving vast volumes of data about time-evolving phenomena in many crucial areas such as economy, science, and industrial processes. Examples in economy include the evolution of every kind of economical indicators at local or global levels, like stock funds, electricity/gas/water consumption, price of basic products, etc. In science, the amount of information collected by researchers is overwhelming and ever growing, including astronomical observations by radio telescopes, space probes, etc. and data collected from experiments in diverse scientific fields.

In order to be useful, this data must be exploited and explained in an understandable way, reporting facts, advice or commands to be performed that use the available background knowledge about the phenomenon under study. These objectives can only be achieved by using natural language, especially if the final information is to be provided by a non-expert. This is clear for example in the case of financial newspapers and scientific publications, in which data is not simply made accessible or summarized as graphics and tables, but arguments and conclusions need to be explained using natural language.

However, there is a lack of tools and means for processing and interpreting all this data using computers. Any organization of data as provided by a computer, either in a numerical, categorical, and/or graphical form, is just a tool that can be employed by human experts to produce an explanation in natural language.

Understandable linguistic descriptions of phenomena are provided by human experts, while computers just provide flexibility in storing and accessing data. In fact, it is becoming easier and easier to collect data, but providing a human being with expertise on a certain domain remains difficult and expensive. This situation clearly poses a problem, since the ratio data/human experts is growing dramatically as a consequence. In summary, there is a clear need for computational systems able to produce automatically linguistic descriptions of data about phenomena.

More specifically, the task of generating easily understandable information for people using human language has been addressed by two fields which, independently until now, have researched the processes this task involves: the natural language generation (Reiter and Dale, 2000) and the linguistic descriptions of data (Zadeh, 1996).

The natural language generation field focuses its efforts on automatically obtaining texts, with the purpose of them being as much as possible indistinguishable from the ones created by humans. The linguistic descriptions of data field, which originates in the soft computing domain, provides summaries or descriptions from data sets using linguistic concepts which deal with the imprecision and ambiguity of language through the use of fuzzy sets.

In this context, we propose in this Ph.D. to research on the linguistic descriptions of data field, covering a group of soft computing-based concepts and techniques, such as linguistic variables, fuzzy operators and quantification methods. For instance, using this kind of solutions, we can obtain quantified sentences such as "most of the students are good" or "A few days with high humidity the temperature is low".

In fact, most of the approaches for building linguistic descriptions described in the literature make use of the concept of "quantified sentence". In this sense, the linguistic description approaches make use of two different types of quantified sentences: type I ("Q of X are A"), as in "several dogs are brown", and type II ("Q of D are A"), as in "a few young researchers have published relevant papers", where X is a finite crisp set, Q is a linguistic quantifier and A, D are fuzzy properties defined over X (Delgado et al., 2014).

Despite its formal nature and its orientation towards providing meaningful information from data, as of today the linguistic descriptions of data field has to face several problems as a novel research domain. First, the sole use of quantified sentences is usually not enough to address real life linguistic description problems. In this sense several approaches have studied the extension of quantified sentences to deal with time series data or to provide reasoned descriptions (see next Section), but their full potential is still unexplored. Secondly, linguistic descriptions do not directly provide textual descriptions ready for human consumption, but rather a set of several linguistic properties which can be assigned to a certain concept or entity. Consequently, in order to use linguistic descriptions in real contexts we do not only need expressiveness, but also a way of translating this expressiveness into tangible solutions for actual human users. Another issue related to the previous problem is that linguistic descriptions approaches have been proposed from both a formal and practical point of view, but applications or systems that make use of these techniques in real life environments are almost nonexistent.

In this context, we are developing this Ph.D. in order to research the use of linguistic descriptions addressing several of the issues that this field currently poses. In fact, we believe that by both providing real solutions employing linguistic descriptions and extending the current theoretical base to consider a higher expressiveness, linguistic descriptions can become a very useful tool as a more human-friendly alternative to other widely used descriptive techniques, such as statistics or data mining approaches.

2 OUTLINE OF OBJECTIVES

The objectives of this Ph.D. are both practical and theoretical. On one hand we intend to explore and address linguistic description needs in real-life contexts. On the other hand, based on the experience gathered in the practical use cases, we intend to achieve a general linguistic description model which can be used to easily develop linguistic description solutions for any application domain. More specifically, we will:

- Study each application domain and create a corpus of commonly used linguistic expressions.
- Extend the computational theory of perceptions (Zadeh, 2000) to include the representation needs previously identified, so that new linguistic resources can be contemplated, such as hypothesis, causality, speculation, conclusions, as well as new kind of quantifiers and operators at a syntactic-semantic level, which allow to establish spatio-temporal relationships among the occurrence of events.
- Research quality assessment criteria for linguistic descriptions. These allow to determine the quality degree of candidate linguistic descriptions in an objective way.
- Create search procedures which select the most relevant descriptions according to the application domain expert's preferences. These procedures may be heuristic (where, in an *ad hoc* way, the expert knowledge is directly included as part of the search procedure) or meta-heuristic (where an optimization function including the expert knowledge guides the search process), depending on the problem characteristics. In the latter case, genetic programming seems an appropriate strategy, since it is oriented to learn grammar instances, which can be used to structure and represent the target linguistic descriptions.
- Research validation methodologies to verify the robustness of the linguistic description generation procedures and the subjective quality of the linguistic descriptions. Although the objective criteria used in the search procedures allow to obtain good quality descriptions, these have a limited validity in this context, due to the high number of possible answers and the subjectivity in their assessment (situation complexity, divergent expert criteria, etc.).
- Ultimately develop a generic linguistic description model which considers and formalizes all the previous described aspects. This model will be implemented as a software library which may serve as the kernel engine of a linguistic description generator authoring tool.

3 STATE OF THE ART

The linguistic descriptions (or summaries) of data (LDD) aim to obtain informative, brief and precise descriptions from numeric datasets and cover a group of soft computing-based concepts and techniques, such as linguistic variables, fuzzy operators and quantification methods. It is a novel field, whose solutions provide information in the form of linguistic terms. Specifically, although preliminar ideas appeared early in the 1980s, it started to develop in the second half of the 1990s, when the advances in the field of fuzzy sets (namely computing with words and computing with perceptions) provided new potential applications, mostly oriented to data mining. Due to its short career and its formal background, many approaches in this field are on the theoretical side, although in some cases practical examples and real life based problems have been addressed.

3.1 Theoretical Work

(Yager, 1982) and (Yager et al., 1990) define the basic linguistic summaries and serve as the starting point in the characterization of linguistic descriptions, introducing the concept of quantified sentences as data summaries. Years later, (Zadeh, 1996), (Zadeh, 2000) and (Zadeh, 2001) introduce the concept of computing with words (CWW) and computing with perceptions (CWP) and highlight the potential of the fuzzy logic to provide a methodology to this concept, including examples which show how this approach could be structured. More recently, (Kacprzyk and Zadrozny, 2005), (Kacprzyk and Zadrozny, 2010) and (Kacprzyk, 2010) introduce some ideas about a potential relationship between computing with words and the natural language field, but do not explore them in depth.

From the ideas and concepts proposed in the previous contributions, the construction of a linguistic description framework which can be applied to any kind of description problem in any domain is perhaps one of the biggest challenges in this field, but it is still far from being achieved. In this sense, the Granular Linguistic Model of a Phenomenon (Menendez-Gonzalez and Trivino, 2011), which has been used as a solution for several practical cases in diverse domains (Menendez-Gonzalez and Trivino, 2011), (Alvarez-Alvarez and Trivino, 2013), (Eciolaza and Trivino, 2011), is the nearest approach there is to an all-in-one framework. It is based on a set of interconnected nodes named perception mappings (PM), which receive a set of computational perceptions (CP) as input. Each PM applies an aggregation function to the input CP (for example minimum, maximum, average or even fuzzy rules) and generates a new CP as a result which can be reused as input to other PM. Figure 1 shows an example of a GLMP model which determines and describes types of climate from temperature and humidity input data. A are linguistic label partitions and W vector of fuzzy fulfillment degrees associated to each label. $R_1 ... R_n$ is a set of fuzzy rules.

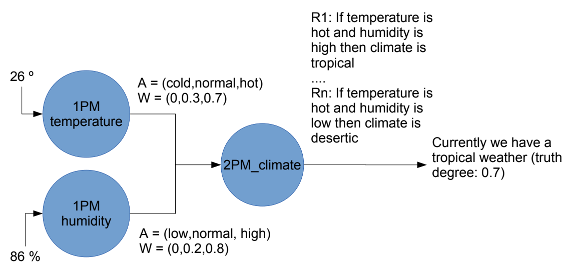

Figure 1: Example of a GLMP model which determines the type of climate from temperature and humidity data inputs.

Other recent contributions explore the use of different quantifiers and develop evaluation criteria for quantified sentences. For example, (Diaz-Hermida and Bugarin, 2011) with several theoretical aspects such as the use of semi-fuzzy quantifiers to model quantified sentences and the description of some generic methods for pattern detection. Furthermore, (Díaz-Hermida et al., 2011), (Castillo-Ortega et al., 2012), (Wilbik and Keller, 2012) and (Menendez and Trivino, 2012) define several and mostly coincident evaluation criteria, such as the data coverage percentage or the sentence fuzzy fulfillment degree. Others were inspired by the conversational maxims in the field of human communication (Gamut, 1991), including the relevance or the length of the description. In fact, when referring to criteria, it can be stated that there is a solid consensus about which characteristics of a linguistic description can be useful in the task of evaluating and ranking candidate descriptions in an objective way.

3.2 Use Cases and Practical Contributions

In (Castillo-Ortega et al., 2011a), the concept of linguistic summary applied to temporal data series is given, which must fulfill the brevity, precision and data coverage criteria. A few algorithms to obtain linguistic summaries are presented. The given example is made on data about the patient inflow in medical centers, from which summaries such as "Most of the days with cold weather" or "patient inflow is low or very low" are obtained. This use case was also explored in (Castillo-Ortega et al., 2011b) using a genetic algorithm approach instead of the standard heuristic algorithms used to generate the linguistic descriptions.

In (Kobayashi and Okumura, 2009), a specific application oriented to the economic domain is described. Nikkei data time series are used, from which several pattern profiles which take into account the curvature and trend of the data series are detected to produce summaries about the evolution of the market in a given date. These descriptions were compared with news reports about that evolution. The obtained descriptions are composed of simple sentences such as "At the end of the session the prices decreased".

(Kacprzyk and Wilbik, 2009) orients the use of linguistic descriptions to temporal series comparison, with the objective of helping human decision taking in an effective way, in this case related to economic investments. The kind of sentences obtained include variation patterns, such as "Among all y, most are constant", "Among all medium y, most are constant" or "Among all moderate y, most are medium and constant".

In (van der Heide and Trivino, 2009), the problem of generating linguistic descriptions for domestic electric consumption is addressed. This work highlights the potential that linguistic descriptions have for an electricity company in order to provide customers with customized information above mere numerical data. In this case, intuitive descriptions are given, such as "About two thirds of the days the consumption in the mornings is lower than the consumption in the afternoons", "Most of the days the consumption in the mornings is lower than the consumption in the evenings" or "About two thirds of the days the consumption in the middays is lower than the consumption in the evenings".

Based on the GLMP model, (Eciolaza and Trivino, 2011) automatically produces linguistic descriptions of driving activity from vehicle simulator data. (Alvarez-Alvarez and Trivino, 2013) also employs GLMP together with fuzzy finite state machines, to create a basic linguistic model of the human gait and to generate a human friendly linguistic description of this phenomenon focused on the assessment of the gait quality, including rules which allow to provide explanations to the descriptions as in "28 days after the knee lesion, the gait quality is very low because the gait symmetry is low and the gait homogeneity is low". (Menendez-Gonzalez and Trivino, 2011) uses this framework to create linguistic descriptions from OLAP cubes in the energy consumption domain, such as "Your behavior is inefficient, due to the high consumption, the quite old devices and the low consumption at the low charge period".

4 METHODOLOGY

The methodology we are employing to develop this Ph.D. is based on a cyclic hybrid bottom-up and top-down approach. This approach consists in studying practical application domains in order to gain knowledge and experience on how to make the best use of linguistic descriptions. From these practical cases, we intend to identify and abstract (bottom-up) a generic model which can gather the techniques employed in these solutions, but which can be applied in other domains as well (top-down).

The use of this model in other domains might not provide enough potential to address the whole problem. In this case, the new linguistic description requirements and concepts would be incorporated into the model in order to extend its capabilities. Figure 2 shows a structural diagram of this methodology.

Since practical problems are the pillars of this methodology, we should also further extend this methodology to deal on how to approach realistic linguistic description needs. Thus, for each problem we must:

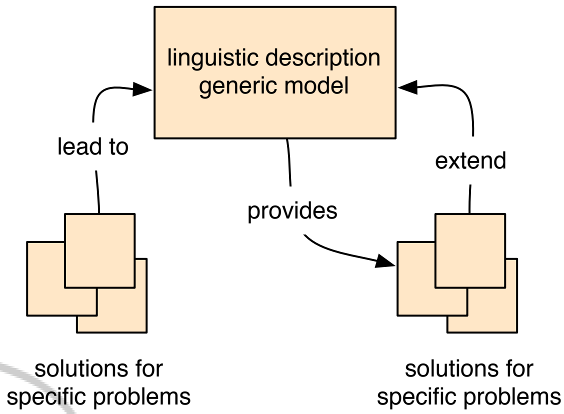

Figure 2: Schema of the Ph.D. development methodology.

1. Study a corpus, if available, of the descriptions elaborated by the domain experts. This allows to identify the structure of the target descriptions and which sort of techniques must be used to create them. If a set of examples is not available, then the experts must define the target descriptions.
2. Design the linguistic description generation approach according to the requirements defined in the previous stage. Determine which operators and techniques must be used in order to extract the relevant information from the source data.
3. Develop prototypes, ideally including a basic natural language generation system which translates the linguistic descriptions into texts which can be reviewed by the experts.
4. Perform a full validation process involving the experts once the results seem satisfactory in order to ensure that the solution can be used in a real context.

These tasks are based on the natural language generation research field, which has addressed many real text generation necessities. Consequently, this way of creating linguistic description solutions seems closer to software engineering methodologies than to a research process. However, the only way of proving the usefulness of linguistic description techniques is to apply them in practical cases, and this usually involves the creation of software systems which must deal with natural language generation, among several other aspects.

5 EXPECTED OUTCOME

We expect to address several linguistic description problems in practical contexts, providing solutions to real life description needs. These approaches will help in the creation of the generic linguistic description model described in the previous section.

More specifically, the experience gathered in the creation of practical linguistic description solutions will allow us to provide a model which is based on real contexts. Consequently, we would achieve a formal model whose usefulness is proven by the practical problems it is based on.

This model would provide solutions for new linguistic description applications, which, in exchange, could also provide new ideas and concepts for the extension of the original model.

6 STAGE OF THE RESEARCH

During the first stage of the Ph.D. we explored the applicability and performance of linguistic descriptions in meteorology, an applied domain which presents several interesting and relevant use cases.

Our first research approach addressed a common task among meteorologists, which consists in the creation of weather reports about the climate behavior. We studied monthly reports about temperatures in Galicia (NW Spain) published by the Galician Meteorology Agency (MeteoGalicia) and elaborated a corpus which served as a base to define the structure and content of the target linguistic descriptions. As a result, we defined the concept of fuzzy nuance and proposed an heuristic procedure which obtained similar reports to the ones issued by the experts (as in Fig. 3). Our results showed that the automatically generated reports were consistent with the original ones.

Shortly after, we researched the application of quantified sentences to create descriptions about the cloud coverage state forecasting. Our approach aggregated geographical time series data from numeric prediction models to provide a global forecast for Galicia. These automatically generated forecasts were compared to the cloud coverage forecasts issued by the meteorologists, showing that, although meteorologists use subjective experience and several information sources to elaborate forecasts, the approach we developed can be applied to provide linguistic descriptions of geographical forecasts. An explanation of the two previous use cases can be found in (Ramos-Soto et al., 2013a), (Ramos-Soto et al., 2012a) and (Ramos-Soto et al., 2012b).

Due to our involvement in the meteorology field

Meteorologist: "Temperatures were high for October, due to the high temperatures which were registered during the first fortnight".

LDD approach: "The temperature was high for October, with very high temperatures during the first fortnight and very cold temperatures during the fourth week".

Figure 3: Example of a temperature report made by a meteorologist and an automatically generated report for the same data source from (Ramos-Soto et al., 2013a).

and the experience gathered in our previous works we created GALiWeather, an application which automatically generates short-term forecast texts in Spanish and Galician for every Galician municipality using a linguistic description approach combined with a basic natural language generation method (see Fig. 4 for a textual forecast example). During this research and development we have been supported by meteorologists from MeteoGalicia, who helped us in the validation process of the application. A preliminary study of the application can be found in (Ramos-Soto et al., 2013b), while (Ramos-Soto et al., 2014a) and (Ramos-Soto et al., 2014c) provide an exhaustive and detailed technical explanation on the final version of GALiWeather. As of today, GALiWeather has been producing daily textual forecasts in MeteoGalicia's test servers for more than half a year. A public release of the automatically generated texts is expected soon.

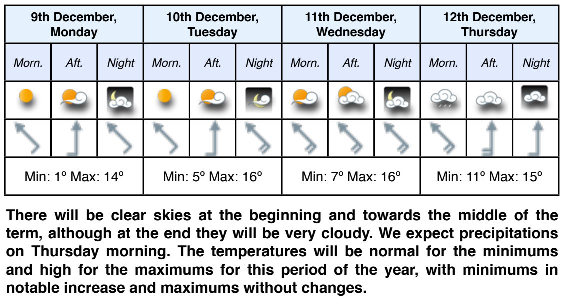

Figure 4: Example of a meteorological short-term forecast obtained by GALiWeather.

While GALiWeather is not as sophisticated as other forecast generators from a natural language generation point of view (such as those described in (Goldberg et al., 1994), (Coch, 1998) or (Sripada et al., 2003)), it focuses on the use of linguistic descriptions in a real context, which provide meaningful information for a wide public.

Currently, we find ourselves in the middle of the Ph.D. development. We are considering new application domains in which linguistic descriptions may prove useful, but are mainly focused on researching a preliminary linguistic description generic model based on our previous experience. We have also recently explored the current state of the art in both natural language generation systems and linguistic descriptions of data in order to ascertain the role that generic LDD approaches (and thus our model) could play integrated into NLG systems (Ramos-Soto et al., 2014b).

We intend to provide a model which can be directly used in practical cases but can also be formally described, thus maintaining both the theoretical and practical aspects of the Ph.D. objectives. Our aim is that this model can create linguistic descriptions from heterogeneous data-sets, although at first we will focus on time series data. We expect to further extend this model to support data with spatial components and also to include new concepts and capabilities as new practical problems arise.

Furthermore, we will also explore the generation of linguistic descriptions using meta-heuristic approaches. This is a task which has been scarcely explored (Castillo-Ortega et al., 2011b) and which may prove useful in the sense of providing a general algorithm for creating linguistic descriptions. This could also be one of the possible extensions to our model.

REFERENCES

Alvarez-Alvarez, A. and Trivino, G. (2013). Linguistic description of the human gait quality. *Engineering Applications of Artificial Intelligence*, 26(1):13 – 23.

Castillo-Ortega, R., Marín, N., and Sánchez, D. (2011a). A fuzzy approach to the linguistic summarization of time series. *Multiple-Valued Logic and Soft Computing*, pages 157–182.

Castillo-Ortega, R., Marín, N., Sánchez, D., and Tettamanzi, A. (2012). Quality assessment in linguistic summaries of data. In Greco, S., Bouchon-Meunier, B., Coletti, G., Fedrizzi, M., Matarazzo, B., and Yager, R., editors, *Advances in Computational Intelligence*, volume 298 of *Communications in Computer and Information Science*, pages 285–294. Springer Berlin Heidelberg.

Castillo-Ortega, R., Marín, N., Sánchez, D., and Tettamanzi, A. G. (2011b). Linguistic summarization of time series data using genetic algorithms. In *Advances in Intelligent Systems Research*, volume 1 - 1, pages 416 – 423.

Coch, J. (1998). Multimeteo: multilingual production of weather forecasts. *ELRA Newsletter*, 3(2).

Delgado, M., Ruiz, M. D., Sánchez, D., and Vila, M. A. (2014). Fuzzy quantification: a state of the art. *Fuzzy Sets and Systems*, 242(0):1 – 30. Theme: Quantifiers and Logic.

Diaz-Hermida, F. and Bugarin, A. (2011). Semi-fuzzy quantifiers as a tool for building linguistic summaries of data patterns. In *IEEE Symposium on Foundations of Computational Intelligence*, pages 45–52.

Díaz-Hermida, F., Ramos-Soto, A., and Bugarín, A. (2011). On the role of fuzzy quantified statements in linguistic summarization. In *Proceedings of 11th International Conference on. Intelligent Systems Design and Applications (ISDA)*, pages 166–171.

Eciolaza, L. and Trivino, G. (2011). Linguistic reporting of driver behavior: Summary and event description. In *Intelligent Systems Design and Applications (ISDA), 2011 11th International Conference on*, pages 148–153.

Gamut, L. T. F. (1991). *Logic, Language and Meaning*. University of Chicago Press.

Goldberg, E., Driedger, N., and Kittredge, R. (1994). Using natural-language processing to produce weather forecasts. *IEEE Expert*, 9(2):45–53.

Kacprzyk, J. (2010). Computing with words is an implementable paradigm: Fuzzy queries, linguistic data summaries, and natural-language generation. *IEEE Trans. Fuzzy Systems*, pages 451–472.

Kacprzyk, J. and Wilbik, A. (2009). Using fuzzy linguistic summaries for the comparison of time series: an application to the analysis of investment fund quotations. In *Proceedings IFSA/EUSFLAT Conf. 2009*, pages 1321–1326.

Kacprzyk, J. and Zadrozny, S. (2005). Linguistic database summaries and their protoforms: towards natural language based knowledge discovery tools. *Inf. Sci. Inf. Comput. Sci.*, 173(4):281–304.

Kacprzyk, J. and Zadrozny, S. (2010). Linguistic data summarization: A high scalability through the use of natural language? *Scalable Fuzzy Algorithms for Data Management and Analysis: Methods and Design*, pages 214–237.

Kobayashi, I. and Okumura, N. (2009). Verbalizing time-series data: With an example of stock price trends. In *Proceedings IFSA/EUSFLAT Conf.*, pages 234–239.

Menendez, C. and Trivino, G. (2012). Selection of the best suitable sentences in linguistic descriptions of data. In Greco, S., Bouchon-Meunier, B., Coletti, G., Fedrizzi, M., Matarazzo, B., and Yager, R., editors, *Advances in Computational Intelligence*, volume 298 of *Communications in Computer and Information Science*, pages 295–304. Springer Berlin Heidelberg.

Menendez-Gonzalez, C. and Trivino, G. (2011). Olap navigation in the granular linguistic model of a phenomenon. In *Computational Intelligence and Data Mining (CIDM), 2011 IEEE Symposium on*, pages 260–267.

Ramos-Soto, A., Bugarín, A., and Barro, S. (2014a). Generación automática de predicciones a corto plazo: Metodología y validación. In *Actas XVII Congreso Español sobre Tecnologías y Lógica Fuzzy (ESTYLF)*, pages 405–410.

Ramos-Soto, A., Bugarin, A., and Barro, S. (2014b). On the role of linguistic descriptions of data in the building of natural language generation systems. *Submitted*.

Ramos-Soto, A., Bugarin, A., Barro, S., and Diaz-Hermida, F. (2013a). Automatic linguistic descriptions of meteorological data a soft computing approach for converting open data to open information. In *Information Systems and Technologies (CISTI), 2013 8th Iberian Conference on*, pages 1–6.

Ramos-Soto, A., Bugarin, A., Barro, S., and Taboada, J. (2013b). Automatic generation of textual short-term weather forecasts on real prediction data. In Larsen, H., Martin-Bautista, M., Vila, M., Andreasen, T., and Christiansen, H., editors, *Flexible Query Answering Systems*, volume 8132 of *Lecture Notes in Computer Science*, pages 269–280. Springer Berlin Heidelberg.

Ramos-Soto, A., Bugarin, A., Barro, S., and Taboada, J. (2014c). Linguistic descriptions for automatic generation of textual short-term weather forecasts on real prediction data. *Fuzzy Systems, IEEE Transactions on*, Early Access.

Ramos-Soto, A., Díaz-Hermida, F., Barro, S., and Bugarín, A. (2012a). Validation of a linguistic summarization approach for time series meteorological data. In *5th ERCIM International Conference*, page 133.

Ramos-Soto, A., Díaz-Hermida, F., and Bugarín, A. (2012b). Construcción de resúmenes lingüísticos informativos sobre series de datos meteorológicos: informes climáticos de temperatura. In *Actas XVI Congreso Español sobre Tecnologías y Lógica Fuzzy (ESTYLF)*, pages 644–649.

Reiter, E. and Dale, R. (2000). *Building Natural Language Generation Systems*. Cambridge University Press.

Sripada, S., Reiter, E., and Davy, I. (2003). Sumtime-mousam: Configurable marine weather forecast generator. *Expert Update*, 6(3):4–10.

van der Heide, A. and Trivino, G. (2009). Automatic generated linguistic summaries of energy consumption data. In *Proceedings of 9th ISDA Conference*, pages 553–559.

Wilbik, A. and Keller, J. M. (2012). A distance metric for a space of linguistic summaries. *Fuzzy Sets and Systems*, 208:79 – 94.

Yager, R. R. (1982). A new approach to the summarization of data. *Information Sciences*, 28(1):69 – 86.

Yager, R. R., Ford, K. M., and Cañas, A. J. (1990). An approach to the linguistic summarization of data. In Bouchon-Meunier, B., Yager, R. R., and Zadeh, L. A., editors, *Uncertainty in Knowledge Bases, 3rd International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems, IPMU 90, Paris, France, July 2-6, 1990, Proceedings*, volume 521 of *Lecture Notes in Computer Science*, pages 456–468. Springer.

Zadeh, L. A. (1996). Fuzzy logic = computing with words. *Fuzzy Systems, IEEE Transactions on*, 4(2):103–111.

Zadeh, L. A. (2000). From computing with numbers to computing with words : From manipulation of measurements to manipulation of perceptions. In *Intelligent Systems and Soft Computing: Prospects, Tools and Applications*, pages 3–40. Springer-Verlag.

Zadeh, L. A. (2001). A new direction in ai — toward a computational theory of perceptions. In Reusch, B., editor, *Computational Intelligence. Theory and Applications*, volume 2206 of *Lecture Notes in Computer Science*, pages 628–628. Springer Berlin Heidelberg.